On mathematics implications of the incorporation of computer tools

Douglas Navarro Guevara
Universidad Nacional de Costa Rica
/Universidad de Costa Rica
navarro.douglas@gmail.com

Abstract

We take into account some non-obvious implications of the implementation of computer tools. For this, we consider the results of the some experiments about the incorporation of a software tool for automatic search of the sum of certain types of power series.

We provide success rate data of students participating in the experiments according to the use of the usual methodology, or the computer facilities. We show the change between the usual mathematical approach and the approach developed for the automatic processing. Then, we consider the point of view of the students, who are immediately ready to change to the tool.

Finally, we consider the situation where the context allows users to believe that the mathematical approach studied in the classroom is the approach used by the software, and so, to believe that the approach is relevant when this is not the case. We also consider the relevance of the preparation the students for the development of the new approaches that eventually leads the incorporation of informatics tools.

The subject of the implications of the implementation of computer tools has received several fundamental remarks from researchers, we remind for example: the "*Non didactical neutrality*" (concerning the incorporation of computer tools) highlighted by Artigue (1998), or else, "*The transfer of credibility to the machine*" highlighted by Trouche (1996).

Here we consider the disagreement that is present among the mathematical approach taught and the mathematical approach implemented in the software used thereon inside and outside the classroom. This discrepancy takes us to the notion of ¨cognitive discrepancy¨ with which we will refer at the circumstance where the actors (teacher and students) now only the traditional mathematical approach and believe that this is the approach pertinent when is not the case.

We consider also the need of the new approaches to continue making mathematics that using the expanding capacity of machines. That takes us to the notion of ¨non mathematical neutrality ¨.

So, we want to present some results of the doctoral thesis (Navarro, 2006) developed by the author around the implications of the use of computer tools in the mathematics teaching / learning process.

For the research work in this thesis we have chosen as subject of study the treatment of the power series (particularly the search for the sum), this because the usual method for dealing with this issue is based on heuristics and it liked possible to make a computer program that could solve this in a significant number of cases. So the first phase was dedicated to developing a methodology for automatic processing and make some programs capable of performing the calculation of the sum, of the convergence radio and some other properties to the types of series usually involved. Then, it was possible to beginning to define experiments devoted to studying various aspects related methodologies that would suggest to students.

Subsequently we present briefly the main experiments and some of the results obtained, then take back the main line of work for this article, *i.e.,* the presentation of the two concepts that we want to propose.

1 A study case: Treatment of power series

We present some results of experiments[1] conducted at the University of Costa Rica (San Jose, Costa Rica) between 2002 and 2003 with two groups of students from the school of mathematics and two groups of students from the Faculty of Engineering.

Brief description of the experiments

Essentially experiments consisted of propose to the students three methods for finding the sum of some power series, namely, through the use of methodology based on heuristic methods usually studied in class, the use of a automate operated as a “black box” and finally using manually a small part of the methodology developed for the design of the automate.

1.1 Regarding methodologies considered

Then we give some details about the methodologies considered.

The usual methodology

The methodology usually used in the classroom for the research of the sum of a power series in term of usual functions is based on a heuristic trial done with tools from a certain "toolbox". This toolbox have between others the following resources: a table of developments in power series of the usual functions (*sin*, *cos*, *exp*, *ln*, *arctan*, etc.); the separation of the series into two new series; the inclusion of missing terms; the variable changes; the re-indexations; the derivation or integration term at term. While, the trial sequence is a research sequence consists of a certain order for the application of the tools, *e.g.*: first try to identify changes in the sign, as shown by the developments of some usual functions; then see if the term a_n has a n or $n!$ in his denominator to identify what are the usual functions concerned; then try to recognize the series of usual functions interspersed among the proposed series; etc.

