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Abstract. In the last decade the integration of technology in the teaching and learning of mathematics has increased substantially. As a result many graphical ideas, some numerical ones, and some good modeling problems have found their way into precalculus books. At the same time, relevant concepts, tools, and approaches, now available via technology, are being ignored. In this paper, we examine how twelve currently popular precalculus textbooks, published from 2000 onwards, have incorporated these concepts. In addition, some preliminary results on possible connections in how these concepts are understood by a group of in-service and pre-service secondary mathematics teachers are also presented.

More than a quarter of the mathematics content taught before the arrival of the scientific calculator is not taught today [Tooke, 2001]. Indeed, technology's influence on today's classroom instruction is readily observable from changes in content, assessment, and teaching methodologies. The multi-representational capabilities of graphing calculators have blurred grade-level distinctions previously associated with various content tasks, providing more students with access to significant mathematics at younger ages [Usiskin, 2007]. The ability to automate cumbersome calculations via technology allows students at various levels to i) use technology to meaningfully explore concepts and problems previously proposed to the most advanced mathematics students, and ii) to extend the breadth and depth at which these concepts are treated. Technology can be used to promote conceptually-oriented instruction by facilitating to include more relevant applications, exploration and discovery in the mathematics classroom. As a result, teaching has become more student-centered, with inquiry playing a more pronounced role in both the delivery and the content of new curriculum. Not surprisingly, assumptions about mathematics curricula made in a time prior to the integration of technology in the classroom are, in some cases, no longer valid. Topics such as optimization, different matrix applications, linear and nonlinear regression, recursion etc. are now accessible to students in secondary and introductory college levels (prior to calculus). More importantly, the numerical and graphical capabilities of hand-held graphing technology can be used to introduce key concepts foundational to calculus at the secondary level,

using different representations, in a way that is similar to how these concepts were developed and how they are better understood, namely, using approximations.

After 17 years of using graphing technology, it is appropriate to ask: Are we taking full advantage of the main capabilities that this technology offers in order to provide precalculus students with the best possible preparation for calculus? To answer this question we first established how the integration of technology expands the study of families of continuous functions at this level. Although some of these topics should be covered in algebra, we decided to analyze how these functions are treated by precalculus textbooks in light of the availability of graphing technology, and the knowledge that pre- and in-service teachers have regarding continuous functions. An inspection of modern precalculus texts reveals a curiously uneven approach to technology use. While many texts make use of technology's visualization capabilities - not only in the scope of the content, but also in the way that many concepts are introduced, and in the type of questions that students are asked - the same textbooks typically fail to exploit technology's powerful numeric and multi-representational capabilities. As we informally perused a number of current textbooks, we began to reconsider the role that technology actually plays in today's classrooms. In this paper we aim to address the following question: Are our textbooks taking full advantage of the main capabilities that technology offers, in order to provide precalculus students with the best possible preparation for calculus? In addition, we also present some preliminary results on possible connections in how these concepts are understood by a group of in-service and pre-service secondary mathematics teachers. One can hardly expect that students master a concept that is not properly addressed in the textbook and it is not well understood by the teacher.

1. How the Integration of Technology Expands the Study of Functions in Precalculus

Before we start addressing topics that can be introduced to strengthen students' conceptual understanding and overall preparation for calculus, it is important to decide on what criteria we shall use to select the concepts, mathematical models, or tools that we include. We believe that any new topic should be tested based on i) the relevance of the concepts that it addresses, ii) its usefulness, as defined by the variety and importance of

applications that the topic can be used for, and iii) its accessibility, based on the student's current level of acquired skills and mathematical maturity [Quesada, 1996].

We list next major possible changes that technology facilitates on the traditional coverage of continuous functions [Quesada, 2007].

- I. The learning of basic transformations, such as $f(x) + a$, $f(x + a)$, $-f(x)$, $af(x)$, $f(ax)$, $f(-x)$, $|f(x)|$, and $f(|x|)$ facilitates the study of families of functions, each with a root or parent function.
- II. The introduction of a technologically balanced approach via the analytical, graphical, and numerical aspects of functions simultaneously. Hence, students can support graphically and/or numerically analytical solutions and vice versa, whenever possible; moreover, they can find some irrational solutions that would not be available analytically [Demana & Waits, 1998].
- III. In addition to the properties traditionally considered for every family of continuous functions, the integration of technology allows us to include:
 - a. finding the range of all functions studied,
 - b. determining irrational zeros, hence all of the real zeros,
 - c. finding local extrema, with intervals where the function is increasing or decreasing,
 - d. using sequences to explore the local and end behavior,
 - e. comparing relative growth of different functions or families of functions,
 - f. considering relevant examples of data that can be modeled via regression by the family of functions studied,
 - g. optimization problems, using different approaches, at different grades [Quesada and Edwards, 2005].
- IV. The toolbox that we provide to the students can now include the use of nontraditional tools such as tables, lists, sequences, and recursion to solve different problems.

It is important to mention that in the same way that the integration of technology in elementary and middle school makes the need for mental calculations and estimation crucial, the ability to sketch (without technology) the graph of continuous functions at the secondary level should become increasingly relevant.

