MATHEMATICS LEARNING STYLES OF BRAZILIAN UNIVERSITY STUDENTS

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Studies on higher education have emphasized the differences among students, and have also been attempting to characterize students' learning strategies and styles (Marton and Booth 1997; Marton and Säljö 1976). While some researchers discuss the existence of general and educable cognitive strategies (Alexander and Judy 1988; Adey 1997), others investigate students' learning strategies in specific domains such as Mathematics (Crawford et al. 1998; Tall 1991, 1997; Pinto 1998).

This paper focuses on learning strategies used by undergraduate Brazilian students. An empirical research using both quantitative and qualitative methodologies (Frota 2002) was conducted with 529 undergraduate engineering students, searching for the methods and preferences they have when studying Calculus. A case study is presented here with the purpose of highlighting the role of a metacognitive approach of learning if one intends to develop mathematical learning strategies and styles.

In the first section I discuss the main constructs broached on this paper: learning strategies, learning styles, metacognition and self-control of learning processes. The second section describes the context of the wider research, from which the case study presented was selected. The forth section details the methodology developed, presenting global results about learning strategies of six students of Calculus 102 (here Calculus II). The fifth and sixth sections group the students according to evidences of a *theoretical* → *practical* learning style, or a *practical* → *theoretical* learning style, describing their cognitive monitoring processes. I conclude with some educational propositions for teaching Calculus, which I have learned with the students along the research process and confirmed some ideas I had.

Learning styles and cognitive monitoring

In this work I deal with two constructs that are actually very close to each one: learning styles and learning strategies. Learning strategies are elements of an active knowledge building process. These elements are continuously developed when a person interacts with objects (mathematical objects in the situation here) and with other people. Literature provides hierarchical classes of learning strategies with various levels of complexity. People could use different learning strategies and have different learning goals. It is also necessary to take into account that people can use the same strategy in different ways. This way of personalizing learning strategies constitutes a learning style. Students usually present learning styles profiles (Frota 2002, 2006).

Research on learning styles constitutes a wide area of investigation, with different approaches demanding for special studies searching for convergences. Zhang and Sternberg (2005) pointed out three controversial issues in the area: 1) styles as traits (stable and unchangeable) in contrast with styles as states (flexible and modifiable); 2) value-laden styles in contrast with value-free styles; 3) different style labels representing different style constructs, or similar style constructs described and named by different terms. Dealing with the third issue, about style terminology, the authors adopted the general term *intellectual styles,* meaning the preferred ways one uses in order to process information and deal with tasks, in other words, the preferred ways one uses the abilities that we have. (Zhang and Sternberg 2005). Cognitive style, problem-solving style, learning style, mind style and thinking styles are some of the major style constructs postulated by research. The authors adopted the general term *intellectual style* as an attempt to encompass all of them. Zhang and Sternberg also consider that intellectual styles have affective, physiological, psychological and sociological characteristics, with a variation in the degrees they are presented.

I adopted the terms *learning strategies* and *learning styles*. I consider that learning Mathematics styles are the preferred learning Mathematics strategies one uses in a special way. Strategies and styles of learning are improved from processes of interaction with the teacher, among classmates, as well as an interaction with the classroom environment. Learning styles merge believes and values and are value-laden, some of them being considered as positive to promote an in-depth learning approach. A person could be classified as presenting a specific learning Math style, but this classification is local or temporal. I consider that learning styles are modifiable states and not traits, in agreement with Sternberg (1997).

Results from this research pointed that self-regulated learning seemed to be the main factor that influences learning strategies and styles of engineering Brazilian students (Frota 2002). So it was necessary to investigate the relation between learning strategies and cognitive monitoring.

Cognitive monitoring is one of the aspects of metacognition. Metacognition is “cognition about cognition” and it consists of knowledge about our own cognitive processes and products as well as and the ability of controlling, monitoring and evaluating those processes and products (Flavell 1979). Studies on metacognition could be classified as studies of knowledge about cognition of cognition, and studies about regulation of cognitive processes (Brown 1987). Monitoring the variety of cognitive enterprises consists on the actions and interactions among the phenomena of metacognitive knowledge and experiences, goals, or tasks and actions or strategies (Flavell 1979).

Cognitive knowledge encompasses different kinds of knowledge: scientific, empiric, emotional, affective, knowledge about people, goals and strategies. Motivation is a factor that could contribute to learning. Students have expectances about their performance based on the evaluation of their own abilities and on the evaluation made by their classmates, teachers and relatives. Students’ expectations are still supported by the importance and value they attribute to tasks and goals.

