On Instrumental Genesis within Procedural and Conceptual Thinking

Lenni Haapasalo, University of Joensuu, Finland
e-mail: Lenni.Haapasalo@joensuu.fi

Abstract. Technology has caused a holistic change in our "mental art", i.e. in the way we think, plan, work and evaluate in a modern society. This implies that educators are forced to shift their views of mathematics and learning of mathematics. On the other hand, fostering students' problem-solving abilities means emphasizing the genesis of heuristic processes and students' possibilities to develop intuition and mathematical ideas. This can be realized when making constructivist views of teaching and learning alive: Concepts and procedures can be at least partially constructed by students themselves - and also the other way around - well-known concepts can be applied in one form or the other one. Solving of authentic problems by utilizing modern technology promotes both of these goals allowing to reflect the relationships of mathematical components under consideration. Opposite to conventional teaching, which often seems to rather contaminate than promote student's metacognitive abilities, progressive pedagogical solutions utilizing modern technology allow us to gain profit from student's natural activities outside the classroom. The article illustrates results and experiences of a ClassPad project from the viewpoint of problem-solving, emphasizing more the making of informal than formal mathematics on the basis of the so called 'Minimalist Instruction' approach and within the framework of eight main activities and motives, which have been proved to be sustainable in the history of human thinking processes and for the generation of new mathematics at different times and within different cultures.

Introduction

Technological development together with a changed conception of knowledge and learning could lead to a paradigm shift: learning of mathematics is more distributive (i.e. independent of time, place and formal modes), socio-constructivist (learning community centred) and technologically enhanced. Technology-based mathematics education has expanded, namely, to include the following solutions (Haapasalo & Silfverberg 2007):

- computer algebra systems (CAS), dynamical geometry (DGS), and dynamical statistics (DSS);
- spreadsheets, drawing programs, and other versatile tools for mathematical modelling;
- online databases of available software, instruction, research, statistics, history, etc.;
- online communication in all of its synchronous and asynchronous forms;
- new kinds of environments to read, write and publish;
- tools for utilizing of the world-wide web: search engines, etc.;
- online experiments and simulations in diverse forms of digital educational content;
- online libraries containing books, learning objects, other teaching materials, etc.
- learning management systems (LMS), which are used to manage students and course materials;
- virtual worlds in the form of three-dimensional immersive environments offering, for example, shared exhibitions or other forms of collaborative functionality.

This potential is used not only via networks or computers but also on calculators and communicators, which students use in informal way on their free time. When using a tool within more or less spontaneous procedural[1] actions, the tool, especially at the beginning, puts certain limitations on what can be investigated and how. Adapting the term of Trouche (2004), I mean by *instrumentation* the process when the tool shapes the actions of the users. On the other hand, users often find their own schemas and schemes to use the tool. In this process of *instrumentalization* not only the use of the tool, but also the objects to be investigated are shaped by the users. Not long ago, students had to make paper-and-pencil manipulation of $1/\sqrt{2}$ to $\sqrt{2}/2$, because the every-man's instrumentation did not allow to make the calculation easily. Today any sophisticated user proceeds faster by using the calculator keys SQR and 1/X. This *instrumental genesis* has

---

[1] I adopt the following characterizations of Haapasalo and Kadijevich (2000):
• *Procedural knowledge* denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
• *Conceptual knowledge* denotes knowledge of particular networks and a skilful "drive" along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

an impact on how mathematical situations appear for a modern citizen. On the other hand, progressive technology in all above-mentioned forms can be interpreted as an orchestra. Therefore it seems appropriate to use Trouche's term *instrumental orchestration* when scaffolding[2] student's instrumental genesis. In this article I restrict myself in considering modern calculators, the original purpose of which is to apply mathematics. Therefore finding a solid pedagogical framework to adopt the device in teaching is often difficult task.

