Teaching Methodology in Mathematics

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Abstract: Beijing Normal University has adopted a reform on teaching methodology of mathematics since 1999. The purpose of the reform is to encourage students to be actively involved in the learning process.

The reform was motivated by students' exam-orientated learning behavior. Students try to obtain high scores by memorizing types of problems instead of ideas. Such orientation let students mistakenly believe that mathematics is made of various types of methods to be memorized.

We initiated a three-semester mathematical analysis course as the first step of the reform. The course includes three parts: lectures, mathematics software labs, and seminars. Our revised textbook consistent with the reform will systematically enlarge the basic knowledge of real number that students learned from high school, emphasize on multi-variable calculus, vector and matrix, and focus on Lebesgue integral in integral calculus.

The result of reform is encouraging. A lot of students start to think mathematics scientifically and become more interested and persisted, though we still face many challenges.

Keywords: exam-orientated, teaching, mathematics major, teaching methodology, learning misconceptions

1. Reform Background

It seems that high enrollment demonstrates Chinese students' talents in mathematics. However, China is still lack of leading mathematicians.

Beijing Normal University (BNU) is the leading normal school to generate high-quality high school teachers.

It is wired why China has an excellent of reputation but lack original mathematicians.

According to our studies [1, 2] on BNU students in mathematics and science major, they are not interested in mathematics, but only in what will be on exams. They are eager to raise questions about the way to solve the problem and strive to memorize steps and answers. However, the students enrolled in BNU are said to be the top one percent of their fellow high schoolmates.

Based on these dilemmas, we decided to study these issues and then design experiments on several undergraduate courses. In this article, we report our findings, survey, methodology and application.

However, the survey needs endless time and energy than we have spent. The results demonstrated in the article are preliminary and still inconclusive. Even though, we should help the kids improved as much as possible.

In the next section, we will accumulate a few findings about our elementary and secondary mathematics education and those among BNU math students (referring as students in the following for convenience); the third section is devoted to the experiment of three-semester course, Mathematical Analysis; our discussion and conclusion are presented in the last section.

2. Findings on Elementary-Secondary Mathematics Education in China and BNU Students

In China, students start to learn arithmetic in their early childhood, as young as two or three years old. Chinese people believe that mathematical abilities were crucial for children's development. A lot of elementary and secondary education is focused on mathematics, even extracurricular activities contributes to children's skilled capability to do exercises.

The drawback is that teachers, as well as parents, think mathematics as different types of exercise problems. The more problems and techniques are done, the better.

In should be mentioned that the recent mathematics education reform encourages students to solve different real problems by memorizing different rules and solutions. Although the topic in classes will be different, the teaching style is the same. An example found recently among our students is that they knew many names in Calculus, but they cannot do computations of elementary algebra and trigonometry. In order to continue the Calculus course, we have to go through basic algebra and trigonometry.

The students have the following misconceptions about mathematics:

Misconception I: Mathematics is just doing exercises;

Misconception II: There is a good way to learn mathematics, which requires no effort;

Misconception III: Mathematics is hard, because it is not related to application. Applications, or problems in applied mathematics are easy;

Misconception IV: Mathematics has been developed logically.

How are these misconceptions related to the mathematics education in schools?

Fact I: Elementary and secondary school students are asked and encouraged to do exercise problems more than enough.

Motivation to Fact I: In order to have a bright future, kids have to go to famous elementary schools, famous high schools, and famous universities. In order to get the admission, kids have to get high scores in entrance exams. In order to obtain high scores, kids have to compete in exercises.

Comments on Fact I: A selection based on exams is relatively fair. There is still no any other better method. To obtain good scores is natural and reasonable. The issues are (1) high scores should not be the only objective of mathematics education; (2) The purpose of mathematical education should be the development of students' mathematical thinking, ability, and techniques.

Fact II: Teachers, in particular, in high schools, are only interested in problems and what kinds of problems would appear in the exams. If someone can figure out one or more problems in a national entrance exam, he or she will be treated as a hero.

Motivation to Fact II: The assessment on a school is mainly based on the scores of students.

Comments on Fact II: Teachers are ordinary people. The priority of their concerns is to lead a good live. The issue is that teachers should know better than public on what the interest of students is and that more exercises do not necessarily imply good performances and decent life.