One cannot ignore the fact that with every new technology a new set of misconceptions and students errors begin to appear. Familiarity with some of the common misunderstandings associated with the graphing technology, namely that “every function seems continuous,” “truncation errors resulting from exceeding the precision of the machine,” “use of inadequate window,” etc. helps the teacher to better integrate technology. For a selection of examples addressing many of these ideas, the interested reader may want to look at [Quesada, 2007].

2. The use of technology in Precalculus textbooks

To date, mathematics textbooks must cover a wide variety of concepts mandated by standards from different states and schools [Seeley, 2003], with the result that many textbooks are extremely large in size. Inconsistencies between the homework and main text, as well as large overall textbook size, with potentially over 100 homework problems per section, gives teachers the perplexing job of deciding what to emphasize in a text [Tarr et. al, 2006].

Since the U.S. lacks a centralized educational system with an established curriculum followed by all textbooks, we developed a rubric, and used it to review a representative sample of precalculus textbooks at the college level, including some of the best sellers in the market. An inspection of modern precalculus texts reveals a curiously uneven approach to technology use. While many texts make use of technology's visualization capabilities - not only in the scope of the content, but also in the way that many concepts are introduced, and in the type of questions that students are asked - the same textbooks typically fail to exploit technology's powerful numeric and multi-representational capabilities.

Twelve commonly used college precalculus textbooks, all published from 2000-2008, were evaluated using a Likert scale of 0-2, both in every family of continuous functions presented in the text, and in adjoining homework sets. Scores were later converted into average percentages, both individually, and as a whole. For a textbook to obtain a score of “1” in either the main section or the homework section, that book had to mention the concept, or present at least one detailed homework problem. To merit a “2”, the textbook had to present a thorough explanation of the topic (in the main text) or have at least two

detailed problems on the topic in the homework. Textbooks were separately evaluated for the criteria for each family of functions. Concepts examined included sketching polynomials, the idea of a complete graph, domain and range, finding solutions numerically via tables, graphical solutions (both $f = g$ and $f - g = 0$ approaches), a technology-balanced approach; use of equations and inequalities involving different transcendental functions or transcendental and algebraic functions, relative growth, use of inequalities, local and global behavior, local extrema, optimization, increasing/decreasing functions, transformations, and sketching.

Results:

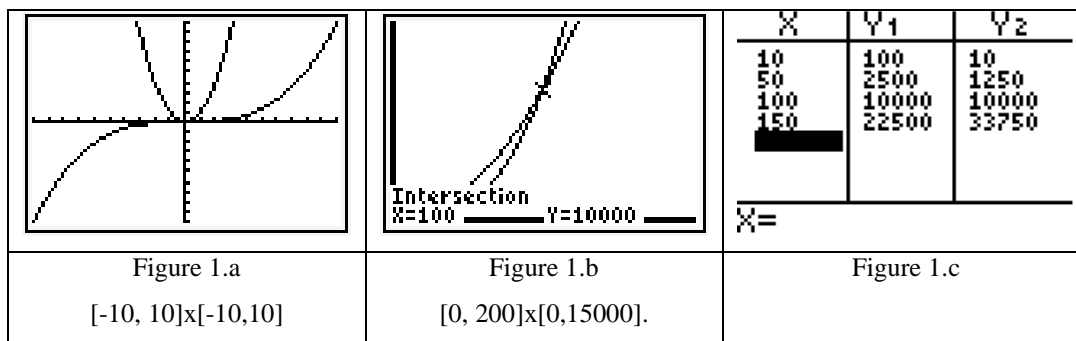
The examined textbooks appear to place varying emphasis on the evaluated concepts. To illustrate, the concept of global behavior is barely mentioned ($< 50\%$) in one textbook, while it is explored extensively (90%) in every family of functions in another text. One quarter of the textbooks score poorly in almost all topics. Some books score high ($>75\%$) on three or four criteria, with only moderate scores (51-75%) in other categories. Scores do not necessarily increase in more recently published books.

Key ideas are normally not revisited with every family of continuous functions. All topics, with the exception of local/global behavior and extrema, were covered most thoroughly in sections on polynomials, with lesser coverage for other families of functions.

In general, weak areas, with average usage less than 50%, included such criteria as numerical solutions, graphical solutions finding zeros, a technology-balanced approach (Quesada, Smith, & Edwards, 2008); the use of transcendental functions; relative growth; problems containing inequalities; optimization; sketching polynomials (turning points, effect of leading coefficient), and the concept of a complete graph. Other criteria were mentioned in almost every textbook and extensively discussed in several, including finding local and global behavior numerically (53%), local extrema (58%), and increasing and decreasing functions (60%). No single textbook addresses every topic; even the highest scoring book barely mentions complete graph and relative growth. While most books emphasize sketching (81%), transformations (67%) and domain and range (68%), they are often presented as detached concepts: few connect these ideas to

the idea of a complete graph. Beyond definitions, little is done on finding the range graphically or numerically for most families of continuous functions, or on the effect of transformations on domain and range. Most textbooks do not link calculator usage with numerical solutions (tables) when determining global behavior, to enable students to unify these ideas conceptually.