From procedural thinking to structures

The philogenesis of mathematical knowledge, also in individual development, shows that procedural knowledge has developed faster than conceptual knowledge, guided by the pragmatic aspects. Students often choose the right thing to do without being able to mention the reason for it. Such behaviours reveal the existence of powerful implicit concepts and theorems that may be called concepts-in-action and theorems-in-action (cf. Haapasalo & Kadijevich 2000, p. 148). According to Rittle-Johnson and Koedinger (2004) and Rittle-Johnson et *al.* (2001), conceptual and procedural knowledge seem to develop iteratively, where a change of problem representation influences their relation. Such a development was assumed in the large empirical MODEM project[3], the framework of which emphasizes the interplay between the two knowledge types, and the links between different representations (see Haapasalo 2003; 2007). Quoting also Schwarz et *al.* (1990) it can be summarized in a somewhat simplified way that to coordinate the process and object features of mathematical knowledge, multiple forms of representation are to be utilized and connected, especially with the aid of modern technological tools. The use of these tools should not reinforce a strictly hierarchical nature of mathematical knowledge but rather promote its flexible network nature (extrapolated from Burton 1999). Furthermore, structures should not be viewed as a priori sets of arrangements but rather as the results of the kinds of activities and actions tools permit when used in certain ways (paraphrased from Confrey and Costa 1996, p. 163).

From pacing to minimalism

Zimmermann's (2003) study of the history of mathematics reveals eight main activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years: *order, find, play, construct, apply, argue, evaluate, and calculate.* Especially the five first activities very often run optimally without any external instruction or demand. Students frequently neglect teacher's tutoring or they feel they do not have time to learn how to use technical tools. Teachers similarly feel they do not have time to teach how these tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be accessed without first reading heavy manuals. The term *minimalist instruction*, introduced by Carroll (1990), is crucial not only for teachers but also for those who write manuals and help menus for the software. Carroll observed that learners often tend to "jump the gun". They avoid careful planning, resist detailed systems of instructional steps, tend to be subject to learning interference from similar tasks, and have difficulty recognizing, diagnosing, and recovering from their errors. I next pick up the following characteristics of minimalist instruction (cf. Lambrecht 1999, van der Meij & Carroll 1998) keeping in mind the fostering of problem-solving abilities. These features of *minimalism* include several varieties of constructivism, offering also instructional assumptions (cf. Lambrecht 1999, Duffy et *al.* 1993)

- Learning is modelled and coached for students with unscripted teacher responses.
- Learning goals are determined from real tasks stressing doing and exploring.
- Errors cannot be avoided and should be used for instruction
- Learners construct multiple perspectives or solutions through discussion and collaboration.
- Learning focuses on the process of knowledge construction and development of reflexive awareness of it.
- Criterion for success is the transfer of learning and a change in students' action potential.
- Assessment is ongoing and based on learners' needs.

The fact that most part of students' instrumentation and instrumentalization very often happens on their freetime, implies that educators should shift the focus from well-prepared classroom lessons on minimalism. Instead of acting like a pace car in a race, institutions should be types of pit-stops to scaffold students' "race" outside the classroom.

[2] Trouche (2004, p. 296) speaks about "external steering of students' instrumental genesis", which we, however, do not find a solid expression within constructivist views on teaching and learning.
[3] See http://www.joensuu.fi/lenni/modemeng.html. The software to illustrate MODEM-framework in detail can be downloaded freely from the appearing link.

The need of quasi-systematization

Emphasizing the genesis of heuristic processes and students' skills to develop intuition and mathematical ideas can hardly be reached without a thorough planning of the problem to be posed and studied – inside and outside the classroom. For this, empirically tested more or less systematic pedagogical models can be helpful. The above-mentioned *MODEM-framework* offers the basis for the planning of learning environments and assessment in systematic way, even though the authentic learning activities are usually far from any systematic approach. When planning a constructivist approach to the mathematical concepts under consideration, the focus is on the *concept orientation* and finding relevant attributes for *concept definition*. On the other hand, when offering students opportunities to construct links between representation forms of the concept, the focus is on the *identification, production* and *reinforcement,* which describe the stages of the quasi-systematic concept building. In learning situations, however, students must be able to choose the problems that they want to learn within continuous self-evaluation instead of relying on express guidance from teachers. The experiences of the *ClassPad project* show (see Eronen and Haapasalo 2006) that this can be realized by organizing different kinds of task types to form a "problem buffet", for example. To go for linear function, one student team initially picked quite a complicated problem series on optimizing mobile phone costs. After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set, which happened to consist of identification tasks – the very lowest level in the systematic MODEM framework. This example shows that sophisticated interplay between a systematic approach and minimalism is possible.

