Reform Mathematics in Bilingual Classrooms:
Flexible Thinking and Language Development in Spanish-Speaking First Graders

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Paper prepared for ICME, the International Congress on Mathematical Education
July 6-13, 2008
Monterrey, Mexico

Abstract

An exploration of young Spanish-speaking Mexican immigrant students' mathematical problem solving in bilingual classrooms shows students' ability to successfully engage in rich problem-solving tasks as required by the Equity Principle in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000). This study of eight first graders continues a problem solving emphasis begun in kindergarten. The study was guided by sociocultural theory that promotes the idea children come to school with the psychological and linguistic tools they need to make sense of mathematics, and the reform approach to mathematics education that emphasizes learning through problem solving. Researchers used problems based in Cognitively Guided Instruction (Carpenter, et al., 1999) to qualitatively examine students' problem solving and communication. Findings showed that students were learning mathematics with understanding as demonstrated in their flexible approaches to solution strategies and the varied representations of their answers. Findings also

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[1] This research was supported by the National Science Foundation, under grant ESI-0424983, awarded to CEMELA (The Center for the Mathematics Education of Latino/as). The views expressed here are those of the author and do not necessarily reflect the views of the funding agency.

demonstrated students' sophisticated ability to explain their mathematical thinking in their first language (Spanish) and second language (English).

Introduction

As young children move from spontaneous home environments into classrooms, their informal numeric activities are replaced by structured mathematical tasks. When the tasks are built around word problems located in familiar contexts, students have the opportunity to make connections between what they know informally about numbers and the formal mathematical concepts they are learning. The aim of this research is to probe deeper into student thinking and communication when they are engaged in formal problem-solving tasks in a Spanish language learning environment.

Many of today's mathematics curricula are linked closely with a reform movement based on the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000). These standards *begin* with the Equity Principle that requires *all* students to have opportunities to engage in quality classroom learning experiences to help them develop a deeper understanding of mathematics (NCTM, 2000). Unfortunately, equal access to quality mathematics experiences is not a reality for many students who live in poverty, come from immigrant communities, and/or speak a dialect or native language other than standard English (Jordan, Kaplan, Oláh, & Locuniak, 2006; Kamii, Rummelsburg, & Kari, 2005; Lubienski, 2000; National Assessment of Educational Progress [NAEP], 2005; Ortiz-Franco, 1999). When these students under-perform, they are placed in classrooms that stress drill and practice at the expense of more conceptually challenging tasks, further removing them from meaningful mathematics experiences (Kamii et al., 2005). Because research has found that many of these same children enter kindergarten and first grade with limited counting skills, limited experience in explaining their thinking, and reduced standard English vocabulary (Jordan et al., 2006; Lubienski, 2000), the notion that they need to build both counting and language skills first has persisted.

While some research has shown that indeed Spanish-speaking Latino students and students from poor communities can fully engage in complex mathematical processes and that their general counting and discursive skills can improve through problem solving activities (Kamii et al., 2005; Villaseñor & Kepner, 1993; Turner, Celedón-Pattichis, Marshall & Tennison, in press), very little research has shown exactly how Spanish-speaking immigrant students engage in these tasks in their native language. This study comes at a time when a perceived achievement gap is being blamed on Spanish language instruction, specifically bilingual education (Collier & Thomas, 2004; Escamilla, Chavez, & Vigil, 2005). For this reason, it is important that educational research demonstrate the potential Spanish-speaking immigrant children have in mathematics when they are given access to high quality teaching and learning activities in their native language.

Theoretical Framework and Review of the Literature

The research presented in this paper focuses on the exploration of young Spanish-speaking students' mathematical thinking through what they do and what they say during problem solving in bilingual classrooms. The emphasis on problem solving and communication comes from reform mathematics and is influenced by sociocultural theory. Particular emphasis is given to sociocultural theory because of its argument for a strong relationship between language and cognitive development (John-Steiner & Mahn, 1996). This research is also informed by social constructivism in mathematics education and the research conducted by Cobb and Yackel (1996) and McClain and Cobb (2001) that explored the development of mathematical discourse and the use of representations as tools for learning in elementary classrooms. It is beyond the scope of this paper to argue for bilingual education. However, the importance of bilingual education, and especially early native language mathematics instruction, is an assumption that underlies this research and is supported by sociocultural theory.

The reform movement in mathematics education has resulted in a shift from the teaching and learning of isolated skills and procedures to a focus on conceptual development and learning with understanding (Hiebert & Carpenter, 1992; NCTM, 2000). Learning with understanding engages students more deeply in the processes of mathematics while solving complex problems. Reform-minded researchers and educators, working together, were instrumental in publishing the influential first edition of the mathematics standards in 1989, followed by the revised *Principles and Standards for School Mathematics* (NCTM) in 2000 where the important areas of content knowledge and process skills for K-12 mathematics education are laid out. Five key processes are emphasized across the content areas including, problem solving, communication, reasoning, connections, and representations.

