

Combining Calculus with Linear Algebra and Ordinary Differential Equations in “Early Bird” - an intensive pre-freshmen course

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Abstract

Mathematical knowledge and abilities are among the most important tools of engineers and are often needed in introductory classes before the relevant topics can be offered in the mathematics classes. The most common compromise consisting of a quick introduction to the relevant mathematics in the non-mathematical classes themselves is neither very satisfying for students and teachers nor is it particularly successful. We introduce the “Early Bird” project allowing interested freshmen to utilize the often unused period between “Abitur” (German high-school graduation) and beginning of studies in the winter term by attending the mathematics modules of “Calculus I” and “Linear Algebra” before the official start of the first semester. As a result students of engineering have the necessary in-depth mathematical knowledge at their disposal considerably earlier. They can reduce the duration of their education in mathematics by one semester by moving forward the advanced courses of higher semesters correspondingly. Teaching both “Calculus I” and “Linear Algebra” within one course allows for a better coordination of the content. Compared to the standard courses taught to freshmen, the curriculum of the course is modified to better address the applications of mathematics in the field of engineering. Additionally, the course covers ordinary linear differential equation as well as an introduction to a computer algebra system.

Index Terms: mathematics, calculus, ordinary differential equations, linear algebra, undergraduate studies, computer algebra system

1. Motivation

Mathematics is one of the most important foundations for engineering. From the first semesters on a sound knowledge of mathematics is necessary for being successful in the basic subjects of engineering (e.g. mechanics, physics, thermodynamics etc.). Problems will occur whenever mathematical knowledge is needed that a thorough mathematical education cannot provide until later. As a typical example, the structure of the mathematics courses usually do not allow for solving ordinary differential equations until the end of the first term, while the first term physics course for the engineering students begins with Newton’s laws and with integrating equations of motion. Given the curricula of engineering courses it is impossible to concentrate even more mathematics in the early semesters. This would also interfere with the legitimate wish of engineering students to identify with the actual engineering aspects of their courses early as the ratio of “non-engineering” classes (not just mathematics, but physics, chemistry etc. as well) would increase even further leaving even less time for the “actual” engineering classes.

In practice the solution most often used is to include a “crash course” of the relevant mathematics in the engineering classes themselves. Due to the obvious time constraints, these introductions rarely provide more than a schematic treatment of specific formulae

required in solving a given problem. This well-intentioned procedure is usually frustrating for students because they do not (and cannot) understand the reasons for applying these specific methods. Similarly, many students that understand these recipe-like introduced concepts belatedly ask the question why the actual connections have not been explained to them earlier.

2. The “Early Bird”-Course

For at least some of the students that begin their studies during winter term (about one third of the students begin their studies in summer term) the time between Abitur and the start of the winter term offers a solution for this dilemma. Most of the engineering students at the Berlin Institute of Technology will attend (with rare exceptions) the courses “Calculus I” and “Linear Algebra” in their first semester. It is possible to offer these modules in a concentrated version before the beginning of the winter term (in the case of first semesters, before actual enrollment). In an introductory compact course of “higher mathematics” the content of “Calculus I” is combined with the content of the “Linear Algebra” course. Additionally, the course covers ordinary linear differential equation as well as an introduction to the computer algebra system Maple to better fit to the special need of the mathematics “end-users”. Since most mathematics classes for engineers are offered every semester most engineering students can now move their second semester math courses forward into the first and as result finish their mathematical education one semester earlier.

This course called “Early Bird” has been offered twice so far, before the beginning of the winter term 2006/2007 and again before winter term 2007/2008. The total number of lessons is the same as that of both regular modules “Linear Algebra” and “Calculus I” combined. This means that the course is not shortened but is held within a shorter period of time. During that time the prospective students are only occupied with learning mathematics without the usual “distractions” by other university classes.

Many universities are offering a variety of “bridge courses” or preparatory courses to bridge the gap between high school and university; many of them are dealing with mathematics or physics. At least at German universities, as far as the authors know, these are additional courses which are intended as a repetition of the subjects that (should) have already been taught and learned at high school. Additionally, some courses are designed to give the freshmen students the opportunity of discovering their strengths and weaknesses [1]. In contrast the aim of “Early Bird” is not offering a preparatory course with main emphasis on the repetition of high school mathematics (cf. [2], [3], [4]), but to provide the opportunity to take standard courses of mathematics for freshmen students.

