HISTORY OF MATHEMATICS MAKING ITS WAY THROUGH THE TEACHER NETWORKS
Professional learning environment and the history of mathematics in mathematics curriculum

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Abstract
This paper will focus on a project led by the author, which aimed to introduce the history of mathematics into the curriculum through a collaborative teaching practice involving both primary and secondary schools in the South East of England. The project has been supported by the National Centre for Excellence in the Teaching of Mathematics and the British Society for the History of Mathematics, and has been running from the beginning of the academic 2006 to date.

The paper presented here explains the philosophy behind each strand of the project rather than following the project chronologically. It looks at the uses of the history of mathematics to initiate two developments: the increased motivation of students, and the creation of a conceptual landscape which would serve as a basis for teachers' continual professional development environment.

1. Background information

The project began in September 2006 and is to be completed by September 2008. Over the course of the project three secondary schools, with a total of fifteen teachers (two of whom were science specialists but taught mathematics to lower ability groups), and three primary schools with a total of three teachers have been involved. More than 450 pupils have been involved in the project at various times, spanning the age range between ten and fourteen (English Key Stages 2 and 3) and covering all ability ranges.

The project has been conceived and led by the author of this article, and is supported by the National Centre for Excellence in the Teaching of Mathematics (UK) and the British Society for the History of Mathematics. An additional Higher Education institution has directly been involved - the University of Plymouth Centre for Innovation in Mathematics Teaching provided the training for all involved teachers in the principles of collaborative teaching practice. Currently eight teachers from three secondary and three primary schools are completing an MA module provided by the University of Plymouth, a programme that is fully funded by the Teacher Development Agency (UK). An additional private consultant has been involved in the project, offering support in the matters of teacher training and the uses of the history of mathematics in development of mathematical pedagogy.

The project was based on the premise that the history of mathematics can improve both the motivation and attainment when used as a contextual background in the teaching of mathematics at this level. At the same time the first round of teachers involved in the project

showed an interest in having an opportunity to develop their teaching and research skills in order to introduce the history of mathematics. The use of modern technologies, especially the Internet and dynamic geometry environments (such as Geometer's Sketchpad) was encouraged.

1. 1 Going even further back - current changes in the curriculum for England and Wales

The National Curriculum was introduced in England and Wales under the Thatcher government in 1988. It prescribed the teaching of all subjects to include 'spiritual, moral, social, and cultural values'. How this was done, and in particular in mathematics, was up to individual teachers, who sometimes struggled to find spiritual values in the teaching of mathematics. While originally the National Curriculum was conceived as an underlying structure in a system in which all pupils were meant to have equal opportunities to learn certain basic concepts, and based on an assumed common agreement of pupils, parents and state alike, it was subsequently recognized to have been quickly transformed into a monolithic bureaucracy which seemed to consume the life energy of the individual teachers and schools by imposing an inflexible and controlling programme which left no space for the development of individual interest or input.[1]

Recently however, this situation has been challenged through a new approach taken by the Qualifications and Curriculum Authority (QCA). This has occurred through the changes made by the QCA to the curriculum which has introduced different sets of values, including a certain amount of freedom for teachers, teacher teams, and consortia of schools to develop their own syllabus in all subjects. The modernising of the curriculum is driven by the need to take into account local needs and needs for different types of vocational training. One of the more positive aspects of this development may be seen in the fact that the local provision of education will have a degree of freedom (not yet defined), and that personalised learning, project based work and mentoring will all have a big role to play in this new vision of education. This opens a valuable opportunity for teachers to demonstrate that mathematics, like any other creative pursuit, is an area where exciting and useful contributions can still be made – both by teachers and by pupils. The need for combining inspirational teaching and the experience and flair of teachers who are interested in pursuing their own academic interest in the subject area is apparent and will be crucial in bringing about a necessary change in this context.

As such, the introduction of the historical element in the mathematics syllabus offers the possibility of developing teaching strategies which do not necessarily provide only historical context, but use the history of mathematics as a tool for discovering facts and exploring mathematical techniques. The new curriculum states that the students should recognise the 'rich historical and cultural roots of mathematics':

> "Mathematics has a rich and fascinating history and has been developed across the world to solve problems and for its own sake. Students should learn about problems from the past that led to the development of particular areas of mathematics,

[1] For a short historical overview of the National Curriculum, see Crawford, (2000).