Sociocultural theory is based on the work of Vygotsky (Kozulin, 1990) and argues that children's psychological and cognitive development is a dynamic process between the social and the individual planes, and that learning occurs when external social experiences are internalized and transformed (John-Steiner & Mahn, 1996). Sociocultural theory supports the notion that humans begin life as intrinsically social beings. All our early thoughts and experiences are incorporated into our innate cognitive structures by the communication we have with our caregivers. As children grow, these early dialogues are internalized and become the basis for understanding new information. In this way, language and culture mediate cognitive development and new information is continually interacting with the old to expand knowledge and refine its structure. Schooling imposes a systematic structure into which students can incorporate their informal knowledge and expand both informal and formal knowledge through connections and generalizations. For young children engaged in the cognitive demands of formal mathematics, there is no doubt that new ideas and concepts are best learned in the native language if these formal concepts are to give structure to the spontaneous concepts developed at home (Van der Veer & Valsiner, 1991).

Problem solving is emphasized in this study. Problems that draw on students' knowledge of the world and provide mathematical contexts for real life situations help students build connections among mathematical ideas. Carpenter et al. (1999) have done extensive research with young children and problem solving and have developed a framework for understanding how children approach problems. Drawing on sociocultural theory, they argue that even young children have a wide range of experiences and knowledge to help them construct their own solutions to word problems in a supportive social learning environment. As a framework to guide teachers, Cognitively Guided Instruction [CGI] (Carpenter, Fennema, Franke, Levi & Empson, 1999) lays out basic problem types and students' strategies. CGI problem types were used as a basis for student activities in this research.

Communication is a second process emphasized in reform mathematics. When students learn to describe their thinking and justify their answers, the result is a greater understanding of their own thinking and reasoning (Cobb et al., 1996; NCTM, 2000). Additionally, when students discuss their solutions out loud, other students have access to their thinking and can borrow these strategies for their own problem solving. From a sociocultural perspective, the source of learning lies in social contexts where students' internal understanding and structuring of ideas is based on the external structures they encounter through social interactions (Sfard, 2001).

Research Questions

In our research we ask, how do first grade Spanish-speaking students communicate their mathematical thinking in their native language during problem solving? What strategies do they use to solve problems? How do they represent the problem situations in drawings? How do they talk about the mathematics of the problems?

This research is based on the assumption that using familiar contexts to engage students gives them the opportunity to explore mathematics from their own linguistic and psychological perspectives. The research also assumes that students' mathematical thinking will be more transparent when the confounding effects of language and culture are reduced.

Description of the Case

La Joya Elementary School[2] is located in a large urban area in the Southwestern United States and serves children from predominantly Mexican immigrant and low-income families. La Joya responds to the educational needs of its 86% Spanish-speaking student population with its kindergarten through fifth grade maintenance bilingual program promoting bilingualism and biliteracy for these students. Data for this study were collected in two first grade bilingual classrooms at La Joya where mathematics instruction was conducted by native Spanish-speaking teachers exclusively in the Spanish language.

The eight first graders in this study were all participants in research during their kindergarten year where CGI was used as a framework to teach problem solving (see Turner et al. in press for a complete description of this study). Six of the students are

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[2] La Joya is a pseudonym.

female and two are male. The eight students were chosen for this study because of their experiences the previous year and because their teachers expressed interest in developing students' problem solving abilities. All eight students speak Spanish as a first language. One student, Omar, is dominant in English, and two, Ana and Gerardo, are bilingual in Spanish and English. The other five students, Dolores, Gina, Briza, Yolanda and Jenna, are English Language Learners. Teachers' formal and informal assessments of these students place them within a range of mathematical ability from low to high.

The research team consists of the first author, a doctoral student and native English speaker, and the other two authors who are university faculty researchers and native Spanish speakers. The research team has been conducting studies around CGI problem solving at this site for two years, exploring both student thinking and long-term teacher professional development.

Design, Methodology, and Analysis

This is an exploratory study of students' mathematical thinking during problem solving based in grounded theory (Creswell, 1998). To promote the development of problem solving strategies, the research team suggested weekly math lessons based on CGI. A typical lesson in each classroom began with an introduction to the problem type in a whole group setting by the teacher. After the introduction to the lesson, teachers and researchers continued the work facilitating small groups. Facilitators asked students to draw their problem solutions and explain their thinking. Data collected came from both in-class problem solving and individual student problem solving interviews. Field notes were taken weekly in the classrooms and student participants were individually interviewed in November 2006 and May 2007 using twelve different CGI problem types (See Carpenter et al., for a list of all problem types). All student interviews were videotaped.

