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U.S. Latino Students' Thinking and Communication on National Assessment Educational Progress (NAEP) Measurement Items

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This study explores Latino[2] students' thinking when solving NAEP measurement problems via task-based interviews. Our study is grounded on a combination of sociocultural and cognitive perspectives in which multiple resources were considered when students represented their mathematical ideas. Utilizing Brenner's (1994) framework, we analyzed video-taped interviews with a focus on students' communication ***about*** *and communication* ***in*** *mathematics as they solved the problems. We discuss themes that emerged from the data, some challenges that students experienced, and the richness in their thinking and communication across two assessment items. General findings indicate that students who were successful were clear in the process that they used to arrive at the solution and could justify their thinking when probed. Further, the students generally knew the mathematical terms that were part of the problem such as 'perimeter, square, and area.' Successful students were proficient in translating between the various representations of diagrams, verbal and symbolic representations. Through these task-based interviews, we found that most students were resourceful in their use of tools and problem-solving strategies and in drawing on previous knowledge (communicating* ***in*** *mathematics), but could not always clearly express their thinking* ***about*** *the mathematics.*

This exploratory study seeks to complement large scale studies that have been done in the U.S. on the National Assessment Educational Progress (NAEP) (e.g. Abedi & Lang, 2001, Lubienski, 2003). The purpose of this study is to shed light on Latino students' thinking and communication as they solve measurement problems from NAEP (1996). Lubienski (2003) reported that in the 2000 NAEP, the greatest gap between white and Hispanic students occurred in the measurement strand. Our effort was to gain a better understanding of how working-class Latino students thought about some of these NAEP measurement items. The goals of this study were: (a) to uncover challenges that selected NAEP measurement items raise for a group of 19 Latino students; (b) to understand their interpretations of the problems; (c) to understand their reasoning and communication of their solutions; and (d) to investigate the role that language plays in their thinking process, especially if the students' first language is not English.

Theoretical Perspectives

This study is part of a larger research agenda that looks at the interplay of mathematics, language and culture among Latino students. Our perspective is essentially a combination of a sociocultural perspective

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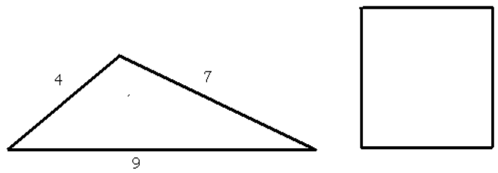
[22] We use the term *Latinos* to refer to the student population in the U.S. whose origins are of Cuban, Mexican, Puerto Rican, South or Central American, or other Spanish culture regardless of race as defined by *The Oxford Encyclopedia of Latinos and Latinas in the United States*, 4 vls, Oxford University Press 2006.

and a cognitive perspective (Brenner, 1998; Civil, 2006; Cobb & Yackel, 1996). Our student interviews were cognitively-based, and our analysis of these interviews was guided by a sociocultural perspective of mathematics cognition and language (Moschkovich, 2002) with an emphasis on students' communication *about, in*, and *with* mathematics (Brenner, 1994). Because mathematics curriculum and teaching standards in the U.S. have come to reflect a model of mathematics learning that emphasizes discourse and communication (NCTM, 1989, 2000), it is important for us to understand the implications for students. Moschkovich (1999) states that in reform-oriented mathematics classrooms, students are no longer grappling mainly with acquiring technical vocabulary, developing comprehension skills to read and understand mathematics textbooks, or solving standard word problems. Instead, students are now expected to participate in mathematical discourse practices (Gee, 1992), such as explaining solutions, processes, describing conjectures, proving conclusions, and presenting arguments.
Brenner's (1994) Communication Framework for Mathematics advocates for students to actively participate in mathematical discourse during classroom discussions. The framework provides categorization of three different kinds of discourse, namely: (a) communication ***about*** mathematics, which entails the need for description of problem solving processes and their own thoughts about these processes; (b) communication ***in*** mathematics, which entails using the language and symbols of mathematical conventions; and (c) communication ***with*** mathematics, which refers to the uses of mathematics that empower students by enabling them to deal with meaningful problems. Brenner emphasizes that all three kinds of mathematical communication are needed in the classroom for developing useful mathematical understanding. The third kind, communication with mathematics, was not part of our coding of the data since our focus was on assessment items, which are overall context-free and carried little meaning for the students outside of school mathematics.

