CHILDREN'S ACTIVITIES WITH SUBSTANTIAL LEARNING ENVIRONMENTS

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Our research design is based on the traditional approach for professional development in Japan, 'Lesson Studies'. The significant point of our project is that anyone can join the research through designing and evaluating Substantial Learning Environments (SLEs). In this paper we report how children (aged 8) developed their mathematical understanding through the number tasks based on the Fibonacci sequence (Bamboo number) described in terms of the SOLO taxonomy. The flexible nature of SLEs makes it possible for teachers and children to explore various patterns of mathematics, and in our pilot study we were excited to observe that children showed great motivation and found/created patterns which teachers did not expect.

INTRODUCTION

Whereas Japanese mathematics education used to have a reputation that students do well, (e.g. the TIMSS study), it now faces challenges in many aspects in the learning and teaching of mathematics. A considerable amount of the mathematical content was reduced as a result of the curriculum reform in 2002, and the latest PISA's result suggests that children and students show weakness in mathematical thinking and problem solving. The goal of our research project is to develop a mathematics curriculum from primary to secondary schools in which students actively engage in rich mathematical activities and acquire a sound knowledge and understanding of mathematics ('Substantial mathematics curriculum'), influenced by the 'mathe2000 project' in Germany (Wittmann, 2005). We consider that working with Substantial Learning Environments (SLEs, explained in the next section) offers educators an ideal context to help children achieve these learning outcomes. We also consider that a network should be established among schools, university initial teacher education (ITE), and continuous professional development for teachers (CPD). This is modelled by the following inter-related four circles (see Figure 1).

The core objective of the network is, like the mathe2000 project, to develop SLEs for primary to university levels, and the three circles around the SLE circle represent the various groups and environments which can contribute to their development. In our project, researchers (and graduate students, trainee teachers, etc.) in universities, and teachers (and children) in schools work together in the process of designing SLEs and lesson plans, and then implement the lessons, share ideas and research findings, and consider how we should improve the current situation in mathematics education. In

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this sense, we consider the network as a 'systemic' one. In this paper, we will report selected episodes from our classroom-based pilot studies undertaken since 2005 in Japan. Vogel (2005) summarises the characteristic operations of interacting with patterns as 'Exploring, identifying, extending, reproducing, comparing, representing and describing' (p. 446). Our research interests are to discuss how students recognise and investigate mathematical patterns, how they communicate with each other, and how they reason/justify their conclusions. We also consider how we could use these findings from classroom-based research to contribute to the curriculum design for primary and secondary schools.

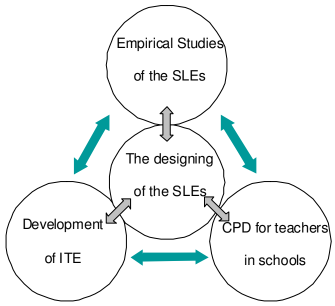


Figure 1: Four interrelated domains of mathematics education research.

DESIGNING SUBSTANTIAL LEARNING ENVIRONMENT

To design a task and learning environment for children and students, we should have a consensus about 'what is mathematics'. In this project, we regard 'mathematics' for children and students as a 'vital science of dynamic patterns' (Wittmann, 2005. See also Feynman, 1968; Hardy, 2000). By this we mean patterns not only restricted to mere number-spotting tasks, but including other types of patterns related to the relationship amongst numbers, shapes, etc., for example, '321 – 123 = 198, 543 – 345 =198, 765 – 567 = 198', patterns found in the multiplication table, tasks with Fibonacci sequence etc.

A task for educators is how we present 'mathematics as science of patterns' to children, and we achieve this by designing SLEs, which have the following features: (1) they represent key objectives, content and principles of teaching; (2) they are related to significant mathematical content, processes and procedures, and are a rich source for mathematical activities; (3) they are flexible and can be adapted to the conditions of particular classrooms; (4) they incorporate mathematical, psychological and pedagogical aspects of teaching mathematics in a holistic way, and offer a wide potential for empirical research (Wittmann, 1995, pp. 365-6). Let us take an example, 'the number pyramid (Zahlenmauern)' (see Figure 2) which we can find in the textbooks *Das Zahlenbuch 1- 4* for primary school children edited by Wittmann and Muller (Wittman, 2005).