Example of a successful instrumentalization

The following example illustrates the simultaneous activation of conceptual and procedural knowledge: the *Geometry Link* –operation of the ClassPad calculator (see http//www.classpad.org) carries a manipulation between the algebraic and geometric windows without any drag-and-drop maneuvre. By applying minimalism, Eronen & Haapasalo (2006) gave students at 8th class opportunity to study voluntarily 9th class mathematics with this device during their summer holiday. This totally new tool was shortly represented to them just few days before their summer holiday. The only duty was to write a portfolio if they worked with the tool. The following sample, taken from the portfolio of a quite average student, shows that she was able to utilize modern technology in sophisticated way on her freetime. She moved from instrumentation to instrumentalization without any tutoring from teacher's side. Note that when varying the constant, the lines does not go up and down (as been taught in school) but moving horizontally. Thinking graphic solution of an equation, for example, this is even more relevant interpretation.

```
6th session on 15th of July 2005. Time 00.27
· I draw a line (cf. a in Figure 1). When drag-dropping, the equation of the line is y= 1.613x-0.5992 (b).
· By changing the equation to y=2x-0.5992 the angle between the line and y-axis is getting smaller (c).
· By changing the equation to y=1x-0.5992, the angle between the line and y-axis is getting bigger.
· I change the equation to y= 1.613x-0.4. I don't see any changes.
· I change the equation to y= 1.613x-4, the line moves to the same direction away from origin (d).
· When changing the equation to y= 1.613x+4, the line moves in the same way, but to another direction on x-
axis with equal distance from the origin.
· I will continue in the morning. Time is now 01.42. I worked 1 h 15 min.
```

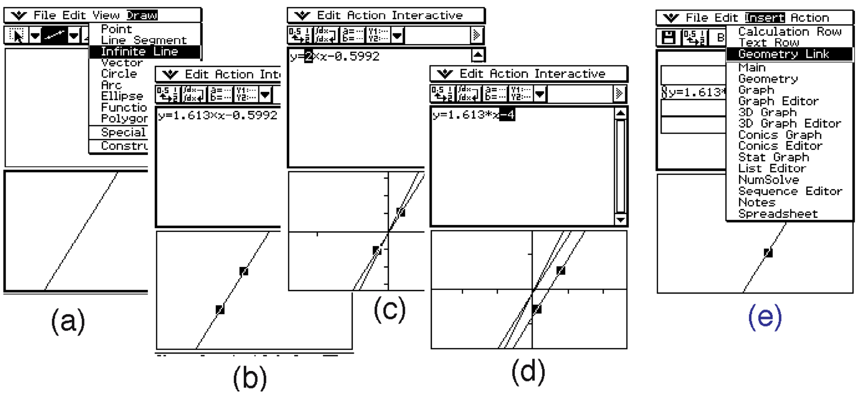

Figure 1. Utilizing SA-method through drad-and-drop technology (a-d) or Geometric Link (e).

Cognitive and affective shifts within instrumentalization

ClassPad project (see Haapasalo and Eronen 2006 or Haapasalo 2007, pp. 3-5) used Zimmermann's idea to model mathematical activities as an octagon and to quantify each activity as follows: the distance of the activity from the center tells us how strong a student thinks this activity is represented by each of the following three *profiles* (see Figure 2 later):

• *Math-profile:* How strong each activity appears to the student when using the term 'mathematics';
• *Identity-profile:* How good the student thinks he or she is performing each of the activities;
• *Techno- profile*: How suitable a computer is in performing each of the activities.

The results suggest that doing mathematics with ClassPad, even during a short period of time outside the classroom, enlarged student's mathematical identity within these motives and activities. As we see from the example above, student's portfolios reveal sophisticated metacognitive skills[4] on students' way to instrumentalization. The paths of student teams, which selected different kinds of problem types from the "buffet" revealed that students did not utilize the quasi-systematic MODEM framework in an optimal way. They often selected tasks more or less randomly. However, ClassPad work without any textbooks within interaction between minimalism and systematization was successful concerning students' cognitive development. Students scored in all test items significantly better after the ClassPad working than in the pre-test. They also showed remarkable procedural skills not only connected to the linear function but to other function types. A postponed test, after 5 months, revealed that this scoring level remained consistent, and for many students it even improved. Students liked the feeling that they had reached action potential, which was described to be one of the main aspects in assessment within minimalism. They also liked the learning without any pre-set goals or tutoring from teacher's side.