Communication is central to our study in that we are investigating students' use of language as they interpret the tasks and explain their thinking; in some cases, our students are bilingual (English and Spanish), but more proficient in one of their two languages. Moschkovich (2002) points out that communication is multifaceted involving gestures, expressions, drawings, and objects as resources to simultaneously communicate mathematical ideas, and they are especially crucial for students who are less proficient in English, but are being educated in the U.S. in an all English instruction classroom, even though the student may be able to use Spanish as a resource. A student's mathematical competence becomes more visible when a sociocultural perspective that allows for multiple resources from the situation to be used. Because language structure of assessment items can present added cognitive demands for the students, especially ELL students (Campbell, Adams, & Davis, 2007), it is pertinent to take into consideration the students' multiple resources for their communication of their reasoning in solving the problems. Thus, our approach focuses on the students' use of resources as they communicate ***about, in,*** and ***with*** mathematics. We advocate that effective communication is the intersection between the kinds of mathematics that are valued and the ways in which culturally and linguistically diverse students express the mathematics.

Methodology

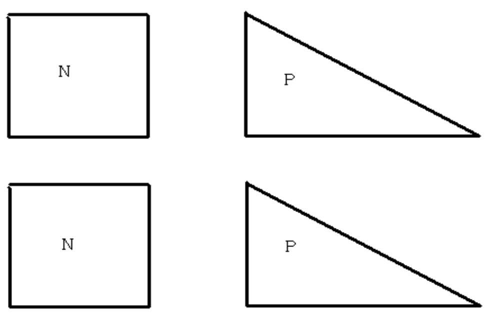
We conducted and videotaped task-based interviews with 19 Latino students in grades four through six. The students were attending elementary and middle schools in predominantly working class neighborhoods. In this paper we focus on two problems from the fourth and eighth grades NAEP assessment pertaining to measurement: (1) "Triangle and Square Perimeter Problem: If both the square and the triangle above (Figure 1) have the same perimeter, what is the length of each side of the square?" (Lengths of the triangle are given as 4, 7 and 9); and (2) "Area Comparison Problem" in which the student is given cutouts of a triangle and a square and is asked to compare their areas (Figure 2). Of the 19 students, a subgroup of 17 students interviewed solving the perimeter problem, and all 19 students interviewed solving the Area Comparison Problem. The students were interviewed in either English or Spanish depending on the students' choice of language.



If both the square and the triangle above have the same perimeter, what is the length of the side of the square?

Figure 1. The Triangle and Square Perimeter Problem

All the interviews were videotaped and conducted in English. The students first solved the problems independently and then they were asked to explain their thinking in their solutions. After they finished their explanations, we asked probing questions based on their responses. In some cases, the students' interactions with the researcher prompted them to revise their initial solution. Our interview scripts for the problems were focused on questions that would externalize the students' thinking.



Bob, Carmen and Tyler were comparing the areas of N and P. They each conclude the following:
Bob: N and P have the same area;
Carmen: The area of N is larger;
Tyler: The area of P is larger.
Who was correct? Use pictures and words to explain why. (Cut outs of N and P are given with the base of P being twice the side of the square.)

Figure 2. The Area Comparison Problem

Analyses of the data consisted of reviewing each interview by problem with an eye for the student's mathematical communication. In the next section we address general findings and then discuss the students' communication of the two problems and their solutions as they participated in discourse ***about*** and ***in*** mathematics. In communication ***about*** mathematics, we coded the students' response on their verbal description of the process that they were using and their justification of their thinking process. For communication ***in*** mathematics, we examined the students' use of definitions like perimeter, area, square, triangle, height, width, etc. and their conception of geometric figures. We coded how well the students could participate in mathematics Discourse (see Gee, 1992). In other words, we observed for the students' performance on how they made sense of the mathematical arguments and conjectures that were being discussed and participate in the discussions in meaningful ways. In the following section, we discuss student understanding and linguistic complexities of the problems in addition to student communication ***about*** and ***in*** mathematics while engaged in solving the problems.

Findings and Discussion

Linguistic complexity can stem from the written form of the problem, especially for English learners. For example, in the Triangle and Square Perimeter Problem, the problem reads, "If both the square and the triangle above have the same perimeter, what is the length of each side of the square?" One of the fourth grade students interpreted the "if" statement as "they do not have the same perimeter." When probed, she said "but they do not because it says IF (emphasis added)." This child was interpreting the "if" statement as a negation statement, therefore, the square and the triangle could not possibly have the same perimeter. Her facial expression indicated that she was faced with conflict and was not able to engage with the mathematics as the problem intended. By the student's interpretation of the written language used in this problem, it is impossible to assess her mathematical understanding of two shapes having the same perimeter. The following is the dialogue between the fourth grade student (St) and the interviewer (Int).