Methodology for automatic processing

We make a brief introduction to the approach used for automatic processing. The automatic processing accurate at first of a finite representation that is why we have identified two types of series, which include most of the exercises of seeking the sum of a power series:

Type I : $\sum_{i=0}^{\infty} b_i \frac{x^i}{i!}$, ***Type II*** : $\sum_{i=\max\{0,-k\}}^{\infty} b_i \frac{(i+k)!}{i!} x^i$ where b_i is a periodic succession and $\boldsymbol{k} \in \mathbf{Z}$

Having made that choice, treatment focuses on the manipulation of the generator sequence of the succession b_i. Initially this sequence is viewed (via the respective homeomorphism) as a vector in space R^n with n equal to the length of the sequence. So we obtain the first tools for identify the sum of a series. In the experiments, they were showed to the students through the following example in which the determination of the sum of the series were reduce to a simple change of based in the correspondent space:

[1] In preparation for this experience, previously we were conducting a pre-experimentation in the Paul Sabatier University at Toulouse, France.

Exemple:
Let's the serie:

$$S = \boxed{6} + \boxed{10}\frac{x}{1!} + \boxed{8}\frac{x^2}{2!} + \boxed{12}\frac{x^3}{3!} + \boxed{6}\frac{x^4}{4!} + \boxed{10}\frac{x^5}{5!} + \boxed{8}\frac{x^6}{6!} + \boxed{12}\frac{x^7}{7!} + \ldots \approx (6,10,8,12)$$

Knowing that the ensemble {ch(x),sh(x),cos(x),sin(x)}≈{(1,0,1,0),(0,1,0,1),(1,0,-1,0),(0,1,0,-1)}is a base of R^4. Find the sum of the serie on question.
So, the problem propose is reduced to find the writing of *(6,10,8,12)* in terms of vectors of this base.
Then, we obtain:

7 (1,0 ,1,0)≈ch(x)
11 (0,1,0, 1)≈sh(x)
-1 (1,0,-1,0)≈cos(x)
+ -1 (0,1,0,-1) ≈sin(x)
(6,10,8,12)

So $S = 7\ ch(x) + 11\ sh(x) - \cos(x) - \sin(x)$

1.2 Regarding experiments

Experimentation has been composed of four activities. In the first three, they ask students to resolve some exercises on the search for the sum of certain power series. In each case, the exercises were treated with different approaches (using the same exercises without putting this fact in evidence).

In the first activity, they suggested the following exercises to be worked out from the usual methodology.

For the following series found their respective sum:

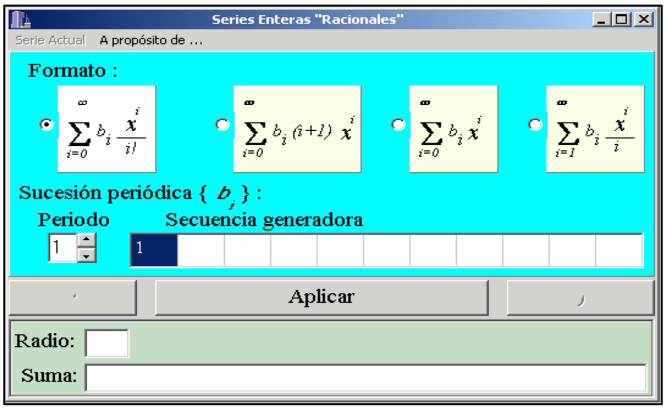
$$i)\ S(x) = 1 + x - \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!} - \frac{x^7}{7!} + \ldots$$

$$ii)\ S(x) = 1 + x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \ldots$$

$$iii)\ S(x) = -x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} + \ldots$$

$$iv)\ S(x) = 5 + 12x - 7\frac{x^2}{2!} + 13\frac{x^3}{3!} + 5\frac{x^4}{4!} + 12\frac{x^5}{5!} - 7\frac{x^6}{6!} + 13\frac{x^7}{7!} + \ldots$$

In the second part, they supply a program specially developed for this activity which helps to determine the sum of some types of power series (all of *Type I*, and for *Type II* which with k =0, 1 or -1).



Picture of the proposed program

In the third stage, they explain the argument the change of base and ask[2] to use this method to search the sum for the exercise number *iii*.

