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MODEL FOR RESEARCH ON MULTICULTURALITY IN MATHEMATICS EDUCATION

1.- Introduction. Objectives.

2.- Model for the Study of Multiculturalism in Mathematics Education. Fundamental Components of Intercultural Mathematics Education. Fundamental Categories of the Model.

3.- Model for the Study of Multiculturalism in Mathematics Education: Object Model (MUMEMO). Conceptualisation, subject matter of field research.

4.- Instrumental components of the model. Categories for the analysis and the production of publications. Formal Categories . Protocols. Conclusions.

5.- References and Bibliography.

1.- Introduction. Objectives

In this document we present a paper which has, as an objective, to guide the reflection and at the same time structure knowledge, relative to Multiculturalism related to Mathematics Education. This is the scientific field object of analysis of the group STG 33 of the ICME 11.

We try to produce a theoretical construct capable of including all the papers that can be qualified as belonging to this field and that permits the study of these papers at different levels. At the same time facilitate the production of new papers both experimental and of field work, such as theoretical studies that develop this line of research within the scientific Area of Mathematics Education.

We also hope that this model permits to analyse, relate and group together the papers sent and conceptually organise the work sessions of the group STG 22 of the ICME 11 while it is being held in July 2008 in Monterrey.

Another objective of the paper that we have carried out by producing a Model for the analysis and classification of the production, and by carrying out its later application in the study of certain productions, is to collaborate in producing “A State of the Art “ about “Multiculturalism and Mathematics Education” that can be further developed in the future by the those researchers interested in doing so.

The genesis of the model and of the study have the same origin, both starting from an initial exploration of different types of documents and which the readers (a group of Master students, working alongside M^aLuisa Oliveras, their “Ethnomathematics” teacher), considered dealt with topics related to the nucleus of our

study: Multiculturalism related to mathematics education. This episode has formed Phase 1 of the work and was carried out between October and December 2007, in the University of Granada, (Spain)

This initial work has permitted me to make certain verifications and reflections, from which some of the instruments for the analysis have arisen. Afterwards, I continued with the work analysing the findings from new points of view, carrying out a structuring, formulating fundamentals and articulation and by creating a model which I present to follow. This period consisted of Phase 2 of the work.

When the model was produced, I try to use it in the study and thus classify two groups of products

a)- Some publications previous to the ICME11 and

b)- All the papers presented at the TSG33 of the ICME11

By doing so, this gives rise to an incipient “State of the Art” as Phase 3 of the work in which other researchers, the members of the Group ST33, have collaborated.

To conclude, we try to create certain instrumental constructs and generate a theoretical framework which helps to develop this field of study.

2. Model for the Study of Multiculturalism in Mathematics Education. Fundamental Components of Intercultural Mathematics Education. Fundamental Categories of the Model

The General Conference of The United Nations Educational Scientific and Cultural Organization, proclaims the declaration of The Principals of International Cultural Cooperation, in which the first article states:

“Each culture has a dignity and value which must be respected and preserved. Every people has the right and duty to develop its culture. In their rich variety and diversity, and in the reciprocal influences they exert on one another, all cultures form part of the common heritage belonging to all mankind.”

However, we often think that this has no bearing on mathematics classes, at any age level, perhaps because of our subconscious absolutist epistemological concept of mathematics, or because we are not aware of the Ethnomathematic perspective (D’Ambrosio 1985, 2006; Oliveras 1996). This perspective, which allows for multiple “living” mathematical approaches (Oliveras 2006), favours a mutual enriching of both majority and minority cultures, as a basic principal of intercultural education. Furthermore, mathematics teachers will teach their science alongside, and by means of mutual intellectual appreciation, tolerance and solidarity.

To this effect, jointly educating people from different cultures is not to favour a possible ‘assimilation’ of the minority, but rather to work in order to achieve a common cultural space which does not cause a loss of identity. It is difficult to achieve this completely without discovering and accepting cultural mathematical practices of minority groups.

Interculturality. Culture and Multimathematics

Let us consider interculturality as: a combination of practices which are generated through the interaction of cultures in a relationship of reciprocal exchange and from a perspective of safeguarding the cultural identity of the participants. This concept is generally confused with multiculturalism, and yet they are different. The latter denotes solely the juxtaposition of various cultures within a society, but without mutual implications.

A wide concept of culture (Oliveras 1996; 2006) leads us to opt for, in multicultural education, an *intercultural* education, which includes valuing cultural pluralism, maintaining individual cultural identity, and considering the cultural mix as a way to general enrichment. Intercultural education from our perspective, includes actions aimed at all students, not only those from minority cultural backgrounds, and its foundation lies in the awareness of the presence of different cultures and a recognition of the particular ways in which each culture develops different areas of knowledge and in particular mathematics.

According to our point of view: *culture* means the way each community is, the way it interprets the world and how it fits into that world. Culture is about values, behaviour, roles in society, that is to say with elements which are changing, dynamic and interacting with reality. It is also about symbolic systems of expression and thought communication, hence, culture includes mathematics.

At first, multiculturalism was thought of, as far as social and educational politics is concerned, as the need to adapt immigrants into the society which has adopted them (assimilation) thereby losing their cultural identity. We have gone from this approach, to understanding the ideal multicultural position as valuing cultural pluralism, officially recognising diversity and accepting the need to maintain cultural identity of minority groups.