> appreciate that pure mathematical findings sometimes precede practical applications, and understand that mathematics continues to develop and evolve."[2]

The current challenge, therefore, facing English teacher-training institutions will be to address the imbalance between the desire to introduce the historical element to the teaching of mathematics and a complete lack of the formal teaching in the subject area (of the history of mathematics). Another interesting issue in this context is the prospect that local communities and schools are able to develop their own curriculum to suit their needs. To further explore this concept, an example of a project supported by the National Centre for Excellence in the Teaching of Mathematics (NCETM) and the British Society for the History of Mathematics (BSHM) is therefore described here, with a view of defining some benchmarks on which it would be possible to base a programme of in-service training for the serving teachers of mathematics. Such courses (at a post-graduate level, offered as transferrable MA modules) can be provided to support teachers in embracing the opportunities that the new curriculum offers.

1.2 The teaching of mathematics in a historical context

The Gatsby Fellowship in Mathematics, that the author of this paper held in the academic year 2004/5 and which was based on incorporating the history of mathematics in everyday teaching for pupils aged 11-18, resulted in a website of teaching and learning resources for the use in secondary schools.[3] Its popularity showed that the interest in this type of teaching and learning of mathematics is considerable and the colleagues, from the first school in which the current project was based, as well as the NCEMT and BSHM, were both keen to support an initiative which would build on the existing resources to enhance the teaching of mathematics in historical context.[4]

By way of background it should be noted that during the same academic year the government initiated an inquiry to assess the state of mathematics education, and made the observation:

> "…there has also been considerable concern about many young people's perception of mathematics as being 'boring and irrelevant' and 'too difficult, compared with other subjects'".[5]

As a consequence, one of the results of the inquiry was a recommendation to 'enhance professional development for mathematics teachers in all education sectors' in the country.[6] Furthermore, the inquiry recommended that 'ACME[7] be provided with enhanced

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[2] Page 4 of the QCA Mathematics Curriculum, accessed 20th March 2008, <<http://curriculum.qca.org.uk/subjects/mathematics/keystage3/index.aspx>>.

[3] See the website <<www.mathsisgoodforyou.com>>, the description of this project on the National Teacher Research Panel site <<http://www.standards.dfes.gov.uk/ntrp/lib/pdf/lawrence.pdf>>, and Lawrence (2006).

[4] At the time when the application for the current project was made to the NCETM the monthly number of visitors to mathsisgoodforyou was in the region of 20 000. Currently it is running in the region of 50 000 visitors per month.

[5] In particular see Point 0.6, Smith (2004), 2.

[6] As pointed on the NCETM website, <<http://www.ncetm.org.uk/about>>, acessed 25th November 2007.

support in order to play an active role in helping to take forward the recommendations of this Inquiry and that a corresponding body be set up to speak on behalf of the mathematics community on strategic issues relating to research and knowledge transfer in mathematics.'[8]

In 2005 the National Centre for Excellence in the Teaching of Mathematics (NCETM) was founded by the Government to fulfil this role, and the project on the *History of Mathematics at Key Stage 3* was one of the first ten projects to get the funding. The project was then based in one secondary school and engaged seven teachers from the same mathematics department (including two non-specialists); now in its final stages and more than eighteen months later, it includes six schools (currently engaging eight teachers) and has changed the name to *What works in the classroom - Collaborative Teaching Practice and the History of Mathematics*. In all participating schools the project has been undertaken through the use of collaborative teaching practice and the development of the teaching methods through historical context, primarily focusing on the teaching of geometry.

2. Collaborative Teaching

There is a small ambiguity contained in the expression 'collaborative teaching' as it sometimes refers to 'co-teaching' - a concept developed to support the teaching of academically less-able and students with special needs. In this latter form, collaborative teaching is deployed widely in the English educational system through the collaboration between teachers and teaching assistants.

Collaborative teaching practice in this paper however, refers to the practice developed in different countries (most prominently Japan, Hungary, and more recently United States and England) and sometimes also closely linked and/or referred to as 'lesson study'.[9] The collaborative teaching practice that has been part of the current project as a way of peer-discussion and collective teaching tool is based on the simple cycle as follows:

- plan the lesson together, discussing the potential research activities/foci of research
- do individual research and share responsibilities for producing resources
- meet or correspond and share the findings of research as well as produced resources
- teach collaboratively or film the session
- assess the filmed lessons as a team, adopting certain resources and approaches for further use and or development, and rejecting others.