Literature highlights three relevant aspects of metacognition: a person's knowledge of his/her own processes of thinking; knowledge of control or self-regulation of actions; beliefs and conceptions that can influence the way someone do Mathematics (Schoenfeld, 1987). Metacognition, beliefs and practices could be considered as special aspects of mathematical thinking (Schoenfeld 1992).

Motivation, expectation, beliefs and affects could guide different kinds of goals, learning strategies and learning styles. Understanding about students’ learning styles could help them know themselves, choosing the course they will take, the learning methods they will adopt, the suitable strategies they will take to learn Mathematics. Being aware of learning styles is also essential for teachers on planning a course, and on proposing different tasks in order to promote the development of diverse learning styles. The study reported, making explicit students’ preferences of dealing with calculus tasks could offer an opportunity to reflect about educational practices of teaching Calculus.

Research context

The case study presented is part of a wide research enrolling undergraduate engineering students from Minas Gerais, Brazil. Quantitative studies pointed out that learning Mathematics students' strategies could be classified as presenting a highly theoretical stress or a practical one. Results were interpreted by using multivariate statistics resources of factor analysis (PAF, with oblique rotation, Oblimin) to deal with a questionnaire answered by 529 undergraduate engineering students. The purpose of this research was searching for characterizing their Calculus study methods, among other aspects such as learning conceptions, motivation and expectations with the course (Frota 2002).

An in-depth qualitative investigation was conducted with eleven students, selected from the total sample. Students participated in a set varying from three to four clinical interviews, each of them taking approximately forty-five minutes. They executed some special tasks concerning Integral Calculus, reflecting on their performance in each task as well as in their study methods. Interviews were conducted through the "thinking aloud" technique. Student's representations of their learning methods were elicited and contrasted with the strategies they effectively used when performing mathematical tasks.

The qualitative study highlighted that students do not adopt a theoretical or a practical learning approach. However, it seems that they make two opposite movements: they move from theory to practice, which I named as the *theoretical → practical* learning style, or they move from practice to theory, which I named as the *practical → theoretical* learning style. This categorization is supported by analyses carried out on two kinds of registers: what students said about their Math study methods and what procedures they adopted to deal with mathematical tasks concerning Integral Calculus.

The mixed methodology adopted made it possible to characterize the population of Calculus students of the University that were investigated, as well as to deeply acknowledge the way engineering students deal with Calculus. If some students have never thought about the methods of learning Mathematics they use (which I named as the *incipient* learning style), others demonstrated having the control of their own learning process.

I will exemplify the learning styles of six students of Calculus II, in order to highlight some evidences of self-regulated learning.

Methodology

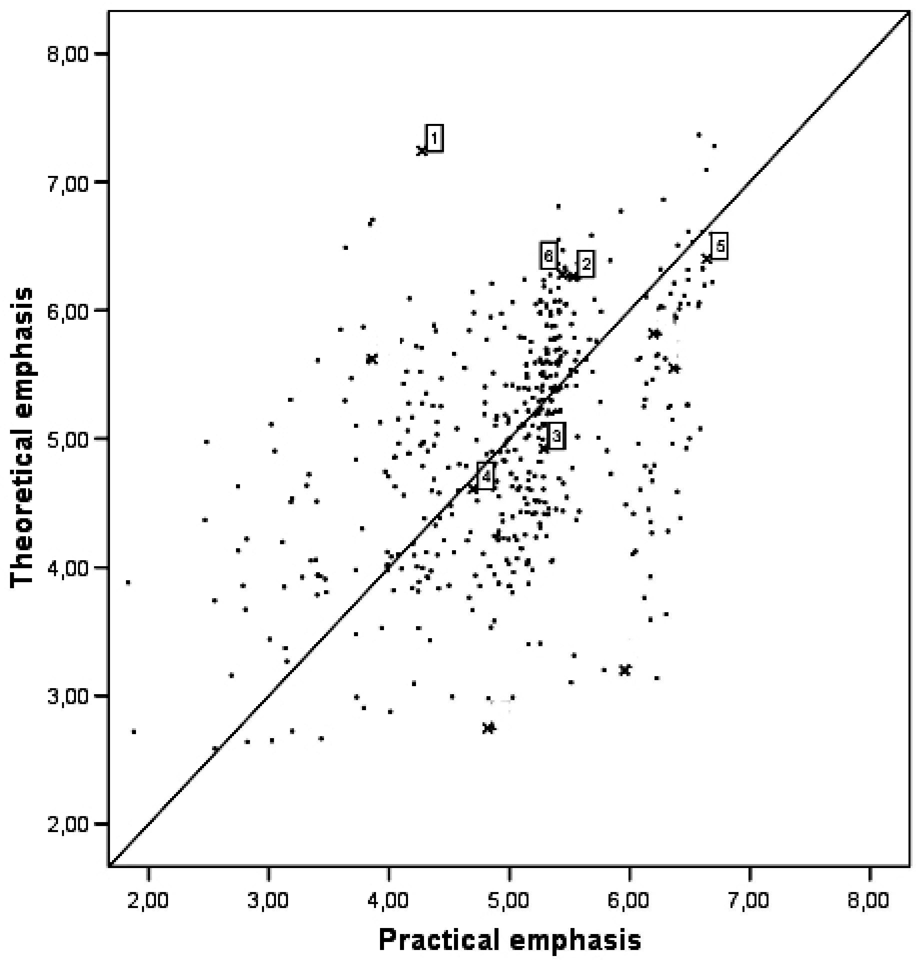
Fabrício, Patrícia, Márcia, Camilo, Ciro and Aluísio[1], students of Calculus II, answered a set of questions. One of the questions inquired about the characteristics of their Calculus study methods. The questionnaire consisted of twelve affirmatives, presented as a five Likert scale, and the students had to indicate in what extent they agreed or disagreed with each proposition.