The regular modules “Calculus I” and “Linear Algebra” consist of two respectively one lecture/s of 90 minutes and one exercise class of 90 minutes per week. In the lectures the more theoretical subjects are presented. In the exercise class the knowledge is expanded, problems are examined and problems will be discussed in detail. While the exercise classes encourage active participation on the part of the students (e.g. by demonstrating their solution at the black board in front of the class) this is neither possible nor would it make sense in lectures with several hundreds of students. Most engineering courses require graded weekly written homework assignments. Due to limited manpower and time constraints, homework can only be accepted and graded for groups of three students. Each module will be completed by an exam. To be admitted to these exams 50% of all points possible in the homework have to be reached. All of the teachers and tutors offer weekly office hours

including homework support.

In the following the setup of the “Early Bird”-course is described. The entire course is completed within 9 weeks (compared to 15 weeks during the semester). During this time there will be one lecture of 90 minutes every weekday before noon. In the afternoon every student is attending a 60 minutes exercise class. The number of students per exercise class is limited to 16 – thus the teacher-student-ratio is much better than in the regular courses during the semester, where the number of students in exercise classes can reach up to 35 or even 40.¹ The university provides standard seminar rooms at the disposal of the students until evening, supporting collaborative homework in groups. Instead of the weekly office hours tutors are present all day as contact persons. In addition, the lecturer and the assistant are offering regular office hours.

At the end of every lecture a homework assignment (shorter compared to regular courses) is handed out. This will be the subject of the afternoon exercise classes and is to be handed in two days later before the lecture. Students will work on this homework in groups of two. The time between the morning lecture and the discussion session in the afternoon is long enough to allow a first glance at the problem set. The number of problems in each assignment is such that the total number of problems at the end of the course is the same as in both of the regular classes combined.

In the second half of the course, i.e. when they had familiarized with the course, the students had the opportunity to get an introduction into the computer algebra system Maple. For special problems Maple could be used for standard calculations, to check their results and for visualization (cf. Figure 1 for a screenshot of a Maple-Worksheet). Additionally, an introductory course into the mathematical typesetting system LaTeX was offered. Both offers were very popular, and many students spent the time between the lectures and their exercise class in the computer pool.

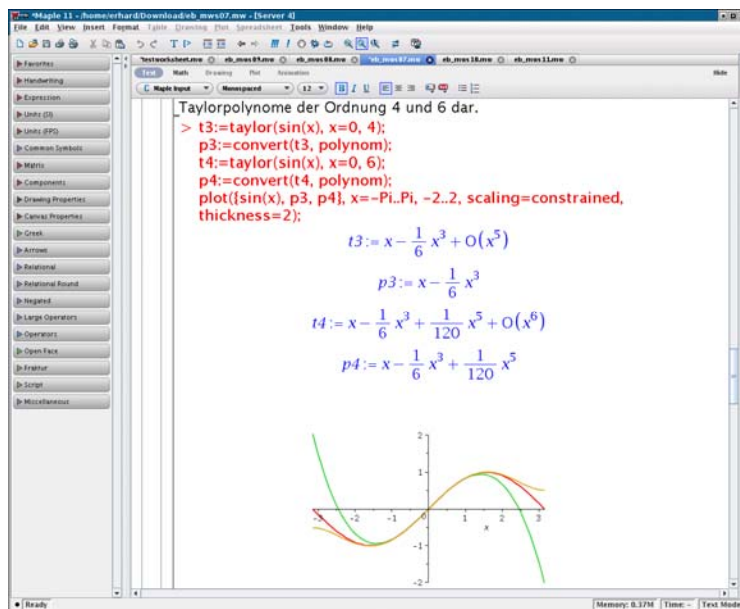


FIGURE 1
SCREENSHOT OF ONE OF THE MAPLE-WORKSHEETS USED IN THE COURSE

The “Early Bird” course is completed by a separate exam for “Calculus I” and “Linear Algebra”

¹ Despite the fact that all lecturers agree on 20 students being the maximum number allowing effective lessons, for reasons of capacity the number of participants in the discussion sessions during the semester is 35–40.

each. “Early Bird” participants attend the same exams that are offered before the beginning of the winter term for students of the corresponding regular modules in the summer term². By that a direct comparison between “Early Bird”-participants and Non-“Early Bird”-participants is possible. Some results are presented in section 6.