The intercultural perspective goes one step further, it starts from an approach whereby the basis is interaction and exchange. Interculturality requires the possibility of affirming one's own culture in relation to other cultures. This is only possible if a process is established whereby everyone contributes and when these contributions are accepted, valued and ultimately united as one, and understood by all as the contents of exchange. I call this "*cultural miscegenation*", an enriching process of change in which we all contribute and we all receive, generating a new cultural being which is stronger and more versatile and with which we all identify because it has been created by all of us.

"Multimathematics", fruit of and the very root of interculturality, makes us reconsider our cultural relationships within an educational and social project which assumes that equal rights and opportunities are possible amongst those who live together in this global and yet individual society.

This is our bold and rich proposal regarding both science and society. We do not fear the future of plural mathematics in which the most beautiful and efficient mathematic thinking will be born that is both differentiated and common at the same time.

Intercultural Education

Education is a social action whose objective is to enable adults to continue and develop a society's own culture whilst fulfilling themselves as people individually immersed in that culture.

This social action is very complex and is an enormous responsibility which is given to teachers, as experts, and to parents or family as the basic social core.

Currently, audiovisual media: internet, t.v., cinema and press, plays a very important role in education given that parents have little time to educate their children and school cannot function beyond the limits of its timetable or its own property except by helping parents and pupils to look critically at what the media presents as part of culture.

When a society is multicultural, education is even more complicated as the question arises as to which culture should be developed alongside education?

I believe that in multicultural societies one should endeavour to enrich some cultures through coexistence with others. A new culture should be generated, a hybrid of the cultures which coexist, a common heritage of all the citizens who live in a territory.

I believe that this involves all of us, given that all education centres are multicultural to some extent because of the presence of multiple cultures, be it on a macro or micro scale. We have various "macrocultures" in those cases where immigrant students or groups of cultural minority students live together, and "microcultures" in which all cases are different given that there will always be groups of students with different characteristics due to their diverse socio-economic backgrounds, their differing ages, interests etcetera which give rise to what I call a "microculture".

How can mathematics education intervene in the multicultural society? On the one hand we must use the general components of education and on the other hand, the specific epistemology of mathematics. This means that we must know and use Models for Education and Curriculum which favour an intercultural education and an epistemological concept of plural mathematics, which recognises and values informal, implicit and living mathematics in all people of all age groups.

The two main components of education are the Education Models and the Curriculum. Regarding education, both should be different according to whether the society of a country is monocultural or multicultural.

The *Education Models* can be summarized as those actions carried out by the teacher and pupils in what is known as "the class" in the school system. They also determine the objectives and general educational aims and they determine the methodology and the resources in order to achieve specific learning, in our case mathematics.

Which Education Models should be used in order for citizens to achieve adequate personal fulfillment in a multicultural society?

If society or the politicians responsible for education have a racist attitude, the answer will be: The dominating culture or that of the majority should be maintained without being 'adulterated', and the Educational Models will be "**Monocultural**".

If society or its education system is not racist, then the answer will be: Educational Models must be “**Multicultural**”, taking into account cultural diversity.

Existing “Multicultural” Education Models are varied and can be classified as follows: “**Intercultural**” and “**Non Intercultural**”.

We call “non intercultural” those which accept pluralism, and that different cultures can coexist, but which consider that they should develop separately and not through interaction.

Those which we call “intercultural”, value cultural differences positively, and promote interaction between different groups as a way of generating knowledge, they are based on socio-constructive criticism. They aim to create a new culture, born from the interaction between the cultures that live side by side and which belong to all of the citizens that occupy a territory.

There are various types of “Intercultural” Education Models which can be placed into three groups:

“*Anti-racist*” Education Models aim to *eliminate institutional or legal discrimination*, by discovering the ways in which the differences between the groups; ethnic, economic, gender, linguistic etc. are used to maintain or establish legitimate inequalities of power and rights of citizens.

“*Transformationist*” Models want to use education to *transform society, changing values and using impartiality as a base*, which includes the right to be different.

“*Social Reconstruction*” Models propose social conscience, using education to fight for social change, to *change the control of resources, of wealth and prestige thus overcoming inequalities*. Material change as well as spiritual change or a change of values

I am in favour of any of these Education Models. Considering models for “Social Reconstruction”, those which are more favourably orientated towards a fairer world, overcoming social and economic inequalities, this model can include the proposals of the “Anti-racist” and “Transformationist” models to achieve even greater social equality.

Teachers, teacher trainers and researchers are not always aware of the education model in which they are immersed. Which of these models is most similar to the teaching methodology of the school in which I am working? This would be an appropriate question for a mathematics teacher to reflect upon, who must know the key points of each model in order to reach an answer.

We will finish our analysis of Models of Multicultural Education resuming in a table (**Table 1**) those key aspects which we feel characterise each model. These are aspects relating to the actions of the teacher, the view and objectives of the pupils, the concept of minority culture, and the time and place necessary to achieve the educational objectives.