Statistical analysis made it possible to classify the representations that 529 engineering students had of their learning strategies. Factor analysis was first conducted using principal components, and then it was made again using principal axis factoring. They

[1] Fabrício and Ciro take Mechanics and Electronic Engineering courses, respectively, in the evening shift. Aluísio and Camilo take the engineering courses of Control and Automation, and Civil Engineering, respectively, in the day shift. Márcia and Poliana study Mechatronics in the morning.

indicated two factors interpreted as a theoretical emphasis and a practical emphasis of studying Calculus. Students with a high theoretical stress agreed or strongly agreed with the use of learning strategies such as: underlining important theoretical points or results on the textbook; making theoretical abstracts; making notes while reading theory and exercises; reading a subject before teacher's explanation, underlining doubt points. On the other hand students with a practical emphasis confirm their agreement with the use of strategies such as: doing, if possible, all the recommended exercises; rereading the notebook and doing exercises; studying solved examples of the book or notebook; rereading the theory on the book and doing the proposed exercises.

Graph 1 highlights the six students of Calculus II, selected from the total sample of 529 engineering students. Cases are labelled with numbers 1 to 6. It is possible to verify that three of them - Fabrício, Patrícia e Aluísio - present a predominant theoretical stress. In the scatter plotter graph they are located over the diagonal line (cases 1, 2 and 6). Camilo (4), Márcia (3) and Ciro (5) have a practical study method stress. In the scatter plotter graph they are located near the diagonal line. This could be representative of a study method joining theoretical and practical emphases, which are more stressed for Ciro, than for Márcia or Camilo.



Graph 1: Emphasis of Calculus learning strategies of engineering students.

Looking at the graph, Fabrício (case 1) appears as an outlier point, and he probably presented the higher theoretical stress style of the total sample (529 cases). Because of this Fabrício's case was selected. Answers of the questionnaire were interpreted as the *representations* he had of his learning strategies. Those representations were contrasted with real *actions* of the student dealing with Calculus tasks. This procedure was made as a kind of a triangulation of quantitative and qualitative data. Fabrício's *representations* and *actions* are not necessarily convergent, as I will be discussing later.

Students participated in a set of four interviews. They had been asked to order and solve a set of exercises, each one presented in a ticket. In Figure 1 the exercises were numbered, only to make it easier to refer to them across the data analysis process.

The exercises were selected satisfying the assumption that they should not involve integral applications, in order to avoid obstacles derived from the area with which the exercises were related. Despite of this fact, students were asked about possible interpretations of the results obtained. Exercises 1 to 6 could be classified as standard ones. It was expected that the students had never done an exercise similar to the seventh one. Students could consider exercises 2 and 3 apparently similar, but exercise 3 demanded special theoretical competence of the restrictions on using *The Fundamental Theorem of Calculus*.

$$1)\int x^3\sqrt{x^4+1}\,dx \quad 2)\int_{-1}^{1} x^3 dx \quad 3)\int_{-1}^{1} x^{-2} dx \quad 4)\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$$

$$5)\int_0^2 xe^{2x^2}\,dx \quad 6)\int \cos x^2 dx \quad 7)\sum_{n=1}^{\infty}\int_0^1 \frac{x^n}{n!}\,dx$$

Figure 1: Exercises proposed to Calculus students (Frota, 2002).