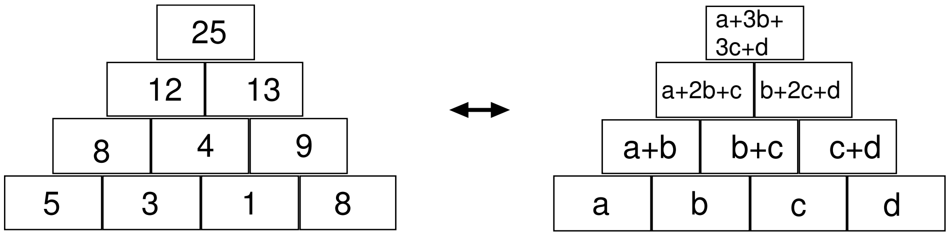


Figure. 2: The number pyramid.

This number pyramid, derived from Pascal's triangle, has a lot of interesting 'patterns', and can be a teaching unit which facilitates the integration of research and practice in the learning and teaching of mathematics. It provides students with an interesting environment for the learning of numbers. Through undertaking problems in the number pyramids, students can practice addition and subtraction in a challenging situation. The design is very flexible and easy to manipulate; we can extend this problem with fractions and decimals, positive and negative numbers, and algebraic symbols which can be used in junior high schools. It also provides teachers and researchers with research questions. For example, it is interesting to conduct research to examine what additional motivation students experience when undertaking 'the number pyramid challenge', or how would children develop calculation concepts through these problems. (With regard to other examples of the SLEs, see Wittmann, 1995, 2000).

RESEARCH DESIGN

In addition to the 'mathe2000 project', our research design is also based on the traditional approach for professional development in Japan, 'Lesson Studies', where teachers work in small teams, carefully and collaboratively crafting lesson plans (Lewis, 2002), however, one problem is that the lesson study often only occurs within schools. The flexible nature of SLEs makes it possible for teachers and children to explore various mathematical patterns. After designing the SLEs and lesson plans, pilot lessons are implemented, and then the research team evaluate pilot lessons. Pilot lessons are mainly recorded by observation with video cameras and field notes. The pilot lessons are analysed by researchers, with teachers and research students often joining the data analysis process. In our pilot lesson Year 4 children (aged 9 to 10) undertook a problem 'Investigate the differences between 3-digit numbers which consist of three consecutive numbers such as 321 – 123'. During the lesson, children not only found the pattern 'the answer is always 198', they also started reasoning by using 'the place value table' with counters.

Our current analysis concerns how children engage in the SLEs and discover and explore mathematical patterns. To capture the complexities of children's cognitive development within classroom dynamics, SOLO (Structure of the Observed Learning Outcome) taxonomy is utilised as a tool of analysis, which enables us to describe how mathematical patterns would develop through the following levels: Pre-structural; Uni-structural; Multi-structural; Relational; Extended abstract. (Biggs and Collis, 1982)

THE STUDY

In this paper, we shall report our findings from lessons with ‘Bamboo numbers’ based on the Fibonacci sequence. Bamboo numbers are made by recursive relations. Those presented for children in this exercise have contrived patterns, e.g. the first numbers are increased by one, and the second numbers are decreased by one, as shown in Figure 3. Bamboos grow rapidly, and the title of the exercise, “bamboo numbers”, is used as a metaphor as mathematical patterns continuously grow in children’s mind. Children are asked to fill in the missing the numbers by using (or not using) these patterns.

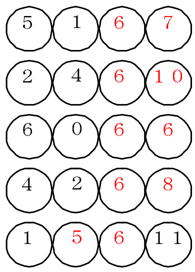


Figure 3: Bamboo numbers.

Four lessons are designed for Year 2 children (aged 7-8), with the aim that children consolidate their addition/subtraction skills, as well as experiencing discovering mathematical patterns, creating their own patterns, and subsequently extending their ‘number sense’ (Anghileri, 2006). After discussions between researchers and teachers, we structure our lessons starting from the introduction of the Bamboo numbers (the 1st and 2nd lessons), and progressing to more complex patterns (3rd and 4th lesson), as shown in Figure 4.