All teachers had, at some point of the project, been involved in planning, teaching, and assessing a lesson as part of a group of teachers. Some of the planning and all teaching

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[7] The Advisory Committee on Mathematics Education, established in 2002 by the Royal Society of Great Britain and the Join Mathematical Council of the UK, supported by the Gatsby Charitable Foundation. Its remit is to 'enable an effective and constructive partnership between Government and the mathematics community' - see <<http://www.acme-uk.org/>>

[8] Smith (2004), 4.

[9] See Lewis (1995), Lewis and Tsuchida (1998), Stigler and Hiebert (1998), and more recently Fullan (2004), (2005).

sessions were filmed and studied by the groups of teachers as part of the analysis part of the cycle (planning-teaching-analysing-refining resources-teaching, etc.), having the learning and teaching strategies as its focal point.

2.1 Collaborative teaching and the networking of teachers

As the name of the project suggests, its primary aim, which is to introduce a historical element to the teaching of mathematics, is pursued through two components: the development of collaborative teaching practice and the trialing of methods and resources through the classroom teaching backed by the evidence-based research to establish pointers for successful practice.

The project has been pursued by practicing teachers with various degrees of experience in the teaching of mathematics (not all of whom are subject specialists), and therefore the question arose of how to create a professional learning environment which would be able to contain all levels of experience and mathematical ability in order to support their participation. Of major interest was the possibility of introducing a model of continuing professional development based on a set of principles which could be replicated elsewhere and which would help teachers develop a range of techniques, and introduce a new element which could help them structure their own learning at the same time as structuring their teaching programme. At the core of this envisaged professional learning environment stood a belief that the interest and personal development can only be achieved in those situations and environments where the professionals themselves find an area of research they would like to further pursue.

Various mathematics educators have seen the different roles the history of mathematics can take through its introduction into the education of mathematics teachers - Freudenthal (1981) for example conceived it as giving a background to the teachers' mathematical knowledge, while others concentrated on offering a possible pathway to the deepening of teachers' reflection capabilities through an in-depth study of the development of mathematical concepts through history (see Arcavi, Bruckheimer, & Ben-Zvi, 1982, 1987; Swetz, 1995). One of the approaches, developed by Hsieh and Hsieh (2000), and Philippou and Christou (1998a, b) dealt with using the history of mathematics as a particular tool and context to develop beliefs and attitudes in mathematics.

The benefit of the use of history of mathematics however, in the context of the described project, can be best seen on the influence in which it created an opportunity for a ***focus*** of cooperation and collaboration as well as a ***conceptual landscape*** which offered opportunities to teachers to develop their individual interests.

This highly individualist approach to the continual professional development of teachers can increase their subject knowledge and enable them, through the modern technologies, to share their experiences and knowledge with mathematics teachers and students from around the world. Our agreed aim was to adopt a creative and individualistic ethos in teaching, providing ample opportunity for bringing the history of mathematics alive to the present generation of school children. Eventually, in practical terms, the defined foci were

enlarged to include, apart from the collaborative teaching practice and the individual research, the creation of a networking platform in the form of web-quests which will be available from September 2008 from the NCETM website.

2.2 Collaborative teaching and the inquiry-led learning

While the first team involved in the project researched the resources in the history of mathematics, the concept of mobile learning and the inquiry-led learning came to the fore as the most appropriate set of tools to inspire the wide ability range of pupils we were dealing with.

The inquiry-led learning is, on occasions, redeployed in contemporary practice with the individualised workshop-type of learning.[10] The basis of the model the project team adopted rested on the modern interpretation of the heuristic method of teaching geometry; one in which pupils are encouraged to discover intuitively some geometrical truth without the resource to the available knowledge to begin with. At an appropriate time, the historical element is being introduced, showing pupils how others dealt with the same or similar problem, thereby

- enhancing the learning process by making connections
- increasing interest and motivation by setting the problem in context
- enriching mathematical understanding through historical context.

The project team was aware of the problems that inquiry-led teaching may contain should it be deployed without the full understanding of the possible drawbacks - unstructured or poorly structured learning environment, the difficulty of leading students to 'discover' complex theories, and the difficulty some teachers may have in adjusting to such a learning environment. We however opted to explore this type of environment helped by the peer network and by the view that learning in a 'mobile' world must change to incorporate not only technology but the ideas of learning that the pupils/children already posses by the time they come to attend school.

Inquiry-led learning also raises a number of questions for the preparation of lessons, teacher training, and finally, curriculum development - all aspects of the described project. Example are:

- at what point does a teacher begin learning with a pupil by his side and how does this affect his or her professional role?
- what kind of training can prepare teachers for inquiry-led learning?

and in our context,

- does the history of mathematics in education at this level and in this context have a wider role to play in setting the curriculum?