On the side of the products-students, we have the following findings [1]:

Finding I: Most (more than 90%) students claim that they like mathematics.

Comments on Finding I: It is difficult to get entrance to BNU mathematics major. However, most of students like mathematics.

Finding II: Students do not know what mathematics is about. Good students say mathematics is just strict, precise, and logic.

Comments on Finding II: The students' impression on mathematics is closely related to their training in pre-university education.

Finding III: Students hope that they can be trained to be excellent both in pure and applied mathematics.

Comments on Finding III: Students worry about that they might not have abilities to apply mathematics.

Finding IV: More than 75% freshmen and 64% seniors prepare for graduate study, 20% freshmen and 50% seniors want to continue their graduate study in mathematics.

Comments on Finding IV: Students are getting comfortable with mathematics and the anxiety on application is reduced significantly. It may also be concluded that many students have been assimilated into the existing mathematical atmosphere and lost their original impetus.

3. The Experiments on Mathematical Analysis

Our beliefs about mathematics education are the following,

(1) Mathematics teachers must *understand* mathematics. The deeper understanding and mastering mathematical formulations, the better performance in mathematics education;

(2) Inappropriate teaching methods are mainly ascribed to inappropriate understanding on mathematics, rather than to certain mathematical techniques;

(3) Mathematics is both a science and an art. But it is not a toolbox to memorize;

(4) Students majoring in mathematics should be able to explain a problem both in the context of mathematical formulations and in real setting.

The instruction of mathematical analysis highlights both the advantages and disadvantages in Chinese higher mathematics education. Principles on the pedagogical methodology are the same as those on pre-university pedagogy, such as memorization on points of knowledge, exercise problems and their answers, and exam.

We believe that such principles are partially responsible for the unsatisfied Chinese achievements in mathematics and its applications.

3.1. The Design on Instruction

The instruction consists three elements: lecture, maple lab, and seminar.

In lectures, students learn what Calculus is about, what basic ideas are behind, what issues are handled, why methods are reasonable, and how ideas and methods can be manipulated. We do assign many exercise problems, but we emphasize on understanding mathematics ideas thru doing problems.

In labs, students learn about what mathematics can do, why logic is powerful, why mastering a tool (Maple in this case) needs understanding mathematical theories, and how an understanding could become a tool (program), or an automaton.

In seminars, students learn how to present a mathematical problem, how to discuss mathematical ideas and proofs with others, and how to judge an argument. We tell students seminars are one of the main techniques for mathematicians and scientists to communicate.

3.2. Methodology in Lectures

We insist on the integrity of logical system in Calculus. That is a reasonable way that students learn what the strictness in mathematics is about, how the strictness is achieved, why such strictness is necessary, why certain ideas and formulations are advantageous, why students deserve working hard on certain topic. Students are told that instructors are guiders, carriers, and coaches, whose duty is just helping them understand mathematics and its application, and encourage them to be excellent in mathematics. Students are required to do many problems and hard exams.

Besides the traditional topics in traditional Calculus and advanced Calculus, we emphasize on the system of real numbers, several variables, and Lebesgue integral.

3.2.1. Real Numbers

The system of real numbers is introduced as the set of all decimal numbers without 9 circulating ones in terms of set and mapping. Such a definition is consistent with modern and high school mathematics. In this way, we do not introduce some other unnatural ways of handling real numbers to students. At the same time, this is a case for students to understand how mathematics achieves its strictness.

The treatments on real numbers in ordinary textbooks include so many axioms helping students and instructors to save time and efforts. The drawback is that students are really confused, is mathematics able to make ordinary numbers clear? If not, what can mathematics do then? Some mathematicians say that how to define real numbers is a type of philosophical problem. Some good students say that mathematics is just to add axioms whenever something cannot be clarified.

It is really sad that a good topic for both mathematics and mathematics education has been developed into a mathematics nightmare. It is Calculus that should clarify how mathematics understands and manipulates real numbers. If we missed the point here, our future intellectuals would mislead next generation.