In the first line we show “Non intercultural” models, and in the following lines “intercultural” models previously described. In the columns are the five key points we have taken for our analysis: Objectives, Pupils, Teacher, Minority Culture, Time,

(Table 1) MODELS OF MULTICULTURAL EDUCATION.				
OBJECTIVES	PUPILS	TEACHER	MINORITY CULTURE	TIME
<p>TYPE 1 CULTURAL ASSIMILATION Objective: Educate to integrate into society equality between people</p> <p>1.1. COMPENSATORY 1.2. SEGREGATED</p>	<p>Must forget their culture and adopt the dominant culture Is considered deficient, incapable (1.1.) Creates negative self image, may cause segregation (1.2.)</p>	<p>Has low expectations of the minority pupil (1.1.). No special attention in the class to pupils from minority cultures (1.1.). “welcome” classes in special minority only groups (1.2.)</p>	<p>Is considered more backward and causes disadvantages for the students. Must be cancelled out through education.</p>	<p>Medium (one year) or Long (two or more years) (1.1.) Short (2-3 months) (1.2.)</p>
<p>TYPE 2 CULTURAL PLURALISM Objective: Preserve pluralism Competence in two cultures</p> <p>2.1. COMPLEMENTARY CLASSES 2.2. INTEGRATED BICULTURAL CLASSES 2.3. BILINGUAL .EDUCATION CLASSES IN TWO LANGUAGES</p>	<p>Must acquire two cultures, the mother culture and the dominant culture, the need to be competent in both</p>	<p>Aware of the need to develop both. The mother culture outside of school time (2.1.) Culture and mother tongue within the school programme (2.2.; 2.3.). Teachers from both cultures working in the centre(2.2.; 2.3.).</p>	<p>Maintaining the mother culture, sign of identity. In educational importance it is the second in (2.1 y 2.2) and first or equal in the two cultures in (2.3)</p>	<p>Long (2.2. y 2.3.) Medium or long (2.1. outside of school hours)</p>
<p>TYPE 3 INTERCULTURAL Objective: Educate including individuals and minority groups and transforming society</p> <p>3.1. ANTI-RACIST 3.2. TRANSFORMATIONIS T 3.3. SOCIAL RECONSTRUCTION</p>	<p>Awareness of own origin, and of cultural diversity and critical comprehension of reality. Analysis and change of attitudes, beliefs, values. Overcome prejudices. Eliminate racist feelings and develop tolerance. Cooperative group learning, balanced personal relations</p>	<p>Eliminate racism. Create conscience of cultural diversity, of the value of all cultures and a critical comprehension of reality. Personal attitudes which are non- authoritarian and pacifist. Use curricular materials for meaningful learning in different cultures. Equal distribution of school achievement. Give equal opportunities.</p>	<p>The right to have a different culture is defended. The minority culture fights and demands to be taken into account.. All cultures have the same value and social importance and in the classroom the advantages of diversity are valued. Respect for the rights of individuals and peoples. Appraisal of the cultural mix And creation of a new culture agreed on by all.</p>	<p>Long And transversal, in other words in all years and subjects of the curriculum.</p>

Perspectives of the Multicultural Curriculum

First let us take a brief look at curricular approaches in order to understand them and distinguish different methods of developing the mathematics curriculum in multicultural classes. Also to place the curricular components of the model we propose.

I believe that the theoretical perspectives of the curriculum allow us to understand and put into practice the curriculum of a subject, in our case mathematics, and give the teachers an explanatory foundation of the most important part of the development of their work as teachers. It shows them the frame work in which their actions take place and it prepares them for the educational consequences that their actions may have.

I believe there are two contrasting curricular paradigms: The “*Classical Curriculum*” and the “*Critical Curriculum*”.

Classical curriculums were established in the twentieth century in nearly all countries and include various types of curriculum such as *cognitive, technological, personalized, rationalist academic, social reconstruction*.

All of these types of curriculum have a psychologically rationalist and cognitive base, typical of an objective world.

I would like to invite those readers who are teachers to reflect upon the curricular focus of their school. To facilitate this reflection, here below are some descriptions of some of the elements of each type of curriculum and some questions to think about:

- Cognitive: this is centred around the pupil and aims to develop intellectual processes and cognitive skills through an active methodology and real experiences and whereby the teacher acts as the guide or facilitator of the activity. It is the process rather than the content which is the essential factor. Both the product and the process are evaluated. The social context is considered to favour mental development.

Is this my school’s curriculum? Do we know the cognitive skills of our pupils? How is the multicultural background being taken into account, given that this favours mental development?

- Technological: The aim is to observe the behaviour of the pupils, the content is the key element, the teacher leads through teaching and instructing and uses technology, worksheets or modules which are scarcely flexible. The pupils learn through external stimulus. They are evaluated according to the contents and skills acquired

What are the outcomes of this curricular approach for the pupil and for the dynamics of a multicultural class?

- Personalized: the aim is to develop the personal potential of each pupil’s actions, skills and abilities. The pupils are the main focus and are active and participate in didactic decisions. The teacher researches the needs and interests of the pupils and employs active methodology which respects the pace of individual learning. Formative assessment, self-assessment and mutual assessment.

What needs must be considered in order to adopt this methodology in a multicultural classroom?