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[10] The inquiry-led learning's main proponents were John Dewey (1859-1952) and Martin Wagenschlein (1896-1988). The main view they propagated was that the understanding must come before knowledge, and some of their ideas form the basis of the 'constructivist' idea of learning. The anti-proponents suggest that the data collected over the past half-century does not suggest that inquiry-based methods actually work. See Kirschner, Sweller, and Clark, (2006) for the latter.

Through the work conducted in the latter part of the project, during the winter/spring term of academic 2007/8, it was possible to make some conclusions regarding these questions.

2.2.1 At what point does a teacher begin learning with a pupil by his side and how does this affect his or her professional role?

An increasing body of research shows that inquiry-based-learning helps create an environment in which the teacher may be required to act in manifold ways.[11] These manifold roles of a teacher relate to the theory of 'Knowledge Manifolds', in which teachers are 'promoted' from teacher/preacher to teacher/consultant and teacher/resource type of roles. Naeve (2005) defined the 'Knowledge Manifolds' as 'linked information landscapes (contexts) where one can navigate, search for, annotate and present all kinds of electronically stored information'.[12] Such open information landscapes have developed with an exponential speed since the founding of Wikipedia (domain launched only in January 2001), and rest on fundamental principles of communal and self-governance in the same way in which Naeve suggests future 'teaching landscapes' will develop. This theory is in concordance with the network theories of knowledge as much as it is with the theory of 'mobile learning'. The described project therefore opted to further explore in practice such as approach to teaching and learning in which teachers are as much learners as their pupils by making parallels between the sets of teachers with the sets of pupils:

- including their colleagues in the learning process through collaborative planning, teaching and assessing
- including personal research and reflection on teaching/learning as a cyclic practice with every new topic
- making sure the subject learning is part of teachers' every-day professional life.

2.2.2 What kind of training can prepare teachers for inquiry-led learning?

The inquiry-led learning as developed through this project grew organically from the collaboration with similary-minded colleagues. Few necessary prerequsites had to be fulfiled - existence of full professional trust and exchange of information and knowledge had to be devoid of all performance management in any of the schools, but the peer network offered plenty of opportunities for exploring the areas of improvement instead. Critical friends were deemed to be colleagues working within smaller groups, and the involvement of the higher education (Plymouth) and national (NCETM) institutions added a dimension to this process through validation and provision of a postgraduate course.

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[11] Naeve describes these roles as that of "*knowledge cartographer* [who] constructs context maps, the *knowledge librarian* [who] fills the maps with content, the *knowledge composer* [who] combines the content into customised learning modules, the *knowledge coach* [who] cultivates questions, the *knowledge preacher* [who] provides answers, the *knowledge plumber* [who] routes questions and the *knowledge mentor* [who] provides a role model and supports learner self-reflection." Described in Naeve (1997).

[12] Naeve (2007), 6.

2.2.3 Does the history of mathematics in education at this level and in this context have a wider role to play in setting the curriculum?

In the description of the other aspects of this project that follows it will be clear how much the history of mathematics helped shape the building of the professional learning environment which then spilt over into the classroom. Setting the scene for development, investigation, research into, and formation of ideas which can be communicated between the peer groups (both teacher and student), was possible to achieve with the history of mathematics for the simple reason that it offers such a rich ground source for exploration.

3. Structuring the self-regulatory Continuing Professional Development through collaboration and research

The project showed how the history of mathematics can set the 'scene' and act as a catalyst in creating a professional learning environment as well as giving a structure to endorse inquiry both in the student and in the teacher. In mathematics, this dimension is or can be, added to any such particular conceptual landscape. This should work with the teaching of any branch of elementary mathematics, but mostly so in the case of geometry. As the subject matter itself deals with understanding of space and properties of spatial elements, this in turn helps and underlines the development of competency in the building of a conceptual landscape of interrelated mathematical ideas. One may say that at the ontological level the building of the networks of concepts underlines the exploratory process of building the structure of learning mathematics, thus making the learning of geometry a truly multi-dimensional knowledge manifold.