The regular modules during the semester suffer from the high number of participants. Obviously, in big classes with several hundreds of students per lecture having a direct contact between the lecturer and every student is impossible. Similarly, this is also an issue in exercise classes of 35–40 people. The larger homework groups of three students each make it harder to determine, who actually contributed to the assignment and who did not.

Due to the improved ratio of students and teachers compared to the courses during the semester it is easier to assure that each participant solves with the problems independently. The low number of 16 students in the exercise classes allows to require continuous and active participation in the exercise classes as a prerequisite for the exam in addition to reaching 50% of all possible points from the obligatory (for all courses of study) homework assignments.

The test-exam in the second half of the course as a preparation for the regular final exam was in direct response to wishes voiced by the students. For the participants this is the first exam at the university ever. During this exam students have the possibility to experience the situation, to get an insight into the usual progress and setup of an academic exam and last but not least to check their knowledge gained up to that point.

3. Advantages of “Early Bird” Over Regular Courses

Instead of the usual 30 respectively 15 lectures of the regular courses “Calculus I”, respectively “Linear Algebra” in “Early Bird” approx. 35 lectures are reserved for calculus including the linear differential equations and the remaining 10 lectures are used for the topics from linear algebra. Even though “Early Bird” deviates from the regular courses as regards content all obligatory topics of both modules can be taken into account in the time-frame of such an intensive-course. The successful participation in the “Early Bird” course is accredited as equivalent to succeeding in “Calculus I” and “Linear Algebra”, i.e. students are admitted to both final written examinations.

In “Calculus I”, differential and integral calculus of one real variable are lectured like in the regular calculus lessons. Additionally “Early Bird” deals with the theory of linear ordinary differential equations. Topics from linear algebra are introduced when required for solving linear differential equations as a supplement or as a specialization. Learning how to handle linear structures is the first learning goal here.

Because both modules are merged within the “Early Bird” course a strict separation of both topics is no longer necessary. This provides the opportunity to teach the two classes in parallel (e.g. separated by weekdays). As a result, it is possible to cross-reference between the topics and to explain freshly introduced mathematical termini under different aspects in both fields. For example the concept of a linear map can be explained with the example of differentiation and integration at hands. This helps the student to develop a deeper understanding of mathematics and its connections.

Another aspect is to familiarize students with symbolic and numeric software. Every

² Early Bird-participants also have to qualify for the exam by reaching 50% of all achievable homework points.

engineering student has to deal with mathematical software packages during his studies, e.g. in virtual experiments or for simulation and visualization purposes, yet most advanced courses do not have the time available for introducing these tools. "Early Bird" offers the opportunity of learning how to use these tools before the other lectures have even started: some use-oriented exercises from calculus and linear differential equations can be solved using the computer algebra system Maple.

We intend to include more application-oriented exercises in this course in the future. The regular lectures in "Calculus I" and "Linear Algebra" for engineers provide numerous examples of use for the introduced mathematical terms and methods. However, until now such examples were rare in the exercises and homework assignments. They require instructors who have good supplementary knowledge in physics and engineering sciences. Moreover, the time frame of a regular class does not allow for the preparation and the treatment of complex use-oriented exercises. Special funds were granted by the university that gave us the opportunity to address this problem. An extensive collection of suitable exercises has been provided, ready to be applied in future courses.

For prospective students "Early Bird" offers an ideal opportunity to get to know life at the university in advance. The German academic system leaves a lot of freedom to the students in arranging their course of studies. Realization of their study goals of course requires a certain degree of determination and self discipline. Many beginning students feel overwhelmed by the change from the well-ordered school system to academic life. The situation is even more severe at a huge university like the Berlin Institute of Technology, particularly in enormous freshmen lectures like "Calculus I" or "Linear Algebra" with more than 2.000 participants. The missing orientation and the distance from the teaching staff in a completely new environment with a plethora of studying opportunities and leisure time activities can lead to delays in the course of study or even dropouts. "Early Bird" participants can test campus life and their own work habits before the actual beginning of their studies on a voluntary basis. In such an intense course it is easier for them to get to know fellow students (from other fields as well) and to form study groups. The better staffing of "Early Bird" ensures that they will always find a competent contact person whenever difficulties occur.