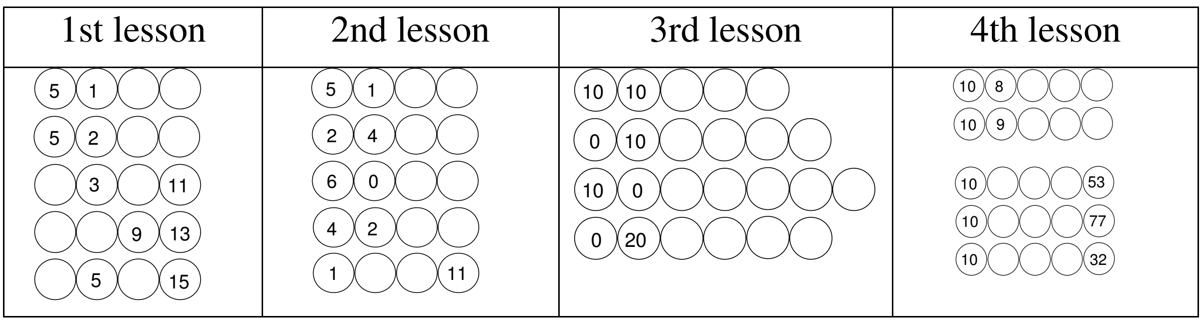


Figure 4: Lesson structure.

Watson (2007) stated that ‘if learners are only offered unistructural situations (simple and obvious relations) they are less likely to develop multistructural understandings’ (p. 115), and we have a similar view. We designed the tasks and the sequence of the lessons so that first children would see the patterns in individual number sequences in the first lesson, e.g. '5-1-6-7', '5-3-8-11' ('Pre-structural' or 'Uni-structural' levels), and then, gradually they would explore patterns horizontally and vertically to complete the patterns in the 2nd and 3rd lessons ('Multi-Structural' level) and more complex problems in the 4th lesson ('Relational' level).

DATA ANALYSIS AND DISCUSSION

In this section, we shall report what happened in our pilot lessons using Bamboo numbers implemented in July 2007, and how children discovered and shared mathematical patterns, and used them to solve more complex questions. The SOLO taxonomy is used to describe and analyse how children's development advanced throughout the lessons, and identify what factors were (not) effective in this development, as described in Figure 5.

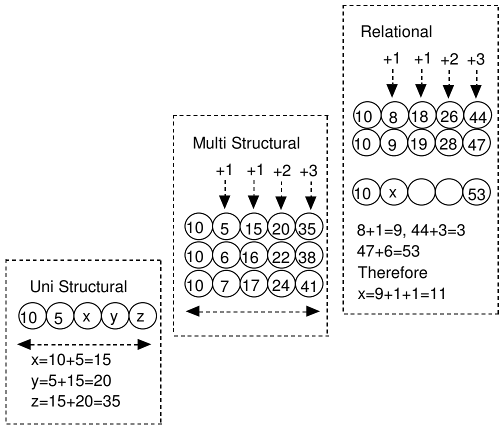


Figure 5: SOLO taxonomy and the Bamboo numbers.

Episodes from the 1st and 2nd lessons

The rule of the Bamboo numbers was easily grasped by all children, and they also successfully made their own patterns by using the numbers 1-9. To encourage children to see the flexible relationship between numbers, two sets of the Bamboo numbers were given to children. Children's statements recorded during the lesson indicate that they clearly advanced to the 'Multi-structural' level in terms of pattern recognition, as almost all children could complete the task below (see Figure 6). However, it seems that they did not fully understand the relationship between the Bamboo numbers, i.e. 'if the first number increased by 1, then the last number is also increased by 1' (although one child stated that 'The right ones are '4-4-8-12' and the left ones are '5-4-9-13' and the difference between 4 and 5 is 1, and that's the same difference as between 8 and 9, and 12 and 13',which might be considered as the early 'Relational' level).