3.2.2. Multi-variables

Multi-variable calculus plays a central role in the mathematical analysis course. It includes vector representations of derivatives, mean-value theorem in integral form, inverse function theorem, implicit function theorem, k-dimensional surfaces in n-dimensional Euclidean space, integrals on k-dimensional surfaces, and Green-Gauss-Stokes theorems. Students are supposed to learn the basics of multivariate tools and understand why linear algebra is necessary.

Multi-variable calculus is the main obstacle when students continue pursuing their careers in mathematics and other fields, especially for those working in applied mathematics. It is quite common that a mathematics graduate gets loss in an applied field. What he can do is just to repeat those terminologies in the applied field.

Multi-variable calculus is indeed a good example that one has to understand mathematics in order to teach well. Some instructors get lost here simply because they do not understand the mathematics in several variables and reduce it to some unrelated exercise problems so that the learners totally do not know what they are doing, and another useless mystery mathematical creature is created.

3.2.3. Lebesgue Integral

The theory of Lebesgue integral is one of the basics of modern mathematics. As the theory of real numbers, Lebesgue integral is simple and powerful, but not natural at beginning, more specifically, countable coverings of cubes are not natural. In the course, we do not abandon Riemann integral, but just retain a relevant part (partition the interval into finite many subintervals, approximation on each subinterval, summation and limit), which is intuitive and possesses a wide range of applications.

Theory of Lebesgue integral provides a powerful tool for absolutely integrable functions; it finishes a huge number of cases in Riemann integral once beautifully. For generalized integrals a further discussion is still needed. It is really convenient to replace Riemann integral by Lebesgue integral in mathematical analysis.

In instruction, we clarified the difference between Lebesgue integral and Riemann integral so that students may comprehend the power of rational over that of perceptual. It did encourage quite a few students to study hard mathematics.

3.3. Methodology in Labs

After introduced basics of Maple in computer room, students are encouraged to solve problems in mathematical analysis. When getting familiar with Maple, students turn to study) more intrinsic functions in the software. Students are grouped voluntarily; each group works on certain topic and reports its discovery to the class. Finally, students are supposed to design a project on a mathematical topic by Maple.

Students are scored by their presentation in class and lab reports.

3.4. Methodology in Seminars

The seminar is open to the whole class and there is no precondition to participate seminar. The materials presented in a seminar are selected by instructors after discussion with students.

In the middle of the second semester, mathematical masterpieces will be introduced in seminars. At this stage, students already learned sufficient amount of mathematics and possess the primary ability to work and discuss on such pieces of mathematical works.

There are only a small group students (10-15 students out of 100) followed the seminar from the first semester to the third. It is really selective that almost all seminar students finally pursue their careers in Mathematics and are among the best in the class both in abilities and in exam scores.

It should be mentioned that the requirements of the course is high, students have to work much more, comparing with those in ordinary mathematical analysis classes, more and harder exercise problems, lab presentations and reports, seminar preparation and presentations. Students do not obtain any extra credit from those extra works. In fact, there were no complains among students for these “unfairness” and our instructors for so much extra work.

3.5. How do students feel?

The instructors and students built up friendship. They support each other. When instructors played the role of coach, students certainly felt pushed and uncomfortable, but students made through and thanked the instructor. The instructors are human beings too; they feel depressed sometime and might not behave so well, while there were few students complain it and the teaching evaluations were always high. After the course, the instructors are still advisor of many students from former mathematical analysis courses.

4. Conclusions and Comments

Students do not enjoy obtaining easy-scores and being treated as kids. They do want to study something challenging, but they should be treated instead of tricked. If an instructor is not strong enough in mathematics, students will be misled, no matter what tricks are introduced.

Students can tolerate defects of coaches, but cannot tolerate being misinformed.

It is still a challenge to verify which teaching methodology is really fit to students' potential.

Reference

[1] Yan Zhang and Zhongdan Huan, A Survey on Learning State of Undergraduates in Department of Mathematics at Beijing Normal University, 2002 (in Chinese) (Published in Higher Education Reform, Issue 7, 2004, p106-116.

[2] Zhongdan Huan, a Reflection on "Double-Basics" Teaching Based on the Mathematics Accomplishment of College Students, (in Chinese) Higher Education Reform, Issue 7, 2004, p116-118.