On the transit from Trigonometry to Calculus: the case of the conceptual breaks in the construction of the trigonometric functions in school

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ON THE TRANSIT FROM TRIGONOMETRY TO CALCULUS: THE CASE OF THE CONCEPTUAL BREAKS IN THE CONSTRUCTION OF THE TRIGONOMETRIC FUNCTIONS IN SCHOOL

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Abstract

This text presents the results of an investigation on the construction of knowledge from the study of the processes present in the articulation of conceptual mathematics systems to what we have called processes of mathematical agreement and mathematical articulation (Martínez-Sierra, 2005).Specifically we are interested in the study of the articulation of the algebra and trigonometry concepts with the elemental calculus concepts (as the function concept). The aim here is to present our analysis, from the point of view of the indicated processes, in order to become aware of the conceptual breaks in the construction of the trigonometric functions(TF) in Mexican upper middle school (pupils between 15 and 18 years old). One our result shows the conceptual breaks are present in two articulating concepts[1]*: the use of radian like angular measurement and the negative angles and angles larger than 360°.*

We are interested in the study of the articulation of the algebra and trigonometry concepts with the elemental calculus concepts (as the function concept). Our principal hypothesis is that such articulation is difficult because there are conceptual breaks between algebra and trigonometry concepts and calculus concepts.

Diverse investigations have been developed which offer explanations on the particularities, as far as their conceptual construction, of transcendental functions as logarithmic functions (Ferrari, 2007, 2004), exponential functions (Lezama, 2005; Martínez-Sierra, 2002) and the trigonometric functions (Buendía and Cordero 2005; Montiel, 2005).

It should be pointed out that our stance that every kind of function produces specific didactic phenomena is in strong contrast to the results and classic theoric developments surrounding the concept of function, because they all make use of the idea that there is *one* process that explains the construction or learning of the concept of function. The works of Sierpinska (1992), Dubinsky (1992) and Sfard (1992) show this, where, for example, Sfard establishes a cognitive process characterized by the stages of interiorization, condensation and reification, based on the epistemological model that function is a dialectic between an operative and a structural facet. Similarly, Harel and Dubinsky characterize mental constructions, necessary for building the concept of function, in terms of Actions, Processes, Objects and Schemas. None of them, though, pay any attention to the different types of functions. We share Ferrari's view (2007, p.

[1] In the construction of the trigonometric functions in school we find the following definition that allows to include the trigonometric functions as real value functions: if x is a real number, (sen x) it is equal, by definition, [sen (x radians)].

33) who mentions that this occurs "because it responds to the current paradigm, the study of the construction of a mathematical object, generally considering that the acquisition of a universal means the understanding of the specific".

In addition, in previous works (Martinez-Sierra, 2005, 2008 in press) we have developed some theoretical notions which have been useful, on one hand, in the explanation of some didactic phenomena and, on the other, in the interpretation of knowledge production processes. In particular, on the knowledge production plane we have provided evidence that certain pieces of knowledge, which we have called *mathematical agreements*, can be understood as the product of a process of mathematical articulation or process of knowledge integration. In this same way, on the plane of explanation of didactic phenomena, we have realized that some of the conceptual breaks at school have their origins in the disarticulation of a certain part of the corpus of mathematics in school (Martínez-Sierra, 2005, 2008 in press).

In the framework of the study of the processes found in the articulation of conceptual mathematical systems which we have called *processes of mathematical convention and articulation*, we have presented our interpretations that explain the conceptual ruptures present in the construction of trigonometric functions in school (Mexican upper middle school (pupils between 15 and 18 years old). Our results show that such ruptures revolve around articulate concepts whose function is to prepare their definition as functions of a real variable: the use of the radian as a unit of angle measurement and the definition of negative angles and angles larger than 360°. In this respect, evidence has been presented through the analysis of the structure of mathematical discourse in Mexican upper middle school by analyzing textbooks and interviews with teachers and students.

Thus, the analysis of the textbooks allowed us to locate some conceptual ruptures associated with the concepts that we have considered articulate: 1) the definition of negative angles and angles greater than 360º is made without any argument to justify their existence, 2), the motives for the sudden appearance of a system of angle measurement such as radians are not made explicit and 3) the *dethematization:* (that is, considering the object of study from the conceptual point of view) of the transit from radians to real numbers as an argument of trigonometric functions. One explanation of such ruptures stems from the consideration that different social practices exist within the structure of mathematical discourse in school, such as: 1) that which considers the measurement of the angle by degrees as "natural" and as a consequence is the chosen unit of measurement within school mathematics activity, 2) that which considers that new and old concepts form a continuum, thus defining negative angles in terms of the direction of the hands of a clock. In contrast, from our point of view, the measurement of angles with radians and negative angles are concepts that articulate, in the mathematical sense, the measurement of angles with degrees and FTs as functions of a real variable. Thus, the contradiction between mathematical meaning and the meaning built through school mathematics discourse is the source of conceptual ruptures.

The foregoing conceptual ruptures strongly condition the different conceptions of Mexican upper middle school students and teachers in relation to trigonometric functions. An example of such conceptions is that which provokes the belief that the domain of trigonometric functions is dimensional with the unit in degrees or radians. This conception makes it impossible to interpret expressions such as $f(x) = x + \sin x$ correctly, since they mix the values of x with real numbers and quantities in degrees. Regarding upper middle school students' conceptions we notice that, in spite of certain formal teaching, students' basic conceptions can be of two types in relation to

negative angles: 1) negative angles do not exist and 2) negative angles are “opposite” to positive angles; where the opposite depends on the type of geometric configurations favored by the student.

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