ICME 11 Mexico 2008
11th International Congress on Mathematical Education

# A one-semester laboratory course in calculus for teachers

**Patricia Baggett**
**New Mexico State University (USA)**
**baggett@nmsu.edu**

**Andrzej Ehrenfeucht**
**University of Colorado**
**andrzej@research.cs.colorado.edu**

# A one-semester laboratory course in calculus for teachers

Patricia Baggett, Professor
Dept of Math Sciences
MSC 3MB
New Mexico State University
Las Cruces, NM 88003-8001
USA
baggett@nmsu.edu

Andrzej Ehrenfeucht, Distinguished Professor
Computer Science Department
PO Box 430
University of Colorado
Boulder, CO 80309-0439
USA
andrzej@research.cs.colorado.edu

http://www.math.nmsu.edu/~breakingaway

## Introduction

At New Mexico State University in Las Cruces, NM, USA, we offer a one-semester calculus course intended for practicing and future middle- and high-school teachers, which is very different in both content and format from most calculus courses that are offered at the college level. (See for example texts such as Stewart, 2008 and Hughes-Hallett et al., 2008, which are often used in college courses.)

## 1. The student population and the organization of the course

Most students who take this course are practicing or future teachers, but the course attracts a broader and more varied audience. Students take it for a variety of reasons; some of them already teach calculus or expect to teach it in the future; others want to extend their math knowledge; and some just need to get some math credit hours. The course has no formal prerequisites except permission of the instructor, so the mathematical background of students who enroll varies. Some have already taken two or more semesters of calculus, and others do not even have the typical prerequisites, such as college algebra and precalculus.

The course can be taken for either graduate or undergraduate credit. It is run in a laboratory format (two 100-minute sessions per week for three credit hours), and most of the work is done in the classroom where students work individually or in groups depending on the task. There are almost no lectures, and there is no textbook for the course. Instead, students are given extensive handouts, so at the end of the course they have a portfolio of materials over 100 pages long.

There are no in-class tests. But each student has to make a notebook of write-ups of selected tasks done in class. The instructor looks over these notebooks biweekly and returns them to students with detailed comments. The quality of a student's notebook determines 70% of the student's grade.

## 2. Content of the course

### a. Applied calculus

The course is centered on applications of calculus. Students are given a task; for example, they are asked to design and construct a physical object having some required properties. They have to create a mathematical model of the situation by choosing and defining appropriate variables and writing appropriate equations. The tasks are chosen so that their completion requires the use of calculus; for example, students might need to find the maximum value of a function. After finishing the mathematical part of the task and carrying out the necessary calculations, students are still required to construct the object. So the final product is a physical artifact. Within such a framework all mathematical concepts are introduced.

b. Examples of tasks
(1) Take a cylindrical soup can and measure its diameter D and height H. Attach to it a string (or a piece of clothes line wire), forming a spiral from the bottom to the top, going around exactly once. Compute its length using an integral formula, and compare this value to the measured length of the piece of string.
(2) You will be given a strip of rectangular poster board having length L and width W, and an index card. You should use the poster board to make a sidewall (lateral surface) of an elliptic cylinder that has axes in the ratio 2 to 1. Its (elliptic) base will be made from the index card. L will be the perimeter of its base and W will be its height.

After you plan the design, I'll give you materials so you can measure W and L, and then do the computation and make your cylinder. The last thing you are going to do is to compute the volume of the cylinder and measure the volume with rice, comparing the two.
(3) (Each student is given a golf ball.) From poster board construct a cone with the smallest volume that encloses the ball.
(4) (Students are given a one-meter-long piece of plastic-coated clothesline wire.) Cut the wire into two parts and form from them a circle and a square. The sum of their areas must be as small as possible. Think twice, cut once!

Not all tasks have a hands-on component:
(5) Compute the volumes of a cone and a sphere using algebraic formulas and integrals.

And not all tasks require calculus:
(6) Write a program for the TI-84 calculator that constructs a k-th degree polynomial P(x) which provides the best fit for n points $(x_i, y_i)$, i = 1, …, n.

Here is an example to show how mathematical concepts are embedded in the tasks:

In order to do task 1 (string around a can):
Students need to learn how to use parametric equations.
They have to write equations for a spiral, x(t) = r*cos(t), y(t) = r*sin(t), z(t) = c*t.
And they have to learn the formula for the length L of a curve defined by parametric equations,
$L = \int_a^b \sqrt{({}^{dx}/_{dt})^2 + ({}^{dy}/_{dt})^2 + ({}^{dz}/_{dt})^2}\, dt.$
(Students were shown an informal but rigorous derivation of this formula.)

It is expected that most students will need some help from the instructor or from more knowledgeable colleagues to carry out the assigned tasks.