3.1 The history of mathematics and the process of reorientation

As Furinghetti has shown (2007) some teachers tend to believe that the style of mathematics teaching they were affected by or exposed to must be reproduced in their own practice. In the case of the described project, this was most evident in the attitudes of teachers who were non-specialists in the subject. Furinghetti showed that the history of mathematics context allows for an exploration of topics in a new light and hence helps teachers construction of teaching sequences. While this was one of the added benefits of introducing the history of mathematics into the collaborative practice, we were also aware of the uses of mathematics in teaching, therefore allowing us to explore the various roles the history of mathematics can take in the classroom practice.

Whilst the history of mathematics in teacher education programmes has been described at some length by Furinghetti (2007), Schubring (Schubring et al., 2000), and Heiede (1996), little has been so far written about the in-service training of practicing teachers in this regard. This project aimed to begin the task by making a sketch of the possible influence the history of mathematics can have on in-service specialist and non-specialist mathematics teachers.

Therefore one of the project's aims became to try to introduce what Furinghetti (2007) calls 'reorientation':

> "…the learners involved in the process … are forced to find their own path towards the appropriation of meaning of mathematical objects".[13]

In this context, the acquisition of meaning was attempted through exposing beliefs about, and the partial understanding of, the concept in question with the new, 'foreign' meaning:

> "A meaning only reveals its depth once it has encountered and come into contact with another, foreign meaning: they engage in a kind of dialogue, which surmounts the closedness and one-sidedness of these particular meanings".[14]

This was found to be particularly effective in two instances:

- the teachers' understanding of mathematical concepts developed through the process of familiarisation towards meaningful context, followed by cognitive expansion, leading in turn to formal definitions, and thereby enabling specialist and non-specialist teachers to acquire:
    - greater competency in the subject matter
    - understanding beyond the basic comprehension of the type 'this is how things work'
    - development of interest to further deepen the knowledge and understanding of the mathematical concept and its relation to other concepts in the knowledge landscape.

- teachers began structuring their understanding of concepts by developing an ability to switch between modes of thinking and behaviours attributable to learner vs teacher, therefore:
    - deepening their own understanding of the process of discovery and learning
    - refocusing from 'how it works' to 'why' as well as '***how*** and ***why*** did ***they*** do it'.

In short, one of the teacher testimonies illustrates these described process thus:

> "… I was… astounded (by)… the depth there is in so many topics we have covered through this project. It has rekindled interest in mathematics in me; students find it interesting as well."

3.2 Scaffolding knowledge for non-specialist mathematics teachers

Naeve's (2005) approach to knowledge which he identifies as that consisting of 'efficient fantasies' and learning as that consisting of 'inspiring fantasies' has a lot to offer in the context of creating a learning environment in which both teachers and students discover new facts and exchange ideas in a more elaborate, creative, and yet mathematically sound ways. Naeve's description of fantasy has a lot to offer in terms of initiating a process of learning not only in the here and now, but one that draws upon the initial interest in the 'fantasy' and how it (the fantasy) occupies a mind of a learner for a longer period of time, offering a prolonged urge to find ever increasingly new content about a subject matter.

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[13] Furinghetti (2007), 113.
[14] Bakhtin (1986), 7, as reported by Radford, Firinghetti, and Katz (2007), 108.

To elaborate on this concept, Naeve sees the education process (in mathematics as well as in information technology - or any other subject area for that matter) as one in which learning is concerned:

> "with exposing the learner to inspiring fantasies and assisting her/him in transforming them into efficient fantasies".[15]

While Naeve somewhat exaggerated the view of the traditional 'learning architectures' being exclusively teacher-centric and consequently his concept of knowledge 'pushing' rather than knowledge 'pulling' may be lacking in subtlety, his intention to shift the focus onto the system of initiation into an interest field, whilst at the same time offering the system of skills to equip a learner with a set of tools to undertake the task of discovery and learning is at the centre of all 'collaborative', 'flexible', and 'personalised' learning concepts.[16]

So far, as in the case of Mariotti (2000), the focus on developing strategies to initiate 'learning fantasies' has been on the pupils. In the new type of learning environment, one in which 'knowledge pulling' rather than 'knowledge pushing' is taking place, teachers and pupils are learners and communicators of insights into mathematical facts at the same time, interchanging roles at different levels. From the experience of our project it became clear however, that some of the roles of the learner and some of the roles of the teacher are interchangeable, whilst others remain strongly rooted in the

a) evolutionary roles and
b) social roles these two groups represent.

3.3 Use of IT in developing CPD strategy in relation to the history of mathematics

> "Much of the schooling is about learning to access parts of the cultural record and to manipulate them using the tools of external working memory such as writing and mathematical notation".[17]

Two aspects of the use of IT were deemed a necessary part of the development of the project:

1. using ICT to support the creation of narratives
2. using ICT to support the exploratory aspect of the learning.