Int: Okay. Now read the first part of the problem again, what does it say?
St: If both the square and the triangle above have the same perimeter, what is the length of each side of the square?
Int: So does that triangle and that square, do they have the same perimeter? (pause)
St: No.
Int: And what does the problem say, the first part?
St: If both the square and the triangle above have the same perimeter...
Int: Okay, stop. What does that mean? What are they telling you?
St: That if both of them have the same perimeter what is the length of each side…
Int: So are they supposed to have the same perimeter?
St: Well, yeah.
Int: According to the question, are they telling you that they have the same perimeter? [student nods head no]
Int: No, how is it? Read that part again.
St: If both the square and the triangle above have the same perimeter.
Int: Okay, stop right there, if both triangle and the square have the same perimeter, are they telling you that they have the same perimeter? [student nods head no]
Int: No, how come they're not?
St: Because they saying IF both...
Int: What does that mean?
St: That means what.. like.. if .. if both of them have the same perimeter.
Int: So, are they saying if both of them, meaning they don't have the same perimeter, but if they did? That's how you interpret the question that they're asking? [student nods yes]

*Communication **About** Mathematics.* The students were able to describe the process when asked to explain their thinking. In a very few cases the students guessed and could not provide a reason. However a majority of the students could provide a reason for their work, even if this reasoning was not complete. They were also able to convey their ideas to the researchers either on their own or with some probing. For example, one sixth grade student used the formula for the area of the triangle and rectangle to conclude that the triangle had a smaller area since it was divided by two. The following is the dialogue that transpired on the Area Comparison Problem with a sixth grade student.

St: Well I thought that Tyler was correct because to find the area of square you have to use the formula of length times width and then for a triangle you have to use the same one but you'll have to use length times width divide by two so ah..

Int: For the… for which one the triangle?

St: Yeah for the triangle.. and so if you have to divide the area of a triangle in half then the triangle is half of a square or a rectangle that's why I would say the square is... has the...has ah...larger area than the triangle..

Here we observe that the student was comfortable with the mathematical terms and was effective in communicating her thinking process to the researcher.

In communicating their solution, some students confused area and perimeter conceptually in their explanations. For example, the students thought that the area of the square was larger than the triangle in the Area Comparison Problem since there were four sides to the square and the triangle only had three. In the Triangle and Square Problem there were students who counted the number of sides or corners when asked to find the perimeter. Two sixth grade students tried to enclose the triangle in a box in the Triangle and Square Problem, and their work indicated that they were trying to find the area instead of the perimeter. In most cases when asked about the meaning of area and perimeter, the students who got the problem incorrect could state that the perimeter was the outside of the shape and the area was the inside.

The students that were successful in explaining their solutions and reasoning through the problem were able to realize higher level thinking and make connections during their discourse ***about*** mathematics. One student in particular, on the Area Comparison Problem, reasoned that the area of the triangle and the rectangle was the same by assigning numbers to the unknown lengths in a way that accurately reflected the relationships (e.g., one side being twice the length of another).

Some students relied on the drawings as their key source of information, yet these were not drawn to scale. A few students requested a ruler for finding the lengths of the sides for some of the figures. Regarding the first problem (finding the length of one side of a square with the same perimeter as a triangle), most students were able to arrive at the correct solution, but those that did not arrive at the correct solution relied on visual approximations. In this same problem on perimeter, some students who were incorrect relied on the use of grids to find the perimeter, and understandably, this strategy can be tied to reform curriculum that emphasizes meaning making, but in this example, we found that such drawings by the students interfered with their reasoning. In fact, this reliance on the grid deserves further attention, as we found it to be not always helpful and part of the confusion with area and perimeter that we address next (also, see Kamii & Kysh, 2006, for their insights on counting squares on a grid to find area of shapes).

Communication ***In*** *Mathematics*. The students who were successful with communication ***in*** mathematics were able to better understand the mathematical ideas through, not only the language, but through the diagram and the symbolic notation of the problem. In the case of the Triangle and Square Problem, they knew how to calculate the perimeter of the triangle and then use this value as the perimeter of the square and comprehend that the triangle and the square had the same perimeter. The students also knew that the square had four equal sides and hence divided the 20 by 4. Further, in the Area Comparison Problem, the students knew the area preserving property and were adept at showing that the square and the triangle had the same area if a cut and paste operation was carried out.

We found that the students who were successful in solving these problems were fluid in their translations between mathematical representations. In another example, a successful sixth grade student simultaneously represented his manipulations of the concrete shapes in problem (2) with rectangles and with the equation 2P=2N ("P" represented the triangle and "N" represented the square). He reasoned that if 2P=2N, then half of each rectangle is the triangle P and the square N, therefore, P=N and the areas were equal. Below is the dialogue with the sixth grade student engaged with the Area Comparison Problem.