To illustrate the Zimmermann- profiles, I pick up a case, when two students worked as peers but radically differed in their learning styles. The conceptually-oriented student (i.e. she wants to come up with procedural thinking but aiming to understand) acted as a peer-teacher, whereas the second one who mainly listened, was classified as a procedurally-bounded learner (i.e. stays on procedural actions). At the beginning, the mathematical identity of the learner was even richer than that of the teacher, especially in relation to finding and argumentation. The more the peer-teacher explained, it was only her own mathematical profile that showed some tendency to expand but only towards the direction of play. This, however, is not as interesting as what happened to both the students' individual self-confidence to do mathematics. After the work, the learner seemed to believe she could do more: find, apply and argue. However, surprisingly, the self-confidence of the peer-teacher seemed to collapse in almost all dimensions.

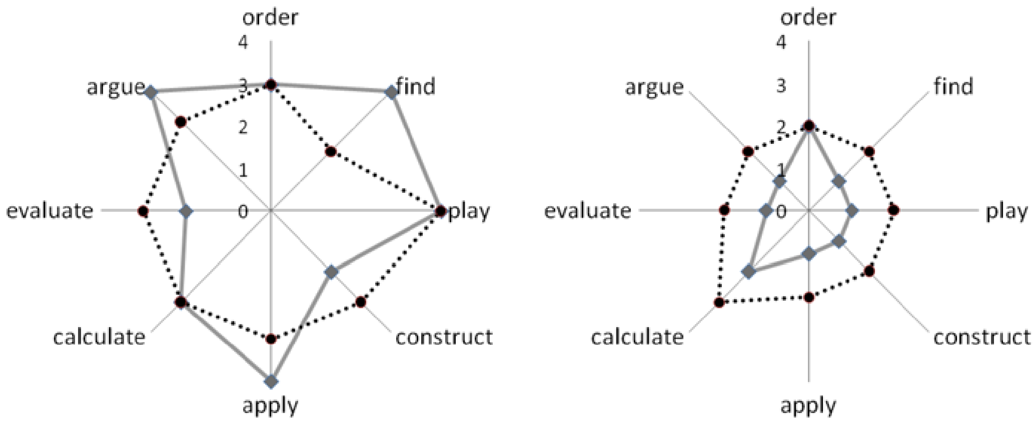

Figure 2. Identity profiles of the peer-teacher (on the right) and her classmate (on the left) at the beginning (dashed) and at the end of the ClassPad working.

[4] Haapasalo (2007) reveals degenerated metacognitions among teachers and students when working with a CBL – program in school.

Closing remarks

I would like to invite the reader to recall how the eighth Zimmermann components are implicitly represented in each of the chapters of this paper. As many researchers emphasize, learners may view technology as a master, servant, partner or an extension of themselves. The instrumentation of a sophisticated impersonal device to instrumentalized personal device is often a long process of instrumental genesis. This forces educators to re-consider the interaction between process and object features of mathematical knowledge. At the same time, to make problem-solving devices to pedagogical tools, intensive co-operation is needed among hardware and software designers and researchers of mathematical learning processes. There is a basic conflict between conceptual and procedural knowledge: how much students should understand before they are able to do, and vice versa. Concerning technology-based learning, the first challenge arises from the structure of the topic to be learned, whereas the other is caused by the instructional variables required for technology use (cf. Haapasalo 2003). Kadijevich et *al.* (2005) highlight the respect of the following two requirements: (1) when utilize mathematics, don't forget available tool(s); when make use of tool, don't forget the underlying mathematics; and (2) to solve the assigned task, use, whenever possible, a process approach as well as an object approach, working with different representations. These demands can be utilized by scaffolding the learning process if (and only if) the teacher has fundamental know-how of the relation between conceptual and procedural knowledge. I hope that I managed to highlight some important attributes of the multi-causal evolution of technology-based orchestration, hopefully resulting in better grounds for further, more detailed examinations of this important topic.

References

Burton, L. (Ed.) (1999). *Learning Mathematics: From Hierarchies to Networks*. London: Falmer Press.

Böhm, J., Forbes, I., Herweyers, G., Hugelshofer, R. & Schomacker, G. (2004). *The Case for CAS*. T[3] Europe & Westfälische Wilhelms-Universität Münster. Retrieved 17 July 2006 from www.t3ww.org/cas/

Confrey J. & Costa S. (1996). A Critique of the Selection of 'Mathematical Objects' as Central Metaphor for Advanced Mathematical Thinking. *International Journal of Computers for Mathematical Learning*, **1**, 2, 139-168.