- Rationalist academic: The aim is the intellectual development of the pupil, who receives information and learns through listening and reading. Attention is centred on the contents, especially data, theory and systemized science information. The teacher takes the role of leader, considering him or herself as the master and communicator of knowledge, giving magisterial classes. Accumulative assessment. The community is not considered as a source for the curriculum.
- Which elements of this curriculum remain in my class? Is it compatible with a multicultural society which young people are preparing for?
- Social-Reconstruction: this is centred around the pupils, and the aim is for them to understand the issues that the community in which they are immersed is facing, and to train them as agents of social change. The teacher is a guide who actively encourages a critical approach. The contents incorporate aspects of both systematised and daily culture. The methodology favours socialization processes with techniques such as group work, analysing problems and researching. The process is assessed via formative assessment and mutual assessment. The social context is considered a key element when designing the curriculum.

Do I identify with this type of curriculum? What preparation does the teacher need, and what resources are required to develop this curriculum in a multicultural classroom?

All of these types of curriculum have a rationalist and cognitive psychological basis typically found in an objective world. Not very coherent with a relativistic epistemology of mathematics. However the *personalized* and the *social reconstruction* approaches try to go beyond the cognitive rationality of the objective world connecting education with the subjective and social world as well.

My proposal is: to participate in the latter two approaches as far as possible and to tend towards a “critical curriculum”.

I am aware that the resources: text books, audiovisuals, computers and materials which can be manipulated such as an abacus or blocks, will condition any curriculum, rendering to some extent the teacher’s intentions invalid. We would need to have the method of analysis and evaluation of resources very clear so that they do not modify our aims and in a concealed way, give out a different message in the form of a hidden curriculum.

Furthermore we have little knowledge of critical curricula, from a mathematical perspective. We believe that they are more closely related to others areas of education rather than mathematics. Let us examine the characteristics together.

The basis of a critical curriculum is a social or anthropological-social analysis of the concepts of the curriculum and of the person considered to be an integrating, productive and transforming agent of society. They have arisen as alternatives to the more conservative, former or classic ones.

From this critical perspective, curricular activity is a social activity to teach the citizens of a new, complex and plural society, an activity which will involve conflict, these conflicts are controlled by dialogue and communication.

The “Critical” curricular proposals are based on:

- Freire’s theories of education, anthropological-social contents (*El curriculum liberador*, and *El currículum como praxis*)
- Apple, Giroux, Flecha and Bernsteins theories of socio-education (*El currículo basado en un código integrado*)
- Habermas’ “Theory of Communicative Action”, as a theory of society (see explanation in *El Currículum Crítico Comunicativo*, Dona Ferrada 2001)

I will quote three critical models in this critical perspective:

- a) The “Liberating Curriculum”, (El “Currículum liberador”)
- b) The “Curriculum based on an integrated code”, (El “Currículum basado en un código integrado”)
- c) The “Curriculum as praxis”, (El “Currículum como praxis”)

These three types of curriculum share the same theoretical basis however, they can be differentiated according to eight factors typically found in all curriculum; aims, pupils, teacher, knowledge, methodology, social vision, the role of the school, conscience and change, which enable you to select one and not another in practice.

By means of the following table (**Table 2**) I will highlight the similarities and differences between the three critical curricular models.

Table 2 CRITICAL PERSPECTIVE CURRICULUMS			
CHARACTERISTICS	Currículo Liberador	Currículo de Código Integrado	Currículo como Praxis
Aims	Build society from dialogue	Integration of knowledge which breaks down the social division of work	Build a reality through meanings
Pupils	They form the core of the educational process. Their position of fundamental oppression makes them agents of social change, of value of their word and action	The pupil acquires a globalized vision of reality	Participate, active Build their own knowledge
Teacher	Equality with the pupils, no role as protagonist, also learns	Guides the learning processes horizontal communication	Participates together with pupils in building knowledge
Knowledge	Reflexive application, questioning, Is produce via interaction between teachers and pupils	Knowledge: organized in weak classification and environments of reference	Distinguishes the social from the natural. Meanings acquired through interaction
Methodology	Socialization, conversation circles	Develop processes of socialization	Participative and questioning socialization
Social vision	Social context is very meaningful, the oppressed society is liberated through speech, dialogue	Social context is important, classes should break away to create educational change	The power is in the dominant ideology, assume and change the praxis
School role	Develops critical reflection, creation, Interaction through dialogue	Break through the limits of different fields of knowledge and connect them	Political character of education For change in society
Conscience and change	Critical conscience of liberation due to encounters with people. Change: Integrates sociocultural diversity	Break through the distribution of power and social classes Changes: in the valid expression of knowledge, in social relations	Praxis (doing) as a process of building social significance. Society is changing built by those it is made up of

With reference to the “Educational Model” through which each type of curriculum can be developed, the three types of curriculum in table 2 correspond with Intercultural Education Models.

For a more in depth analysis of these curricula, new concepts are required, like *the diverse types of rationalism* of Habermas, which in some way allow us to

validate the concept of “Ethnomathematics”, appropriate for these critical concepts of society and education.

Both Mead and Habermas think that a person’s learning has two regulatory mechanisms, one internal and one intersubjective, which regulate the action. The connection between both is produced through language. In my opinion, education is the intermediary between languages.