The mathematical word problems given to students during classroom lessons and individual interviews were all based on problem types defined by Carpenter et al. (1999). These problems were carefully developed to reflect contexts that would be familiar to the focal students. Problem types included join, separate, compare, part-part-whole, multiplication, partitive division, measurement division, and multi step problems. For example, a measurement division problem would ask: You [the student] have 18 cookies and some bags. You can put three cookies in each bag. How many bags of cookies can you make?

Analysis of the data included a detailed examination of students' use of language in relation to problem solving, student work from both the classroom and individual interviews, and classroom field notes. Analysis began with the selective open coding of trends across all the data (Emerson et al. 1995) for the eight focal students. After open coding, axial coding (Creswell, 1998) was used to consolidate codes, create categories and develop themes. Themes revealed the variety of ways students approached the problems, the strategies they used, and how these connected to their explanations. Finally, selective coding led to the development of themes influenced by the theoretical connections outlined in the literature review.

Findings

While students' quantitative success in problem solving is not the focus of this study, it highlights valuable information on students' overall mathematical achievement and the effectiveness of problem solving activities. By the end of the school year, the focal students were able to successfully solve even the most challenging problem types for their age group including addition and subtraction problems with the starting value unknown, compare problems with a referent number unknown, and part-part-whole situations (Carpenter et al., 1999). According to Carpenter et al., these problems are challenging for students because they are difficult to directly model. Conversations with the teachers confirmed that these problem types were not present in the first grade curriculum. Results for the November 2006 individual interviews showed an average of 82% correct solutions for the focal students, and in May 2007 they averaged 91% correct solutions. Twelve problem types were used in these interviews including, join and separate, part-part-whole, multiplication, comparison, partitive and measurement division, and multi-step.

A major theme that emerged from the analysis of data showed that students were approaching problems in various ways and demonstrating flexibility in their thinking. The flexibility that students showed in strategies and representation showed they were learning mathematics with understanding (Hiebert & Carpenter, 1992). A second theme demonstrated that the language they were using to explain their thinking was increasingly sophisticated and reflected the way they were thinking about the problems. The following examples demonstrate their problem solving and communication abilities.

Gina applies base ten thinking. An interaction recorded in field notes with Gina showed her ability to apply base ten thinking to larger numbers. Transferring successful strategies from smaller numbers to larger numbers is challenging for first graders (Spelke & Tsivkin, 2001). The first author asked Gina the following question: You had 67 pieces of gum and your friend Reinata had 76. Who had more? How many more? Gina wrote the numbers on her paper, thought for a few seconds, and then said "nueve [nine]". When asked why it was nine, she said because if it was 10 more then her friend would have 77 pieces of gum, so it had to be nine. When I asked her about who had fewer pieces of gum and how many fewer, she knew immediately that if her friend Reinata had nine more she would have nine fewer. In this example, Gina dealt with large numbers by applying what she knows about the patterns and relationships across the number system. After writing 67 and 76 on her paper, she was able to recognize the pattern and realized that if she added 10 to 67 she would have 77. Hence, her answer had to be nine.

Ana and Omar show flexibility of thinking in their representations. Omar used coin notation to solve a comparison problem between 31 balloons and 18 balloons. He wrote "d" for dime (10), "N" for nickel (5), and "P" for penny (1). He wrote 18 as "dNPPP" and 31 as "dddP". He even expressed his answer to how many more balloons 31 is than 18 in this way, writing 13 as "dPPP." In a problem with three boxes of crayons with ten crayons in each box and an additional 17 crayons, Ana drew three squares with the number "10" in each, five sets of tally marks, and 2 single lines. To find how many crayons in all, she counted by tens, then by fives, and finally by ones to get 47. Similar to Omar's method, Ana was able to express the answer using coin notation, writing "DDDNNNPP." Although tally marks, coin notation, and counting by tens had been encouraged in their classroom curriculum, Omar and Ana applied these representations to the novel problem situations the research team introduced during classroom visits.