Students were also asked to read: 1) an example from the Calculus book, chosen by themselves, trying to discuss the solution process; 2) a short text, presenting a theory they had not studied yet, explaining their understanding of the ideas presented, and highlighting the difficulties they had.

Data was analysed providing 31 indexes that could classify students' actions dealing with the tasks and used to identify the learning style presented. Indexes 1 to 9 characterize a *theoretical* $\rightarrow$ *practical* learning style, consisting on: selecting a group of exercises according to the classifications inherent to a specific content; ordering exercises according to the level of complexity initially considered in conformity with previous theoretical categorizations; searching for theoretical support in the text book or in the notebook; skillfully reading with competence a new Math text, explaining the ideas in their own words, and drawing analogies with previous studies; effectively dealing with exercises demanding the use of a new theory not studied yet; making synopses and justifying the procedures adopted when doing exercises.

Indexes 8 a 23 are evidences of a *practical* → *theoretical* learning style, such as: clustering exercises from a practical view, superficially mentioning theory; making mistakes on ordering exercises by the level of complexity, or clustering exercises of different levels of difficulty; explaining the order chosen to solve exercises only from usual aspects of practice; taking advantage of strategies of "trial and error", instead of using standard procedures; lacking of competence to find a subject in the text book; presenting difficulties to deal with new questions demanding a previous study in the book; difficulty to summarize theories; difficulty to choose an adequate mathematical integration formula from a table made by themselves; a constant demand for comparing new proposed exercises with solved exercises. A learning style was considered *incipient* when students presented indexes numbered from 24 to 31, such as: using step-by-step procedures; presenting difficulty to classify exercises from theory or from practice; ordering exercises without any explanation; failing to choose the appropriate formula from a table of integrals; repeating strategies of trial and error; presenting problems with previous knowledge; demonstrating problems with reading theory in a textbook.

Some indexes, like indexes 8 and 9, could characterize more than one style: explaining the procedures when doing a task, or participating in a theoretical dialogue developed from an exercise. Those characteristics, for example, could be found among students presenting a style with a theoretical emphasis or a practical one.

The analysis of all the data of the questionnaire and the interviews made it possible to classify Fabrício, Patrícia, Márcia, Camilo, Ciro and Aluísio according to learning Mathematics styles, based on the indexes identified across the qualitative study.

Student	Index	Learning style
Fabrício	8, 9, 11, 12, 14, 15, 16, 20, 23, 24, 28	*practical → theoretical*
Patrícia	1, 2, 3, 4, 5, 6, 7, 8, 9	*theoretical → practical*
Márcia	5, 7, 8, 9, 10, 11, 16, 17	*practical → theoretical*
Camilo	12, 14, 20, 21, 23, 26, 27, 28, 29, 30, 31	*incipient*
Ciro	1, 2, 3, 4, 5, 6, 7, 8	*theoretical → practical*
Aluísio	1, 2, 3, 4, 5,7, 8, 9	*theoretical → practical*

Figure 2: Students of Calculus II indexes and learning styles.

Results are gathered according to some evidences of learning styles, and presented now from a perspective of detaching some evidences of students' self-regulated learning processes. The case of Camilo will not be discussed because during the interviews it was impossible to identify the aspects of self-regulated learning I am intend to highlight.

Evidences of a *practical → theoretical* learning style: the case of Fabrício and Márcia

Fabrício separated exercises in two groups, clustering the indefinite integrals 1, 2 and 6 in a first group, and exercises 2, 3, 5 and 7 of definite integrals in a second group, but he could not justify the classification adopted.

Márcia separated exercises in three groups, according to the degree of difficulty, as she explained. She clustered exercises 1, 6, 2 and 3 in the first group, exercises 4 and 5 in the second one, and exercise 7 in the third one.

Fabrício solved exercises 1 and 4 using the method of integration by substitution and tried to use the same procedure to deal with exercise 6. He consulted the textbook, searching for a similar exercise. He proposed to use the trigonometric relation $\cos^2 x = \frac{1}{2}(1+\cos 2x)$, but he failed, explaining that he had studied all the integration techniques separately, but he had not tried to establish a theoretical synthesis of the subject. Márcia proceeded as Fabrício did, solving exercise 1 with no difficulty and having some with exercise 6. Both of them could explain the synthesis they had done with the method of integration by substitution.