### c. Mathematical concepts

Defining mathematical models of physical objects is very time-consuming, and it is difficult for most students, independent of their mathematical background. So the amount of time that might have been spent on discussing mathematical concepts is limited. As a result the only concepts that are fully investigated are the concepts of derivative, integral, and anti-derivative. Also the selection of elementary functions that are studied is determined by the applications. Because most applications deal with physical objects, exponential and logarithmic functions are practically absent, and trigonometric functions are overrepresented.

But the main difference between this course and other calculus courses is in the way the concepts of derivatives and integrals are introduced. Usually the derivative is described as the slope of a line tangent to a graph of a function, and the integral is the signed area between the graph of a function and the x-axis. (See Hughes-Hallett et al. and Stewart, cited above; and also older books, eg., Wylie, Jr., 1953.) These definitions create serious conceptual problems when calculus is applied. In applications we use denominate numbers, and we require that students always use appropriate units. But in task (1), the integral is used to compute a length, and therefore the result is measured in centimeters or inches. If the integral is introduced as an area, the result is expressed in the wrong units.

Such discrepancies are endemic, but they can be avoided when we use more abstract definitions of derivatives and integrals which are not tied to any specific physical quantities. So we may define the derivative as the rate of change of one variable relative to another, when the first one is a function of the second. And we may define the integral of a function f(x), where x varies between a and b, as the average of values f(x) times b – a. After working with several different applications of derivatives and integrals, even students with a weak math background have found these definitions intuitive and not difficult. But it is worth mentioning that this concept of integral is equivalent to the Lebesgue integral (Burk, 2007; Bressoud, 2008), and not the Riemann integral, which is usually used in undergraduate calculus courses.

### d. Computations

Most applications require a considerable amount of numerical calculations. In this course TI-84 calculators are used. The features used most often are nDeriv, fnInt, Solver, Matrices, and the graphing facilities. But the extensive use of numerical computations also influences which calculus topics are stressed and which are omitted. The use of numerical methods almost completely removes the need for hand computation of derivatives and anti-derivatives. So these topics, which usually form the bulk of a calculus course, are practically absent from this course. On the other hand, it is better when students know how the calculator carries out its computations, so such topics as Newton's method, which underlies Solver, are included in the course.

## 3. Students' comments and opinions

This course has been offered three times (and will be offered again fall 2008), and each time students have given it a grade of A, on a scale from A to F, so the total number of negative comments has been very small and has reflected students' individual dissatisfaction with some aspects of the course (for example, grading and the time the class meets, i.e., late in the evening). Positive comments have varied. Teachers and future teachers often mention the value of a portfolio of lesson plans, which contains materials they can use in their own classrooms. Some students, who have taken calculus courses before, have said that this course made calculus meaningful. Some students liked to work with calculators. Many liked creating actual artifacts.

There have been almost no complaints about the lack of a textbook (only one such complaint during three semesters), and several students have mentioned it as a positive aspect of the course.

We expected that better-prepared students who have taken the course at the graduate level would find it rather easy, and students with a weaker mathematical background would find it difficult. But the distribution of judgments about the difficulty of the course has not been different in these two groups. Students work in groups, and better-prepared and less well-prepared students are seated together, with instructions that stronger students should help weaker students. This may explain the lack of a difference.

## 4. Observations and evaluation

Not having prerequisites for this course, and the resulting spread of students' math backgrounds, happen to be less troublesome than we expected. A background in algebra, geometry and trigonometry is needed mainly for creating mathematical models of physical situations. But even students who have taken all such courses have only the technical skills required; they have had no experience in using these skills in real situations.

Based on anonymous course evaluations, the class has been successful in presenting calculus as meaningful and interesting and often surprising.

But it would be better if it were stretched into a two-semester course, which is not unreasonable because the course doesn't require prerequisites whose fulfillment may require two semesters.

Teachers and future teachers have received portfolios of materials that they may use in their own classrooms.

But the course does not prepare students for taking typical calculus tests, which require knowledge of many formulas and skills in computing derivatives and anti-derivatives by hand.

## References

Bressoud, D. (2008). *A radical approach to Lebesgue's theory of integration*. Washington, DC: Mathematical Association of America and Cambridge University Press.

Burk, Frank (2007). *A garden of integrals*. Washington, DC: Mathematical Association of America.

Hughes-Hallett, D. et al. ( 2008), Calculus: Single and Multivariable 4th Edition. Hoboken, NJ: John Wiley Publishers.

Stewart, James (2007). *Calculus: Concepts and Contexts Enhanced Review Edition* 3rd Edition. Pacific Grove, CA: Brooks Cole Publishing.

Wylie, C.R. Jr. (1953). *Calculus*. New York: McGraw-Hill Book Company, Inc.