Brisa solves seven bags of ten marbles plus an additional six single marbles. In this problem Brisa used base ten thinking skills to recognize that she could simply count by tens for the bags and then count by ones to add in the single marbles. In her drawing she included all the marbles in the bags, not as an aid to problem solving, but possibly as an aid to visualization. Her reasoning is clear in her explanation as she relates her problem solving strategy to the structure of the story. Because there were ten marbles in each bag, she counted the bags by ten and then counted in the six single marbles. She said, "Porque puse diez…había…Ashley tenía siete bolsitas de canicas, en cada bolsita tenía diez. Y conté de diez en diez hasta 70 y habían seis sueltas y le [sic] conté 70 hasta 76. *[Because I put ten…there were…Ashley had seven bags of marbles, in each bag she had ten. And I counted by tens up to 70 and there were six singles and I counted from 70 up to 76.]*

Gina solves how much money Marian needs to buy a doll that costs $18 when she only has $9. Gina chose interlocking cubes to find her solution. As she manipulated the cubes to make 18 with a group of ten and another group of eight, she saw that she could remove one cube from the ten rod, add it to the eight rod to have two groups of nine. From this model Gina saw that if Marian has nine dollars, she needs nine more to make the $18 she needs to buy the doll. Gina explains, "Porque ella tiene nueve… como si le ponen con los demás es diez, y le van a quedar ocho, y como ella tiene nueve le quité uno y se lo puse aquí, y ahora le quedan nueve. *[Because she has nine…because if they put it with the rest it is ten, and they are left with eight, and because she had nine and I removed one and put it here, and now she is left with nine.]*

Yolanda uses mental math to subtract 15 cookies from 35 cookies. In this example, Yolanda is focused on manipulating the numbers and employing her knowledge of grouping and counting by fives. Yolanda has demonstrated a sophisticated number sense in past interviews and it appears that she has a clear numerical image in her mind about the number relations involved in this problem. This may be why she does not need to use cubes or paper and pencil to find a solution. She uses her fingers only as an aid in keeping track of her counting. Her explanation focuses on the nature of the number relationships instead of the structure of the problem. She says, "Usé los dedos y puse 35 menos diez es treinta…oh ((shakes her head)) menos cinco es treinta y menos otra cinco es veinticinco, y menos otros cinco es veinte. [*I used my fingers and put 35 minus 10 is 30…oh minus five is 30 and minus another five is 25 and minus another five is 20.]*

Gerardo uses a diagram to find the answer to how many marbles his friend gave him if his friend started with 12 and wound up with five. Gerardo made a clear connection between his diagram of 12 circles and the way he explained his solution strategy in English. He said, "Because when I put twelve, um, I started counting them and where I got to five, right here where I started, ((indicating drawing), I put a line and then I keeped [sic] going until over here so I could know how much did he give me and then I started counting the ones that he give me." During this explanation, he showed how he counted five of the circles, made a line to separate the five from the rest, and then counted these remaining lines to get seven. This example shows the value of students developing modeling strategies to aid in both problem solving and explanations of their reasoning about the problem.

Conclusions

The above data show that students have the psychological tools they need to tackle challenging problems and the linguistic sophistication to both comprehend problem situations and describe their thinking and solution strategies. Problem solving that encourages students to explain their actions and justify their solutions gives young children the opportunity to build both conceptual knowledge and mathematical discourse. Students' access to their native language in this process clearly gives them an advantage they would not otherwise have if they had to make sense of both a second language and new mathematical ideas.

In addition, the data show that students' problem solving strategies and the ways they communicate their thinking is rich and varied. Students are moving beyond direct modeling to find solutions and are using the more advanced strategies of counting, recalled facts, derived facts, and trial and error which show an increasing sense of number (Carpenter et al., 1999). Students are using a variety of representations to make sense of the problems and also to help them communicate their thinking more clearly. Students' oral communication is a critical step in learning for understanding and helps students organize and consolidate their thinking about the problems they have solved (NCTM, 2000).

Implications and Recommendations

The implications for these findings relate directly to bilingual education and equity in mathematics. We see that Spanish-speaking first grade students learning mathematics in their native language gives them access to their full range of social and cultural experiences to make sense of new concepts. These students from Mexican immigrant, Spanish-speaking families are impressive problem solvers. Bilingual education and the opportunity to learn early mathematical concepts and number sense through challenging problem solving have provided these students with an equitable learning environment.

Because students are thinking about problems in a variety of ways and communicating their thinking in the strategies they use, the way they talk about the problems, and the representations they create, assessment needs to take into account multiple forms of expression to better understand how students are thinking about problems. Students need multiple exposures to concepts and repeated opportunities to practice with a variety of challenging problem situations so that their growing knowledge base can become both flexible and secure (NCTM, 2000).

Questions for Further Study

This study has only begun the exploration of students' mathematical thinking within bilingual contexts. Further analysis should probe more deeply into the language students are using to explain their thinking, how this mathematical discourse is developing over time, and how the discourse relates to the strategies students use to solve and represent problems.

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