As a result, "Early Bird" not only serves the purpose of teaching mathematics in time for application in other classes, but it also serves as a valuable orientation help for future students.

4. Organizational Aspects

All participating students had applied at the Berlin Institute of Technology, but did not know if they would be admitted at the beginning of the "Early Bird" course. The participants were chosen with a high chance (judging from experience) of receiving the final admittance to the Berlin Institute of Technology. The risk that some of the applicants will not or cannot be accepted has to be taken by both sides. Nevertheless, the offer seems to be so appealing to the participants that they are willing to take this risk. On the other hand we believe that we can convince the applicants to start at our university even if they are accepted at other universities as well. As a side-effect participants can see for themselves if the intended course of study suits them. A loss from dropouts is considered acceptable because it is considered to be better even for the students to see as early as possible if their intended study suits them or not. That is why we take the risk that it is possible for participants not to

enroll at the Berlin Institute of Technology after the course.

The usual number of participants of the “Early Bird” course was 350–400. This is about the usual number of students in one of the regular lectures during the semester (the approximately 2000 students participating in the regular courses are divided over several parallel lectures, each held by a different teacher). This means that no additional lecturer is required for offering the course. As a result a concentrated course like this can be held anytime without the requirement of additional staff. Only one of the regular parallel lectures and the associated exercise classes need to be moved forward and shifted into the semester break.

For the required better ratio of teachers and students special funds of the university have been requested because the high workload of the students was to be balanced by excellent support. Currently, the department of mathematics got increased funding for hiring tutors for all courses (including the regular calculus and linear algebra courses) anyway, resulting in similar cost per student.

The “Early Bird” course has yet another positive side-effect: By redesigning a lot of study programs, among other things the switching to the (for the German academic education) new Bachelor/Master-system but also by the growing number of beginning students in the engineering sciences the number of participants in the modules “Calculus I” and “Linear Algebra” has grown so fast, that in the winter-term alone up to 2.500 students have to attend these classes. Students attend separate, “parallel” lectures that are scheduled not to conflict with the schedules of their other courses [5].

The content of all lectures is the same and up to five are offered in parallel. The largest auditoria of the TU Berlin are required to fit that many students into one lecture. “Early Bird” helps to reduce the number of students in the regular modules to “only” 2.100–2.200. Without “Early Bird” up to six parallel lectures would be necessary. Given the limited number of available seminar rooms, the associated exercise classes would also be more difficult to organize were it not for “Early Bird”.

5. Criticism and Discussion

A few colleagues criticize the “Early Bird” course. Lecturers that teach the “regular” courses during the semester fear that only the less talented - or at least less interested - students will be attending their lectures. They fear that the grade standards, already characterized as quite low, will decline even further. They feel confirmed by the better performance of the “Early Bird” participants.

The fluctuations in the exam results of the last years are too big to examine a decline of standards as a result of missing “Early Bird” participants. Our opinion is rather that the success of “Early Bird” participants can be tracked back to an excellent ratio of teachers and students, a more focused approach to the study of the mathematical topics and last but not least to the positive almost family-like environment.

We are convinced that in big classes like the mathematics modules of the first semesters for engineers a big improvement can be achieved by strongly improving the ratio of teachers and students as was done in “Early Bird”. Some other concepts and effect of “Early Bird”, such as concentrating all classes in one field in a shorter period of time cannot be transferred to other, regular classes and might not be suitable for all students because a concentrated course also means a high workload for teachers and for students.

6. First Results

The “Early Bird” course was offered twice so far with a high degree of success, first before the beginning of the winter term 2006/07 and in the following year before the winter term 2007/08. The participants attended this 9 weeks course and afterwards took the same exams as students that attended the modules in the summer term and took their exams before the beginning of the winter term.