Application of the linear algebra at problem [continuation]
Knowing that…
The ensemble $A_2=\{-\frac{1}{2}\ln(1-x^2), th^{-1}(x), -\frac{1}{2}\ln(1+x^2), arctg(x)\}$
$\approx\{(1,0,1,0),(0,1,0,1),(1,0,-1,0),(0,1,0,-1)\}$
is a base for the space of this type of series which the generator sequence has longer 4. Using the methodology proposed (linear combination of members of the based A_2) to search the sum of the following series

iii) $S(x) = -x-\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}-\frac{x^5}{5}-\frac{x^6}{6}+\frac{x^7}{7}+\ \dots \approx (1,-1,-1,1)$

1.3 Brief description of the results

Below, there are summaries some the results of the most significant exercise, the exercise *iii*.

Exercice *iii*	Usual Methodology	Black box	Methodology for the automatic processing
Group 1	100%	100%	100%
Group 2	29%	86%	88%
Group 3	0%	79%	64%
Group 4	0%	84%	83%

It can be felt as the automate intervention cause much more uniform results than those obtained with the usual methodology, but so are those obtained with the manual application of the methodology developed for the automate.

1.4 Some notes about the students' work

Through the analysis of the tests we found many aspects of great interest with respect to the implications of the employed methodologies, then write down a couple of them[3].

Imprecision of the usual methodology

One interesting aspect in the students' work was the fact that the usual methodology tried to be imprecise, for example, in the sense that it admit multiple solutions for a given exercise.

$$-arctg(x)-\frac{1}{2}\ln(1+x^2)$$

$$-\ln(1-x)-arctg(x)-th^{-1}(x)-th^{-1}(x^2)$$

$$-\int\frac{1+x}{1+x^2}dx$$

$$-\ln(1+x^2)+arctg(-x)+th^{-1}(-x)+th^{-1}(x^2)+\int\frac{1}{1+x}dx$$

Different solutions to exercise *iii* encountered by students

[2] During the development of activities some questions were also submitted to students.

[3] In the thesis work, a detailed analysis of diverse aspects can be found. In such analysis, an important resource was the methodology of “changement de cadres” of Douady (1986).

High sensibility

We also found that this methodology is very sensitive to small changes in the exercises. For example the exercises: *i*, *ii*, *iv*

$$\begin{array}{|l|}\hline i)\ \mathrm{S}(x)=\cos(x)+\sin(x) \\ ii)\ \mathrm{S}(x)=ch(x)+\sin(x) \\ iv)\ \mathrm{S}(x)=-\cos(x)+6ch(x)+12.5\sin(x)-0.5sh(x) \\ \hline\end{array}$$

are all, linear combinations of the same functions, but when they are treated with the usual methodology has been taken very different results, as a minimum of 88% for the first two to 4% for the latter.

1.5 Students point of view

There are some comments from the students once showed the results of the automatic processing:

Students from UPS[4]:

- With the computer it is more simple and effective.
- I prefer to use the computer, there is no point in doing it by hand.

Answers for the UCR students to the question:

Do you want to know how the software got the answer?

- Because the software has a more mathematical calculation (because it gives a symbolic response). Myself, I would be really interested to know how it does something that I can do myself.
- No, because when you know how to do it, it is not worthwhile to know how others do it.

We can see how students generally take a very positive position about the use of computer tools.

2. Cognitive discrepancy and non mathematical neutrality

Subsequently we present a little more detail of the methodology developed for the automatic processing with the aim of showing the distance between these methodology and the usual methodology. To then raise more specifically the notions of ¨cognitive discrepancy¨ and ¨non mathematical neutrality¨.

2.1 Methodology developed for the automatic processing

We present as illustration the full processing of exercise number *iii*.

$$S(x) = -x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} + \ \dots$$

The procedure for the resolution focuses on simplifying the sequence generator. It should be noted that in this case, we consider the generator sequence as: $\overline{1,-1,-1,1}$ instead of $\overline{-1,-1,1,1}$.

Consider then the following operators:

ChB : Operator of base changing, change the canonical base to base 1,0,1,0), (0,1,0,1), (1,0,-1,0), (0,1,0,-1)}.

Ctr : Operator of contraction, when the numbers do not void that the ranks that are multiples of *p*, it removes the ranks of non *p* multiples.

[4] During the pre-experimentation.

CSgA : Operator alternating sign changing of the sequence.
Spl : Operator simplification to a minimum sequence.
Derv : Operator of circular shift (derivation term at term).