According to Fabrício and Márcia, exercises 2 and 3 are similar. Both of them resolved exercise 2, but used *The Fundamental Theorem of Calculus* improperly on dealing with exercise 3. Differences occurred during the dialog. Márcia solved exercise 2 with no difficulty and could verify that the solution was correct, making a graph and discovering that the integrant function odd, remembering that an odd function integrated in a symmetrical interval has as result zero. Márcia also realized that the solution presented to exercise 3 was wrong, but she did not found a proper way to deal with it.

Fabrício demonstrated having much difficulty with exercises involving definite integrals. He spent a long time consulting his book, looking for solved exercises of definite integral. It seems that he did not have the habit of making annotations during classes, because he did not use his notebook. During all the interviews he used only the textbook, demonstrating some difficulty to find topics in it.

Fabrício always started studying solved examples, by comparing standard procedures, so he could not solve exercise 7. Despite being selected because of a high theoretical stress, which is clearly evident in the dispersion diagram (Graph 1), Fabrício demonstrated a preference for a practical learning style, which could be noticed during the individual interviews. The movements Fabrício made from practice to theory seemed to be very irrelevant, and justified by the fact that he was not studying as much as it was necessary and that should be done, as he himself pointed out. It seems evident that Fabrício was conscious of the problems he had when studying Calculus, explaining that he had not systematized all the integration techniques the teacher had introduced. He assumed that answering the questionnaire he was projecting the way he would like to conduct his course, and his answers emphasized learning strategies with a theoretical stress. I interpreted this explanation as a kind of self-control that the student presented.

During the interviews Márcia talked about her study method, explaining that she used to do the exercises until the end and checked out the answers from the book, because she used to deal inadvertently with signs. She said: “ I like to do just a little of each one (referring to degrees of exercises), begin from the easiest ones (exercises), and go on increasing the degree of difficulty. I like to do it this way.” (Márcia, I 01) She demonstrated knowing to locate information from the book to solve exercise 7, connecting the mathematical contents of series and integrals. During the activities of reading solved exercises in the book and reading and talking about a new subject in the textbook, Márcia revealed some characteristics of being able to keep up a dialog with the researcher about issues related with the tasks. She seemed to study Calculus practicing, and she pointed out that the method she used to study depended on the teacher.

> “I am sure I am learning when I can… just reading the question, the exercise… guess its solution […]” “I did not study very much last semester. The teacher presented things in a more detailed way, and the exercises he did in class were similar to those in the exams. During this semester, I think the teacher’s method is like… you have to want to learn and win […] You have to search for information in the book, reading slowly... And you have to solve enough exercises.” (Márcia, I 03)

Márcia seemed to appreciate the way her teacher taught them Calculus, encouraging students to do different kinds of exercises. That practical learning emphasis adopted by the teacher seemed to be in accordance with her own way of studying.

Fabrício and Márcia were explicit about their particular way of dealing with Calculus, which supports a *practical* → *theoretical* learning style. They revealed some awareness as to the course they were taking, making their preferences and difficulties very clear.

Evidences of a *theoretical* → *practical* learning style: the case of Aluísio, Patrícia and Ciro.

Aluísio, Patrícia and Ciro presented some characteristics when dealing with tasks that one could interpret as supporting a *theoretical* → *practical* learning style.

Aluísio strongly agreed with the statement of the questionnaire that when studying Calculus he tried to learn a subject before the teacher's explanation. During the interviews, he posed that it was very important to have an idea about the subject that the teacher would be discussing in the next class, reading theory first, although sometimes it was impossible for him because of the great amount of topics he had to study.

Aluísio considered that in general doing exercises was important if one wanted to learn Calculus, but he was always searching for a theoretical support: "I read everything first and then I write the main points of each topic. Afterwards, I have a quick look at my notebook, and if there is something there I haven't noticed I complete my notes and do some exercises (Aluísio, I 01).

Aluísio selected and ordered the exercises by adopting the strategy of searching for similarities. He established four groups: (a) the first was formed by exercises 2, 3 and 7; (b) the second one encompassed exercises 4 and 5; (c) the third and fourth groups were formed by exercises 1 and 6 respectively. Patrícia exhibited only two groups, the first being formed by the easiest exercises, 4, 1, 5, 2 and 3. In the second group she isolated exercises 6 and 7, because she was in doubt about how to solve them. Ciro grouped exercises 1, 6, 5 and 4, because he thought that he would be able to solve them changing variables in the integrals. Ciro defined a second group formed by the easiest exercises, 2 and 3, and isolated exercise 7 in the last group.