This allows a direct comparison between “Early Bird” participants and “normal” students. The “Early Bird” students performed outstandingly well in both exams, “Calculus I” and “Linear Algebra”. In 2006 approx. 78% of the “Early Birds” passed in “Calculus I” (compared to 54% of the Non-“Early Bird”-students) and 89% in “Linear Algebra” (compared to 73%). The passing rate was higher by a factor of 1.44 in “Calculus I” and a factor of 1.22 in “Linear Algebra” (see Figures 2 and 3). The success of the course concept could be confirmed in the following year 2007 with similar factors of 1.45 and 1.16, respectively (see Figures 4 and 5).

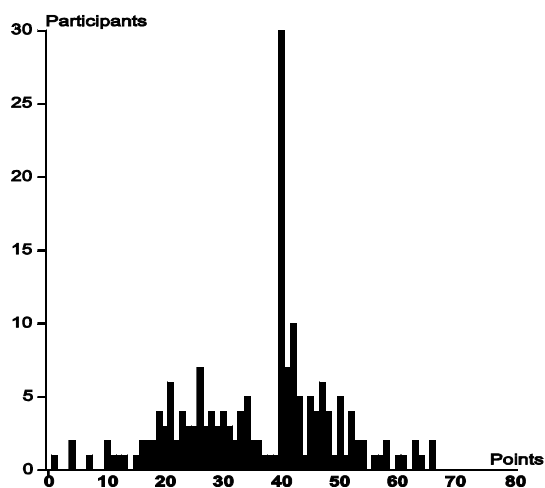


FIGURE 2

CALCULUS I EXAM (09.10.2006), 40 POINTS NECESSARY TO PASS,
NON-EARLY BIRD-PARTICIPANTS, 182 PARTICIPANTS, 99 PASSED (54.4%)

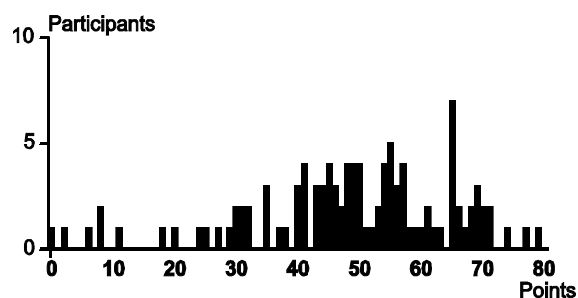


FIGURE 3

CALCULUS I EXAM (09.10.2006), 40 NECESSARY TO PASS,
EARLY BIRD-PARTICIPANTS, 106 PARTICIPANTS, 83 PASSED (78.3%)

The teachers involved in the course remarked on the very pleasant atmosphere during the course and the motivation of the students remained very high throughout the course. One reason for this could be that the participants spent their holiday time for studying voluntarily and so were more ambitious anyway, but the good ratio of teacher and students in the exercise classes almost certainly also contributed.

Nevertheless we do not want to leave unmentioned that the course was very strenuous for the teachers as well as for the students. The workload was very high so that it was necessary to respond with sensitivity to the prospective students' worries (“Can I do all these problems?”, “Will I finish this course successfully?” etc.). A barbecue session after the test exam contributed to the sense of community and made the common goal to pass the course clear. Between freshmen students and teachers as well as among students a good climate developed, which rarely happens in such big classes. The outstanding performance of the “Early Bird” participants indicates the success of this course.

Despite the course's scheduling during summer break there were no difficulties in getting teaching staff for the course. All teachers were freed from their teaching obligations in the subsequent winter term by teaching this class. In this way scientific assistants can use their free time during the semester for research. Tutors can use the free time for their studies, internships or to spend one term abroad, while staying employed at the university and getting paid as well.

All of the lecturers/tutors enjoyed the family-like spirit and the focused work with the students a lot. This was the reason for them to show considerably more commitment and dedication. In addition to the normal tasks and duties teaching staff voluntarily offered additional services like additional office hours, recapitulating-lessons for selected chapters or a study trip during the week between the end of the course and the exams. Surely the high motivation on both sides, students and teachers, is contributing to the success of the course.