[The student takes the two triangles together to form a rectangle and the two squares on top of that and Looks up at interviewer]

Int: Okay, so you put these two triangles together…that's interesting, and then what?
St: I put these squares over it so I can measure, see if they are the same (okay) so they were the same (okay) [plays with the shapes that he made]
(thinks then writes)…two Ps equals two Ns
Int: Can you tell me…I mean this is a very nice…you know algebraic expression …can you tell me what it means?
St: Okay, two of these which are Ps [holds up triangle] equals two of the Ns which are squares [holds up the square] because as you can see, you put these together…like that [the triangles]. If you put these together, it makes a rectangle, and if you put these on top, then it makes the…the squares [repeats making the two triangles], put the squares on top, and it's the same size.
Int: Okay…same size, okay, so that's nice, so 2P=2N, so how can you use that information if you can …tell me about the area of one N and one P?
St: They are the same.
Int: And why?
St: Because…because it's like if you are saying…like if you were to cut this in half [the rectangle], if you cut this off right here [puts the triangle and the square together and then indicates a cutting motion with his fingers], you could put it right here [points to where the extra part of the triangle would fit into the square], (okay) and if you were to do that then you could do it on this side too [holds up the other triangle and square] this one…which are the same [brings the things that he is holding in his hands together] so they are the same size (okay) …well N and P are the same area.

This sixth grade student was able to conclude that N and P have the same area through his manipulation of the cut-out shapes and symbolic notation that aided in his thinking. In general, we found that successful students were able to explain their reasoning with more linguistic precision than those who struggled, who tended to use lots of pronouns with no clear referents.

In one particular case, a sixth grade student was being queried about his method in solving the Triangle and Square Perimeter Problem. His initial solution was "4." Below is the dialogue between the interviewer and the sixth grade student.

Int: Okay do you want to explain your thinking?
St: Ahh, I added, I added, ahh 4, 7 and 9, I got 20 and then I divided 20 by 4 and I got (hesitates) I got…oh, I messed up [erases his choice of 4]
Int: Oh, you are changing your answer.
St: Yeah I messed up on this one.
Int: Why did you change it? What happened?
St: Because its 20 the the [indicates all around the square] the (unclear) whole perimeter of the square is 20 so I know umm, there are four umm…divided by 20 equals 5, so it's 5 for each side.
Int: Okay. Tell me about the math sentence that you just mentioned…umm…you added this and got 20 [points to the triangle] and then tell me what you did over here?
St: I, umm, divided 20 by 4.
Int: Oh, okay and 20 divided by 4 …

St: Equals 5.
Int: Equals 5. Okay, I see what you did. Umm, initially when you got the 4, what were you thinking? Did you just …
St: Divided by 5 instead of 4; I messed up.
Int: Oh okay, and how did you know to divide by 4?
St: Well because I know that 4 times 5 equals 20.
Int: Okay, umm, how did you know to take the 20 and divide it by 4 ?
St: Because there are 4 sides.
Int: Oh, what do you know about the sides of a square?
St: It has 4 …4 equal sides.
Int: Ahh …equal, okay, I see your thinking.

The student showed a good understanding of participating in mathematical discourse. Even though he made an error, he was able to rectify his error independent of the researcher and knows the proper terminology for the terms that he is describing. He is also able to justify the steps that he took in solving the problem.

Closing Thoughts

Proper mathematical discourse (Gee, 1992) was displayed in the interactions with students who successfully solved the two problems. The students who were successful were clear in the process that they used to arrive at the solution and could justify their thinking when probed. Further, we found that, in general, the students knew the mathematical terms that were part of the problem such as 'perimeter, square, and area.' Successful students were proficient in translating between the various representations of diagrams, verbal and symbolic representations.

Examining the overall results from the interviews, the linguistic complexity was an issue that seemed to be prominent with this group of Latino students, especially in the case of the Triangle and Square Perimeter Problem. The students struggled with the language of this particular problem and this, in turn, seemed to impact the students in using visual strategies to solve the problem. There was a variety of visual approaches that were used by the students that either consisted of 'seeing' that the sides were equal to using grids or by looking at the numbers of sides or corners in a square or a triangle.

The Area Comparison Problem was more accessible to the students and there were fewer issues of linguistic complexity observed. This could be attributed to the lower complexity in the language used and the presence of cut-outs. These shapes helped mediate the students' thinking and communication with the researcher.

Through these task-based interviews, we found that by asking the students questions about their thinking, most were resourceful in their use of tools and problem-solving strategies and in drawing on previous knowledge (communicating ***in*** mathematics), but could not always clearly express their thinking ***about*** the mathematics. Our findings indicate that most Latino/a students were able to participate meaningfully in mathematical discourse to some degree in communicating their solutions and conveying their thinking through various resources once they were probed and encouraged to share their thinking.

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