Aluísio solved exercises 2 and 3 very rapidly, applying what he called "a direct integral formula". Although he was told he usually did not use of graphic representations to think about exercises, I asked him to draw the graph of integrant functions of exercises 2 and 3. He made the graphs without any problems, and perceived his mistake solving exercise 2 (the first result he found was not zero). A discussion concerning exercises 2 and 3 guided him to realize that despite of a false appearance of similarity, exercises 2 and 3 were different. He perceived that the function $f(x)=x^{-2}$ presented "problems" in the zero point. He recognised the impossibility of using *The Fundamental Theorem of Calculus* to solve exercise 3, mentioning the necessity of using procedures for improper integrals.

Patrícia also solved exercises 4, 1 and 5 without any problems, using the method of change of variables. Like Aluísio and Ciro, she failed to solve exercise 3. Patrícia and the researcher developed a long dialogue, until the moment that she concluded that the integral proposed was classified as improper. Patrícia always gave theoretical explanations, analysing data, thinking about the possibilities, examining the coherence between results and theory.

Dealing with exercise 7, Aluísio justified that he had included exercise 7 in the first group because 0/n! was a constant, and it could precede the integral. He was not very sure about this procedure and went on explaining that he was trying to relate the exercise with the study of series. He could not see another way to solve the exercise. Patrícia consulted the textbook to resolve the task recognizing, as Aluísio did, that it was related with the famous representation by series of the function e^x. Ciro could not find a way to solve the problem, because he was just beginning to study series.

By different ways Aluísio, Patrícia and Ciro adopted different strategies of selection, ordering and making synopses, presenting a speculative and prospective approach when dealing with Calculus. They presented a theoretical focus of study, establishing some theoretical considerations first, before executing mathematical tasks. They performed a learning style classified as *theoretical* $\rightarrow$ *practical*.

The way Patrícia and Ciro dealt with the new text on double integral, which I proposed that should be read, caught my attention. Patrícia could keep up a dialogue on the new reading, drawing some analogies between the interpretations of simple integrals as areas, and double integrals as volumes. Ciro discussed many details, including the introduction of the chapter. He seemed to be very curious about the applications of double integrals, mainly those related with Mechanics. Ciro considered it was very a very interesting enterprise reading the subject in the book; an opportunity to discuss details of the definition of simple and double integrals related, for example, with partitions and its norms. This kind of attitude seems to be not common among engineering students in Brazil.

Implications for educational practice

Reflecting on this research outcomes I focused on some educational questions: What kind of learning strategies do teachers choose when teaching Calculus? Are teachers aware of their own strategies of learning Mathematics, in special strategies of learning Calculus? Are they aware of the role they play in fostering students' development of Mathematics learning styles?

This research points out the need for university teachers to know about the learning styles of their students. Being more aware of the ways students deal with Calculus, teachers could improve methodologies that take the differences between the students under consideration, and promote a self-regulated learning.

I can say that I have learned with the students whose strategies I have been investigating in order to teach Calculus, and classified them as easy to implement, and strategies that presented some problems to implement.

The first group encompasses strategies which did not demand substantive changes of teachers and students, such as: 1) asking the student to justify procedures adopted in solving exercises; 2) soliciting the student to classify and cluster exercises by similarities of solution; 3) asking students to make themselves a list of exercises that was useful on revising some subjects; 4) inciting students to analyse and correct mistakes; 5) reading an exercise with students, trying to delineate possible and valid strategies of solution; 6) proposing more challenging exercises.

Teaching strategies of a second group depend on deep changes of teachers and students. It seems necessary to develop a new conception of teaching and learning Calculus, pointing out the importance of: 1) inciting and guiding students in reading the textbook; 2) directing students to make theoretical summaries; 3) proposing tasks which demand a research; 4) promoting discussion about students' methods of learning.

In my current research project I am searching for new strategies of using the textbook, as a dialogue instrument. Students have been encouraged to make theoretical synthesis, learning to locate the main ideas. They were expected to become authors of their own didactical text. All those changes and strategies assume some knowledge of more convergences and frictions between learning strategies and styles, and between teaching strategies and styles (Vermunt and Verloop, 1999), forecasting some ways of making the transition from "teacher's regulation of learning" to "student learning self-regulation" concrete.

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