It is already possible to see that this course is very popular. Before the first run a lot of effort had to be undertaken to inform about this opportunity. Viral marketing did the better part of this work in the second run: the Early Bird course before winter term 2007/08 has been recommended to most of the participants by friends and relatives. Because the endowment with staff is not unlimited only about 2/3 of the applicants could be admitted to the course. Especially students already enrolled that wanted to use this course as a revision course had to be rejected.

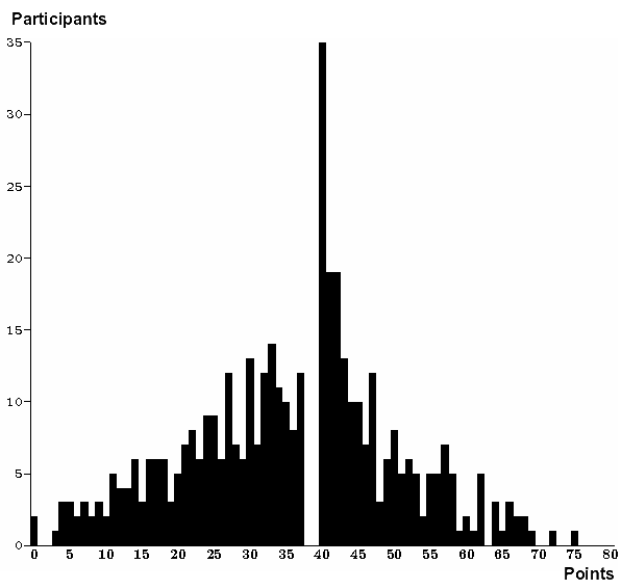


FIGURE 4
CALCULUS I EXAM (08.10.2007), 40 POINTS NECESSARY TO PASS,
431 NON-EARLY BIRD-PARTICIPANTS, 204 PASSED (47,3%)

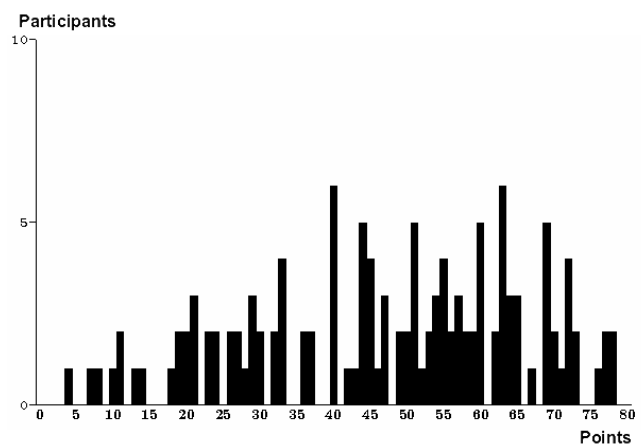


FIGURE 5
CALCULUS I EXAM (08.10.2007), 40 POINTS NECESSARY TO PASS,
128 EARLY BIRD-PARTICIPANTS, 88 PASSED (68,8%)

7. Conclusion

Altogether “Early Bird” was received very positively by freshmen students as well as lecturers/tutors. Freshmen appreciated the intensive support compared to the regular courses during semester; study groups collaborated more closely than their counterparts within the “regular” lectures. Teachers and tutors say that the atmosphere with very dedicated students in small groups is more pleasant than in the other courses. Nevertheless we should keep in mind that the course is very strenuous for all participants. For nine weeks lecturers and

students are exclusively doing mathematics. Yet over 95% (2006) respectively 98% (2007) of the students said that they would recommend this class to others. In the end the excellent performance of “Early Bird” participants speaks for itself, where already the same performance as that of the regular students would have to be seen as a success because the students have the mathematics at their disposal when it is actually needed in other classes in the first semester.

We are convinced that this course together with its didactic concept on the one hand supports students in finding a deeper understanding of mathematics and mathematical relations and on the other hand provides them some understanding of the tools they need to solve practical problems in their further studies and their professional life.

Whether the participation in the “Early Bird” course affects students’ success in the long run remains to be analyzed. This analysis will be performed after the next semester, when more data of the participants’ progress in later courses becomes available.

The course is so popular that many there are many applications in advance. Because of the great success the Department of Mathematics decided to offer the next “Early Bird” course before winter term 2008/09 without special funds by the university.

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