Solution to the exercise *iii*:

$ChB(S_{[-1,\ \overline{1,-1,-1,1}]}(x))=$	$S_{[-1,\ \overline{1,0,-1,0}]}(x)=$	$Ctr(S_{[-1,\ \overline{1,0,-1,0}]}(x))=\frac{1}{2}S_{[-1,\ \overline{1,-1}]}(x^2)$
		$\frac{1}{2}CSgA(S_{[-1,\ \overline{1,-1}]}(x^2))=\frac{1}{2}S_{[-1,\ \overline{1,1}]}(-x^2)$
		$\frac{1}{2}Spl(S_{[-1,\ \overline{1,1}]}(-x^2))=\frac{1}{2}S_{[-1,\ \overline{1}]}(-x^2)$
		$\frac{1}{2}S_{[-1,\ \overline{1}]}(-x^2)=-\frac{1}{2}\ln(1+x^2)$
$+$	$-S_{[-1,\ \overline{0,1,0,-1}]}(x)=$	$-Derv(S_{[-1,\ \overline{0,1,0,-1}]}(x))=-S_{[0,\ \overline{1,0,-1,0}]}(x))$
		$-Ctr(S_{[0,\ \overline{1,0,-1,0}]}(x))=S_{[0,\ \overline{1,-1}]}(x^2)$
		$-CSgA(S_{[0,\ \overline{1,-1}]}(x^2)=-S_{[0,\ \overline{1,1}]}(-x^2)$
		$-Spl(S_{[0,\ \overline{1,1}]}(x^2)=-S_{[0,\ \overline{1}]}(-x^2)$
		$-S_{[0,\ \overline{1}]}(-x^2)=-\frac{1}{1+x^2}$
		$-\int\frac{1}{1+x^2}-arctg(x)$

Thus, the mathematical approach has been a significant change, in fact, it becomes systematic. Once the development carried out, is very simple to build arbitrary examples that would highly complex to be solved with the usual methodology but which are immediate with the use of the automate.

2.2 Cognitive discrepancy

We propose the term ¨cognitive discrepancy¨ to represent the situation that is leaving students in a teaching context that allows to believe that the approach discussed in the course is the approach used by the tool when it is not the case.

In the case study above mathematical approaches used to determine the sum of a power series are significantly different, however, into the comments of the students is possible to detect the influence of his prior knowledge on the use of the usual methodology when judging the appropriateness of the methodology of the automate. We repeat two of his comments (before knowing anything about the methodology applied by the automate) where there is this kind of bias:

> ¨ *I would be really interested to know how it does something that I can do myself*¨.
> ¨*No, because when you know how to do it, it is not worthwhile to know how others do it*¨.

If we consider that in this type of case we continue to teach nothing but the usual methodology and that even the teacher know nothing about the method used by the machine then we have some important drawbacks: First, we work on false and then with the machine the students may have some answers that may escape the usual method. So, we consider very inconvenient the possibility that students could have access to answers from a machine, which are out of reach of knowledge of professor, because of discrepancy between the mathematical approach known and the mathematical approach used for the development of the tool.

A very important example where this phenomena arise is the searching of the primitives underscored by Philippe Elbaz-Vincent in (Guin and Trouche, 2002) where we continue teaching methods dated back of Newton when the methods have changed tremendously (Bronstein, 2000).

2.3 No mathematical neutrality

In the case considered of processing power series, the incorporation of tools means, as we have seen, a significant change to the mathematical approach of the subject. In fact, it has changed not only the representation if not actually working methods (particularly focuses on simplifying the sequence generator using the concerned operators).Therefore, we consider in such circumstances that the incorporation of computer technology is not mathematically neutral.

In fact, this mathematical non neutrality more than change the effectiveness on the subject could mean deep changes on the possibilities of the topic in question. Thus, for the long-term incorporation of computers in teaching and research on mathematical we must consider that at least for a part of future trade, it will need a deeper understanding of the possibilities of algorithmic methods to make more effective the exploiting power of computers but it probably will require a better understanding of the nature of computable functions[5].

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[5] In the computer sense of the term (Hennie, 1977).