Both authors agree, and I agree with them, about the importance of “others” in a person’s own learning. This is related to the intercultural education that we are analysing and proposing as the most appropriate way of teaching in multicultural societies.

“Becoming a person is something which develops and which emerges through experience and social activity...”

‘I’ is the organism’s reaction to the attitudes of others.

The structure of the complete person is therefore a reflection of the complete social process”

(Mead 1993, p.167-202. cited by Ferrada, D. 2001)

We have analysed education models and types of curriculum as part of the foundations of Multicultural Education, showing Intercultural Mathematic Education as the ideal which we should aim towards.

In this framework, mathematics as part of the contents of the curriculum are part of the foundations of all intercultural mathematic education. The epistemology focus has already been outlined: relativist, complexist, ethno (living). The thematic is singular in order for it to be coherent with the aforementioned foundations.

As for the selection criteria for the educational contents, the critical focus and the social reconstruction focus propose taking into consideration topics which arise from the intersection of three “theoretical worlds” in which the pupil is immersed.

the subjective world → local life domain

the social world → local life and global life domain

the objective world → global life domain

The three worlds come together in the two domains of life in which the pupil lives: the global domain or “world” of the whole human race in general, and the local life domain which refers to the cultural group that the pupil belongs to, be it town, city, autonomous community or country.

Thus, I also propose on the one hand interdisciplinary themes, which bring together the three worlds for writing books, projects and other curricular products. On the other hand, I propose nuclei or work groups to generate “contextualized mathematical sense” based upon Ethnomathematics or Multimathematics (Oliveras 1995a, 1995b, 2005,2006). These nuclei will take their inspiration from the different cultures which are present in the classroom, the pupils’ own cultures which constitute their social world, their local life, their local contexts of knowledge (Wittgenstein), but which at the same time form part of a global world.

The topics will be treated with multiple scientific intervention according to the differing materials of the corresponding curricula, from a subjective and objective perspective, which unites rationalized culture of a universal nature and local daily life culture through mathematical activities.

In conclusion, Interculturality and Society, Culture and Multimathematics. Intercultural Education and the Perspectives of the Multicultural Curriculum, are theoretical elements upon which educational and social actions relating to multiculturalism in mathematics education are built. For this reason they are considered as the foundations for our model, using the most basic elements as a guide or “Fundamental Category” for our model, these being: SOCIETY, CULTURE, EDUCATION, MATHEMATICS.

These fundamental abstract categories acquire values from the aforementioned theoretical movements: relativist, socio-constructivist and critical. The concrete categories are interlaced, creating the conceptual basis for the model, or framework of reference, which consists of:

- a. *fair Multicultural Society* or intercultural
- b. *Hybrid Culture*, of micro and macro cultures coexisting within a common context
- c. *Living mathematics in context*, cultural multimathematics, ethnomathematics
- d. *Intercultural Education and Critical Curriculum*, in mathematics education, with models of social change and curricula like Praxis which promote social interculturality, connecting with heading a, and closing the circle of a fair and just society in which mathematics are a tool of knowledge in the hands of all groups and all people.

3. Model for the study of Multiculturalism in Mathematics Education MUMEMO. Model objective. Conceptualization and subject matter in this scientific field.

We are referring to a model which allows us to investigate Multiculturalism in Mathematics Education. What do we mean by model?

According to Bachelard, the model is the intermediary between a phenomenal reality and a difficult field of study.

In the phenomenon of mathematics education being inserted into social multiculturalism, it is useful to find an intermediary to use when analysing, given that this field of study is difficult.

According to Artigues, the model is the third component of a ternary relationship. On the one hand, this component relates to reality, and on the other, to the science that this reality is researching. It is a wider vision of the model, which shares a mutual relationship with both parts, reality informs the model and the model is an eyeglass with which to be able to see this reality. Science lends the model its constructions to constitute the eyeglass and the model enhances these constructions on returning them. This comparison is useful in our case.

Bunge defines the model as a representation of reality. If it is a matter of schematic conceptual representation, we have a conceptual model. If it is a question of a physical representation, it is called an “object-model”.

In order to study Multicultural Mathematics education, some kind of representation is necessary, owing to the complexity of the topic and the many variables which intervene, because through these representations we can highlight hidden components of social and educational reality or those which are difficult to process. For this reason, we have constructed a model, a conceptual construction with a “Schematic Representaion” of the components that constitute the nucleus of the study of Multiculturality, furthermore we have chosen a physical representation of this creation or “object-model” (Bunge)

Our “object-model” is the pyramid of Chichén Itzá, with its four sides and baseline foundation, and its steps which progressively take us towards the summit. From this summit, we can see the intercultural landscape around us –amongst solid columns, snake heads and raised heads of the gods twisting around its edges. The four foundations that run up and down the four sloping sides, with two scales, one for the humans and one with steps larger than a human, for the gods. The human scale is steep, central and so high, that it represents just how far we still are from the ideal of multiculturalism.

Let us take a look at its physical representation, the pyramid of Kukulcan, the most impressive monument of the Maya ruins of Chichén Itzá (Mexico), which we have chosen as object-model for Intercultural Mathematics Education, and let us climb up it to the summit of all the problems that it contains, which are as beautiful as the monument and as hard as reality.