To satisfy the former, a collection of web-quests[18] is being developed - self contained websites for each of the lessons taught and studied as part of the project. These fulfil many roles, one of most important being the development of the base of knowledge in the history of mathematics which is multi-dimensional and usable not only for the purpose of one lesson but available for re-use and individual study by pupils.

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[15] Naeve, (2005), 4.
[16] All part of the national strategies on 'Every Child Matters', 'Personalised Learning' and 'Extended Schools'. See related sections at the <<http.//www.standards.dfes.gov.uk>>.
[17] Donald (1991), 329.
[18] Web-quests are self-contained websites which can mutate over time, and incorporate all elements for the study of a topic from introductory remarks to worksheets, and the possibility of submitting work for assessment. For further information and a collection of examples see << http://webquest.org/>>.

It is an undisputable fact that one of the most basic aims of all education is to instruct a learner into the sets of conventions. In the case of mathematics however, there is a danger that the convention may be confused with the invention - for example the fact that one (1) is not a prime number is often seen by a non-specialist teacher as a convention rather than a mathematical fact. The children too can have a difficulty in distinguishing the two important but entirely separate concepts, which can lead to the misinterpretation of all mathematics as a field consisting entirely of the various compilations of conventions. The historical narrative in this respect also offers a role in distinguishing the two. The process may be enriched by the narrative purposefully designed to satisfy the students' need to encompass all natural phenomena in a self-contained world of ideas thus also satisfying the mind's ability to form a dialogue between "particular 'episodic' events and the general 'theoretic' models".[19]

This however, offers the greatest challenge to a working teacher - finding the time and reliable resources which they can use to enable them to:

- construct such a narrative
- construct the resources or create an environment to support the inquiry-led learning.

Although in this respect the field of exploration widens considerably when the history of mathematic is offered as such one possible landscape, the necessity for the discovery and re-discovery of mathematical facts remains the question that needs to be addressed. In the current project not all work was based on the dynamic geometry environments which can be offered as such tools, but their use is one which offers not only one-dimensional tools of discovery such as exploratory (although investigative, tightly structured worksheet based) tasks, but can add to the multi-layering of the discovery process by allowing the development of the individual research process by students.

Shaffer & Kaput, (1998) and Kaput (1986) put a great deal of belief in the significance of the role that the computational media in mathematics can have as a way of helping build relationship structures between different representations of the same situation/problem/concept. Whilst the dynamic geometry software offers this in a literal way, through giving visual and manipulative representations of dynamic systems and relationships contained within them (as well as with other systems), the relationships between problems/concepts still remain lacking in 'meaning' and context. In the case of non-specialist teachers this is further exacerbated by their slowness in recognising the potential to form links between different concepts or between the different aspects of the same concept. This was one of the drawbacks of using such methods.

Nemirovsky (1998), Noss and Hoyles (1996) and Wilensky (1991) addressed elsewhere the new pedagogical approaches to teaching of mathematics in this new 'virtual' culture, and with the aid of dynamic geometry. Of concern to us is the extension of these approaches to incorporate the meaningful associations and to build upon this to further support the development of a long-term interest in mathematics and the mathematical cultures through a web of what Naeve (2005) calls 'inspirational fantasies' which sustain the desire and drive to

[19] Shaffer & Kaput (1998), 101.

learn rather than the ‘knowing’ (a set of meaningless formal rules). This is the main focus of the last part of the described project.

4. Conclusions

The project is yet to be completed and will finish with the production of two types of resources:

1. teacher oriented guidance on developing a continuing professional development learning environment which may overlap, relate to, and develop within a classroom, containing all the above mentioned ideas and their influence on the learning and teaching of mathematics
2. student oriented resources with the aim of developing an interest in mathematical topics through narration and activity-based materials.

It is hoped that the findings of the project will not only be useful in the ways of disseminating the materials developed through the project, but that they will offer a sound basis for developing a post-graduate Masters level qualification for the teachers already in service. The training in a collaborative teaching environment, with an emphasis on the history of mathematics, has proved through this project to be able to provide groups and networks of teachers with a set of skills as well as a multitude of ‘learning fantasies’ which could ‘re-ignite’ their interest in the subject and its pedagogy, and give them more opportunities to develop the new non-prescriptive parts of the mathematics curriculum.

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