Drijvers, P. (2000). Students Encountering Obstacles Using a CAS. *International Journal of Computers for Mathematical Learning*, **5**, 2, 189-209.

Duffy, T. M., Lowyck, J., Jonassen, D. H. & Welsh, T. M. (Eds.) (1993). *Designing Environments for Constructive Learning*. Springer, NY.

Eronen, L. & Haapasalo, L. (2006.) Shifting from Textbook Tasks to Mathematics Making. In T. Asunta & J. Viiri (Eds.) *Pathways into research-based teaching and learning in mathematics and science education*. University of Jyväskylä. Department of Teacher Education. Research Report **84**, pp. 75-92.

Gjone, G. (2004). Process or object? Ways of Solving Mathematical Problems Using CAS. *Teaching Mathematics and Computer Science*, **2**, 1, 117-132.

Haapasalo, L. (2003). The Conflict between Conceptual and Procedural Knowledge: Should We Need to Understand in Order to be Able to Do, or vice versa? In L. Haapasalo & K. Sormunen (Eds.), *Towards Meaningful Mathematics and Science Education.* University of Joensuu: Bulletins of the Faculty of Education **86**, pp. 1-20

Haapasalo, L. (2007). Adapting Mathematics Education to the Needs of ICT. The Electronic Journal of Mathematics and Technology, **1**, 1. Internet: http://www.radford.edu/~scorwin/eJMT/Content/Papers/eJMT_v1n1p1.pdf http://www.radford.edu/ejmt

Haapasalo, L. & Kadijevich, Dj. (2000). Two Types of Mathematical Knowledge and Their Relation. *Journal für Mathematik-Didaktik*, **21**, 2, 139-157.

Haapasalo, L. & Silfverberg, H. (2007). Technology Enriched Mathematics Education. In E. Pehkonen, M. Ahtee, J. Lavonen (Eds.) *How Finns Learn Mathematics and Science*. The Netherlands: Rotterdam, Sense Publishing, pp. 163-180.

Kadijevich, Dj., Haapasalo, L. & Hvorecky, J. (2005). Using Technology in Applications and Modelling. *Teaching Mathematics and its Applications*, **24**, 2-3, 114-122.

Lambrecht, J. J. (1999). *Developing Employment-Related Office Technology Skills (MDS-1199).* Berkeley: National Center for Research in Vocational Education, University of California. Internet: www.tc.umn.edu/~jlambrec/technology/index.htm

Lokar, M. & Lokar, M. (2001). CAS and Slovene External Examination. *The International Journal of Computer Algebra in Mathematics Education*, **8**, 1, 23-44.

Rittle-Johnson, B. & Koedinger, K. (2004). Comparing Instructional Strategies for Integrating Conceptual and Procedural Knowledge. In D. Mewborn, P. Sztajn, D. White, H. Hiegel, R. Bryant & K. Nooney (Eds.), *Proceedings of the Twenty-fourth Annual Meeting of the North American Chapters of the International Group for the Psychology of Mathematics Education.* Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education, pp. 969-978.

Rittle-Johnson, B., Siegler, R. & Alibali, M. (2001). Developing Conceptual Understanding and Procedural Skill in Mathematics: an Iterative Process. *Journal of Educational Psychology*, **93**, 2, 346-362.

Schwarz, B., Dreyfus, T. & Bruckheimer, M. (1990). A Model of the Function Concept in a Three-fold Representation. *Computers & Education*, **14**, 3, 249-262.

Trouche, L. (2004). Managing the Complexity of Human/Machine Interaction in Computerized Learning Environment: Guiding Students' Command Process through Instrumental Orchestrations. *International Journal of Computers for Mathematical Learning*, **9**(3), 281307.

van der Meij, H. & Carroll, J. M. (1998). Principles and Heuristics for Designing Minimalist Instruction. In J. M. Carroll, (Ed.) *Minimalism Beyond the Nurnberg Funnel*. Cambridge, MA: The MIT Press, pp. 19-53.

Zimmermann, B. (2003). On the Genesis of Mathematics and Mathematical Thinking - a Network of Motives and Activities Drawn from the History of Mathematics. In L. Haapasalo and K. Sormunen (Eds.) *Towards Meaningful Mathematics and Science Education*. University of Joensuu: Bulletins of the Faculty of Education **86**, pp. 29-47.