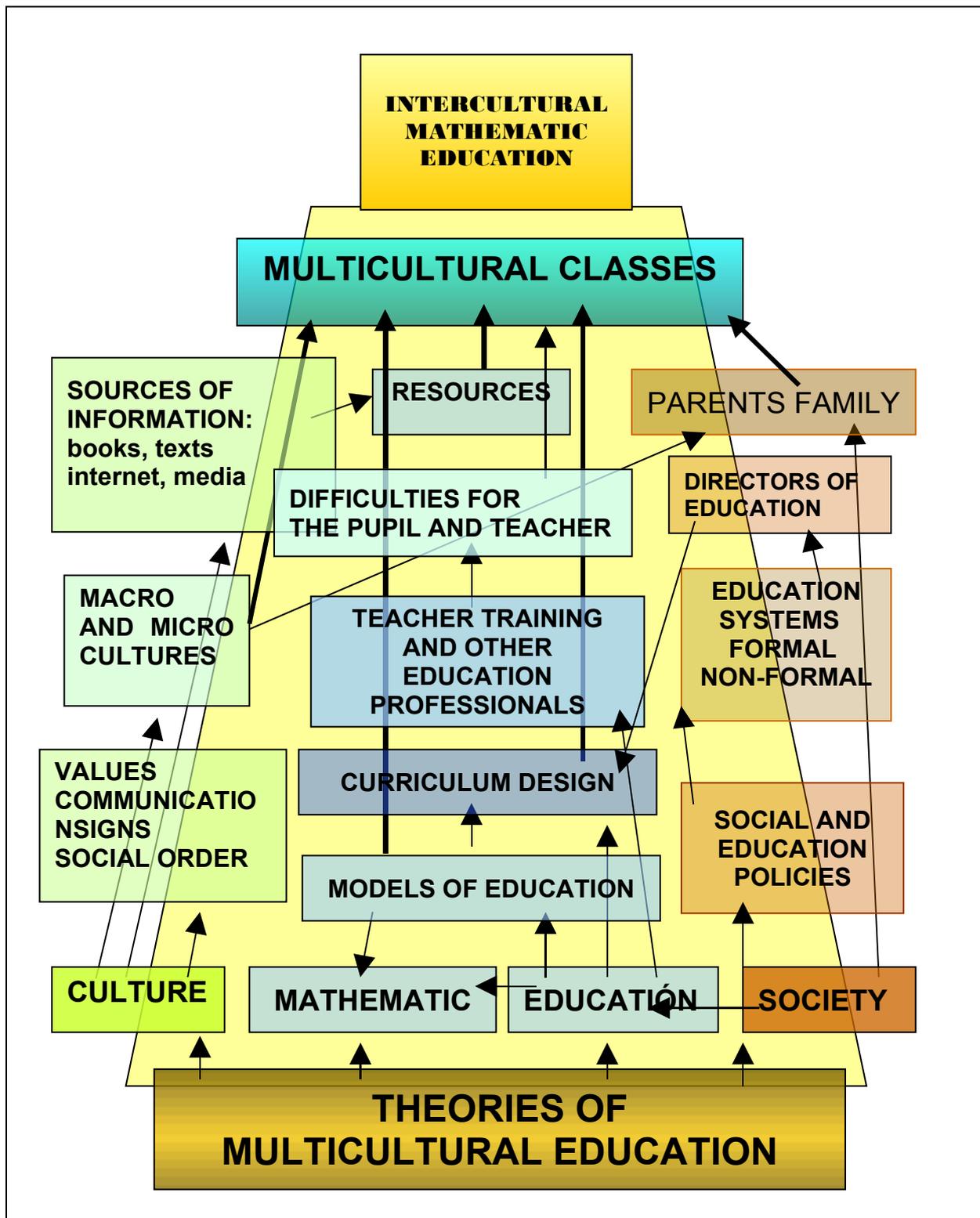
Figure 1



This pyramid with its quadrangular base, was built for astronomical purposes and it is an excellent example of astronomical architecture. It stands in the centre of a vast plain, and it is surrounded by many other monuments.

This solid structure is an appropriate metaphor for our creation of a model of analysis and classification of the products of this line which we can call **MUMEMO** “Model for Multicultural Mathematic Education”. Let us look at the schematic representation:

Figure 2



All of these words represent aspects which are to be studied in Multicultural Education. They constitute the “Fundamental Categories” of the MUMEMO Model which facilitates analysis. In the diagram, these aspects are connected via arrows which represent the relationship between real facts. They allow us to discover and understand “areas of development” of Multicultural education. They also allow us to pinpoint in these “areas” some of the problems that actually exist, making it easier to study them and find solutions.

In a practical sense of applying the model to study public works, we call it “Syntax of the Model”. The model MUMEMO, works by applying instrumental categories which allow us to identify various properties and classify in accordance with them, and to group works which are related to the categories and separate those which diverge.

Let us now examine the categories and protocol that we can use to express the characteristics of the works that have been studied and which constitute the tools of the syntax of the model.

4.- Instrumental components of the model. Categories to analyse and produce publications. Formal Categories. Protocols. Examples of analysis. Conclusions.