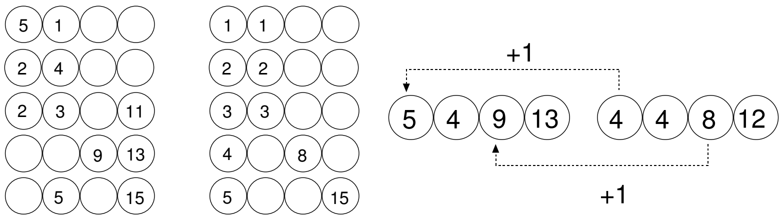
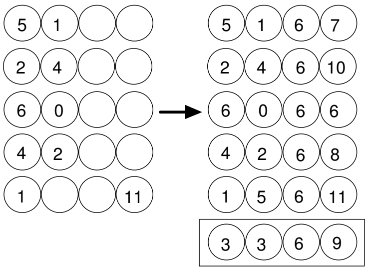


Figure 6: Children's understanding of the Bamboo numbers.

In the second lesson, children continued to undertake tasks with the Bamboo numbers and shared their findings via interaction among the whole class, which is summarised in the table below (Figure 7). In particular, they started paying attention to the relationship between numbers vertically, and could create another pattern '3-3-6-9'.

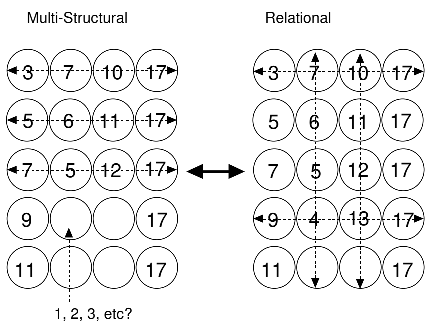


What children noticed …

- There is no 3.
- There is no 9.
- The 2nd numbers are 0, 1, 2, 4, 5, 6
- If you make '3-3-6-9' then the pattern will be complete.

Figure 7: What children noticed in the 2nd lesson.

By the end of the second lesson, most children in the class still stayed in the 'Multi structural' level, as it was observed that some struggled to complete the last two patterns ('9-?-?-17' and '11-?-?-17'), shown in Figure 8. However, some children showed progress from the 'Multi structural' to the early 'Relational' level through their pattern recognition and interactions with the teacher, and some children clearly indicated that they worked with both the vertical and horizontal patterns to complete the last two, i.e. the second numbers are 7, 6 and 5, so the last two should be 4 and 3. Also, the third numbers are 10, 11 and 12, so the next numbers should be 13 and 14. One child also said that he first found these numbers vertically and then checked these numbers also 'worked' horizontally, described as the table below.



T: How did you solve this?
Ps: I tried several numbers… I did it in my head...
T: I heard someone said that it is easy if you notice something… So what is 'something?'
P1: A pattern!
T: A pattern! So what do you notice?
P2: 7-6-5 … (indicating the 2nd column)

Figure 8: Children's discussion about problem solving strategy.

Episodes from the 3rd and 4th lessons

In the 3rd lesson, the task below (Figure 9) was given to children to provide opportunities with them to look for patterns not only horizontally but also vertically, to encourage their further understanding about the relationships between the Bamboo numbers. Children first undertook the task individually, and found they could extend the patterns to the left by looking at the numbers diagonally, shown in Figure 9 below. By the end of the 3rd lesson, they found that the patterns are actually the same.