This model MUMEMO is a construct with which to carry out Didactics of Mathematics research of the papers published by our community of researchers, curricular organisers and assessors who are interested in the social and educational phenomena which are produced in Multicultural environments and which concern Mathematics. We are going to show the “instrumental components” of the model, which make up the syntaxis or rules and tools for the practical functioning with which we can express the characteristics of the papers studied.

They are formed by the set of **“Categories”, “Subcategories of order 1 and 2”, “Protocols” and System of Codification”**.

“System of Codification” consists of characterising each article using a bi one to one code which will enable its later treatment using the Instruments of “Categorisation, Subcategorization and Protocol”, with no need to express its complete name. The codification system is simple, using ordered pairs, giving a natural number to each document in the same order as the alphabetical order, which is established by the surname of each author, matching the number and the letter, for example: (1,A) (1st Dámbrósio, U), (2,A) (2nd Ascher, M),..., (13, O) (13th, Oliveras, ML.). This application can be shown in a table, for example:

Table 3- Table of Codification of the Papers studied

Num. paper	1	2	3	4						15	
Author	D´AmbrosioU.	AscherM.								OreyD.	
Code	(1 A)	(2 A)								(15 O)	

A list of alphabetically ordered authors is included along with this table. Here it is easy to find who corresponds to the code (n,y) since the second component of the ordered pair makes reference to the surname which are strictly put in order.

Until now we have produced several Categories and Subcategories, paying attention to different variables which are not independent but some of which can be included within others and so be taken into account at the same time. This makes the categorisation instrument progressively more specialised. The Papers can be analysed from the point of view of their fundamentals and their actions in addition to the object on which they are acting and also on which topic the action was working, and so on until “n variables”. This generates an n-tupla of characteristics of the papers analysed with the model which provides a great deal of information about these papers and permits research into them.

With respect to the variable: “Actions and their objectives”, we have defined the following categories:

INSTRUMENTAL CATEGORIES RELATIVE TO ACTIONS

“Actions carried out and their objectives”

A.- RESEARCH

A.I. Field Research:

- A.I.1-Projects or actions of formal teaching in multicultural groups
- A.I.2-Specific teacher training projects or actions
- A.I.3-Specific projects or actions of non formal education
- A.I.4-Development and analysis of resources that permit and favour intercultural education

A.II. Basic Research:

- A.II.1-Bibliographical compilations
- A.II.2-Text analysis and productions on the topic
- A.II.3-Projects and Proposals of intercultural teaching to be developed
- A.II.4-Projects and Proposal of teacher training to be developed
- A.II.5-Analysis of theories and existing models

B.- EXPERIENCES

- B.I. Descriptions of teaching experiences relative to multicultural groups-classes
- B.II. Descriptions of political-educational systems related to multiculturalism
- B.III Descriptions of teacher training programs
- B.IV Descriptions of didactic resources that favour intercultural education

C.- THEORETICAL STUDIES

- C.I. Elaboration or analysis and explanation of Theories and Models of “MULTICULTURALITY AND MATHEMATICS EDUCATION”
- C.II. Elaboration and explanation of Theories that form part of the system of Foundations of Intercultural Education

With respect to the variable ”Subjects or objects of research or actions”, considering “subjects” to be the elements about which the research has been carried out even if it does not concern people. They are the object of the research, teaching or other action studied, we have the following subcategories that we call ORDER 1:

**INSTRUMENTAL SUBCATEGORIES OF ORDER 1
RELATIVE TO THE SUBJECT:**

“The elements (subjects or objects) about which the actions have been developed”:

A-THE PEOPLE AND THE GROUPS

A.1. TRAINERS/MATHEMATICS TEACHER TRAINING, THE CULTURE OF THE TRAINERS AND THEIR VISION OF THOSE TRAINED AND THEIR CULTURES

A.2. EMIGRANTS. SECOND GENERATION, OR NATIVES WITH DIFFERENT FAMILY ORIGIN, (CHICANOS ETC.)

A.3. EMIGRANTS. FIRST GENERATION, OR BORN IN OTHER CULTURES WITH DIFFERENT ORIGINS

A.4. NEIGHBOURING COMMUNITIES WITH DIFFERENT CULTURES THAT EXCHANGE OR CLASH

B-ACTIONS THAT ARE CARRIED OUT AND DESCRIBED

B.1. REGULATED PRIMARY OR SECONDARY MATHEMATICS TEACHING

B.2. REGULATED HIGHER EDUCATION IN MATHEMATICS

B.3. TEACHER TRAINING

B.4. ADULT EDUCATION

B.5. NON REGULATED TEACHING

C-THE OBJECTS OR PRODUCTS AS THE CENTRE OF THE ANALYSIS

C.1. TEACHING RESOURCES: DIFFERENT TYPES OF RESOURCES

C.2. TEXT BOOKS (THEY ARE SPECIFIC RESOURCES)

C.3. TRAINING PROGRAMMES OR SCHOOL CURRICULAR DESIGNS

C.4. TESTS OR EXAMS

**INSTRUMENTAL SUBCATEGORIES OF ORDER 2 RELATIVE TO
CONTENTS**

“Specific contents (mathematical or educational, linguistic, anthropological) studied in mathematical contexts”.