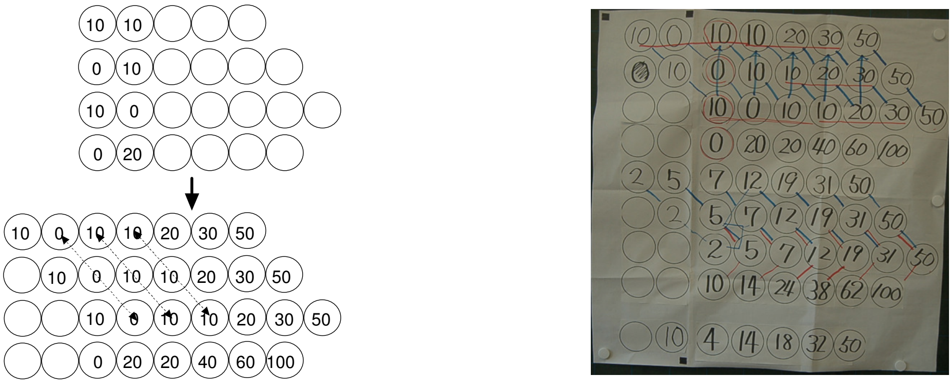


Figure 9: What children noticed in the 3rd lesson.

The children's recognition of patterns in the 3rd lesson really encouraged us to give the further challenging task in the 4th lesson. Our hypothesis was that children would be able to complete the task effectively if they had a good understanding about the relationship, 'If the first number is increased by 1, then what would happen to the other numbers? If the second number is increased by 1, then …' By using this relationship, the pattern '10-?-?-?-53' can be solved if one notices that '47+6=53', so the second number should be '9+2=11' etc. (see Figure 10)

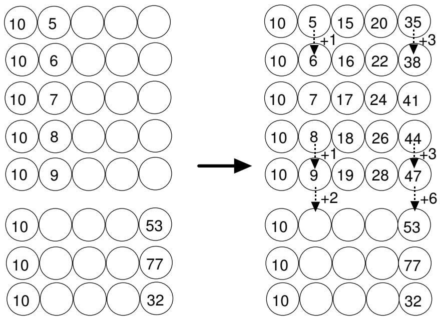


Figure 10: 'Relational' understanding.

The first five patterns were completed easily by all children, and their findings were shared. They noticed that 'the last numbers increased by 3', 'the second column is 5, 6, 7, 8 and 9', etc. They also noticed that 'the third column is increased by 1' and 'the fourth column is increased by 2'. After sharing these findings, children undertook the last three tasks. From what children stated and shared, we expected that many of them would soon utilise the patterns which they discovered, i.e. progressing to the

‘Relational’ level of understanding about the Bamboo numbers. Not all children however used the patterns; several just tried some numbers randomly, i.e. these children could not apply the patterns which they had just shared in the class. By the end of the lesson, 18 of 40 children could complete all patterns, and 7 managed to complete all but the last two ’10-?-?-?-53’ and ’10-?-?-?-77’.

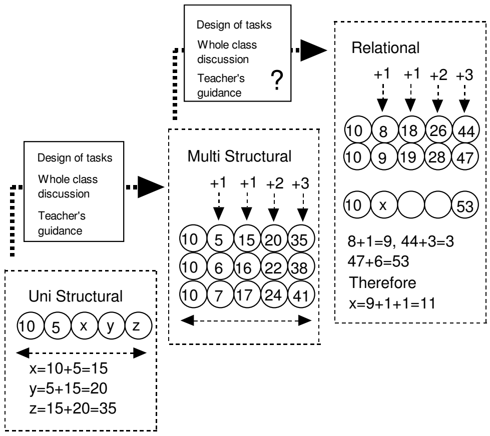


Figure 11: Bamboo numbers and level of understanding.

Considering the age of children and difficulty of the tasks, what we have observed from the lessons suggests some positive results, i.e. the design and organisation of this series of lessons could encourage some children (62.5% of the class) to advance from ‘Multi-Structural’ to ‘Relational’ levels in their understanding of the Bamboo numbers. However, while 100% of children could advance to ‘Multi-Structural’ level, our observation implies that advancement from the ‘Multi-Structural’ to ‘Relational’ level is a big step for many children, even after they actively undertook the given tasks, and repeatedly shared their findings in whole class interactions with their teacher’s guidance. It is still uncertain what other factors would be key for progression from ‘Multi-structural’ to ‘Relational’ levels, and this is clearly one of our future tasks, including re-designing the tasks and lesson structure (that is why ‘?’ is added between ‘Multi-Structural’ and ‘Relational’ levels in the figure above). Studying this matter is our next task for designing the substantial curriculum in primary schools and secondary schools.

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