A.- THE MATHEMATICS CURRICULUM

B.- ASSESSMENT

B.1. SCHOOL PROGRESS IN MATHEMATICS: SUCCESS, FAILURE

B.2. TESTS ADAPTED TO DIFFERENT CULTURES

B.3. DIFFERENTIATED OBJECTIVES AND STANDARDS

C- SPECIFIC MATHEMATICS DIFFICULTIES

C.1. RELATED TO THE DIFFERENT CULTURE, EXPRESSION AND SYMBOLIZATION

C.2. CONCEPTUAL, OPERATIONAL DIFFICULTIES, ETC

D- PROBLEM OF THE DIFFERENT LANGUAGE

D.1. IN THE TEACHERS, THEIR ABILITY

D.2. IN THE TEACHERS, IN THE ACTIONS IN CLASS

D.3. IN THE BOOKS AND OTHER RESOURCES

E- THE CUSTOMS AND FORMS OF CULTURAL EXPRESSION
E.1. ORAL CULTURE AND ORAL MATHEMATICAL LEARNING
E.2. VISUAL CULTURE AND MATHEMATICS LEARNING
E.3. COMPUTER CULTURE AND MATHEMATICS LEARNING
E.4. RESOURCES ADAPTED TO THE DIFFERENT CUSTOMS RELATIVE TO
THE LEARNING AND THE CULTURAL RELATIONS

These subcategories can be called “Thematic Subcategories” (or “Contents Subcategories”) or of ORDER 2, and in mathematical didactic research they are of great interest to study school failure, learning errors, assessment and others by focusing the studies on specific topics of the curriculum. For example, the difficulty of translating from the native language of an area in Brazil to the Portuguese of mathematics terminology, analysis of the comprehension of the associated mathematical concepts.

Categories can be developed more by creating other new ones that arise from those emerging from the papers analysed and cross them with the subcategories with which they are compatible, for example. A possible type of paper could be a “Field Research about a teacher training project where their own resources are designed for formal education in a specific multicultural group”.

The papers found in our bibliographical search respond to some of these types. We have added others which conceptually complete the existing possibilities, from our perspective. Therefore we are dealing with completed emerging categories which logically were missing and are possible.

In order to use these categories and subcategories, this article is initially designed using a bi one to one code as we have previously said and later their own codes from the different Categorisation and Subcategorisation Instruments in the Protocols, are assigned.

In relation to some of the classifying categories produced and presented in this section, we have designed some instruments for the analysis of the papers which permit us to characterise each paper in a more detailed way and so be able to continue its analysis and classification. We are going to call the instruments “**Protocols to Analyse Documents**” about Multiculturality, Interculturality and Mathematics Education.

At present we have prepared four of these “Protocols”.

PROTOCOL I. Applicable to documents relative to Field Research;

PROTOCOL II. For documents relative to Basic Research;

PROTOCOL III. Applicable to different Documents that describe work which is not research such as: classroom experiences, projects, creative elaboration of resources, bibliographical review, tests.

PROTOCOL IV. To characterise documents conceptually by situating them in the Fundamental Categories.

We present the aforementioned Protocols, where Protocol 3 is applied to a case:

PROTOCOL I. Field Research

Title of the research paper	Code and Authors	Language (s)	Place of Publication	Year Published	Journal or book where it is published	ISSN ó ISBN	Length of text in n° of words or of pages	Subject of research	Cultural group to which the subjects belong	Research Methodology	Sub-categories	Categories (Key words: categories)

PROTOCOL II. Basic Research

Title of the research paper	Code and Authors	Language (s)	Place of publication	Year Published	Journal or book where it is published	ISSN or ISBN	Length of text in n° of words or of pages	Research Methodology	Subcategories	Categories (Key words: categories)

PROTOCOL III. Documents that are not research (classroom experiences, projects, resources, bibliographical reviews, tests).

Code and title of the research paper	Authors	Language (s)	Place of publication	Year Published	Journal or book where it is published	ISSN or ISBN	Length of text in n° of words	Kind of Work	Subcategories: Subjects or objects	Categories
(1 KM) Personal epistemology and mathematics: A critical review and synthesis of research	Krista R. Muis	English	Estados Unidos Washington	2004	Review of Education Research Tomo 74, N°3	00346-543	29.795	Review	Actions B1 Contents B1	Mathematics Epistemology Beliefs, Student behaviour, Mathematics education, Teaching methods.

In order to characterise documents with respect to the Fundamental Categories we have the following:

PROTOCOL IV. FUNDAMENTAL CATEGORIES

CODIFIED PAPERS	CULTURE	MATHEMATICS		EDUCATION		SOCIETY
		EPISTEMOLOGY, BELIEFS,	MATHEMATICAL TOPICS	EDUCATIONAL MODEL	TYPE OF CURRICULUM MATHEMATICAL RESOURCES	
(13, KM)		X		X		

Conclusion

The set of “Categories”, “Subcategories of order 1 and 2”, “Protocols” and “System of Codifying”, that we have shown make up the Instrumental components of the MUMEMO Model we have produced.

This model is a theoretical construct with which to carry out Didactic-Mathematical research about the documents which our community, interested in the social and educational phenomena that are produced in Multicultural environments and which concern Mathematics, publish. Such are the papers presented in this Group ST 33 of the ICME 11 and whose presentation we will carry out by grouping together and putting in order based on this Model.

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