In general, textbooks tend to ignore or omit discussions of relative growth that focus on numerical aspects of functions despite the fact that such explorations, via tables, is straightforward, efficient, and conveys the numerical sense of which functions grow faster or slower. In our experience, many students find it easier to use tables than graphs to answer relative growth questions. Problems requiring students to find intersections of graphs lying outside the standard calculator viewing window were noticeably absent, despite the fact that such problems require knowledge of relative growth. For instance, when asked to find all intersections of the functions $f(x)=x^2$ and $g(x)=0.01x^3$, students who rely exclusively on the standard viewing window (Figure 1.a) typically fail to identify the intersection (100, 10000) shown in Figure 1.b. As seen in Figure 1.c, using the table to evaluate both functions for increasing values of x , allow the students to easily determine not only which function grows faster for arbitrarily large values of x , but also to find a reasonable interval where to look for the hidden intersection. Students need to recognize when the algebraic approach is easier to use, and how the theory of polynomial equations helps to determine the number and parity of possible solutions.



Most books do not address these ideas in a recurrent way with algebraic and transcendental equations. Moreover, more books emphasize graphical solutions involving the intersection of functions (59%) rather than graphically finding zeros of the difference

of these functions (46%). A few books describe both methods, and then allow students to select their preferred method when solving homework problems. Yet, finding the zeros numerically or graphically tend to be easier graphically than finding the points of intersection.

Table I: Analysis of 12 College Precalculus Textbooks published 2000-2008

TEXT AVERAGES	HOMEWORK AVERAGES	Text-HMK	Combined Average	TOPIC
0.81	----		0.81	Sketching Polynomials (turning pts, leading coeff.)
0.74	0.55	.19	0.65	Complete Graph
1.45	1.28	.17	1.36	Domain and Range
0.25	0.03	0.23	0.12	Numerical Solutions
1.22	0.98	0.24	1.18	Graphical Sol: $f=g$
0.83	1.00	-0.17	0.92	Graphical Sol: $f-g=0$
0.68	0.63	0.05	0.65	Technology-balanced approach
0.71	0.89	-0.18	0.80	Eq & Ineq involving different Transcendentals/Alg functions
0.85	0.66	0.19	0.75	Relative Growth
0.77	0.91	-0.14	0.84	Inequalities
1.13	0.99	0.14	1.06	Local and Global Behavior
1.14	1.18	0.04	1.16	Local Extrema
0.62	1.05	-0.43	0.83	Optimization
1.26	1.15	0.11	1.21	Intervals where f is increasing/decreasing
1.52	1.15	0.37	1.34	Transformations
1.42	1.82	-0.40	1.62	Sketching rational and trig functions
Likert scale: 0= No; 1 = mentioned; 2 = well explained with an example/several homework questions)				

Only 17% of the textbooks emphasize the use of inequalities with all families of functions; other authors briefly work with inequalities in just one or two sections, primarily in chapters on polynomials. Solving equations rarely integrates numerical, graphical, and algebraic approaches, or contains equations involving transcendental and algebraic functions, or different transcendental functions. Most books do not include inequalities involving transcendental functions, yet inequalities make students think graphically!

Although 33% of the textbooks found numerical solutions via tables; only one examines this topic in detail. While most place an emphasis on transformations and sketching of functions, many do not relate these ideas to those of the complete graph, while over 66% do not discuss the idea of the complete graph of a function in detail. However, an awareness of when a window displays all the important features of the graph of a function is essential for approaching problems graphically.

Placement of Concept:

The presentation of most concepts was evenly balanced between main text and homework, but there were some surprising findings. A few concepts (particularly optimization and sketching) were presented mainly as homework, rather than in the main text; they were prominently placed at the beginning of homework sets. In contrast,

TABLE II: Placement of relative growth problems in six precalculus textbooks

Textbook	Page Number	Problem No. /Total in section
A	323	# 78/82
C	241	37/38, 52/70
D	106	40/45
G	360	80/81, 81/81
L	309	88/108
S	91	66/69, 67/69

relative growth problems (38%) were usually placed at the end of long homework sets, typically as extension problems, rather than being discussed in the main text (Table II); relatively few homework exercises require the knowledge of relative growth to solve problems such as finding hidden intercepts. In conclusion, these textbooks offer varying content on the concepts studied, with often inconsistent reinforcement of concepts throughout the body of the text.

3. Possible Connections to Student/Teacher Knowledge

We are currently researching the preparation of in-service and pre-service teachers via a 90 minute test consisting of 33 problems addressing the same concepts used as criteria in the textbook evaluation. Textbook ratings were compared to test scores of 85 in-service teachers and 14 pre-service teachers.

TABLE III: Comparison of textbook and preliminary pre-service and in-service teacher scores on key concepts

Topics	Textbooks	Pre-service	In-service
Properties of functions (range, extrema, inverses...)	62.1%	47.7%	46.5%
Sketching functions (polynomials, rational, trig...)	60.8%	39.9%	58.1%
Equations from graphs/properties	52.5%	53.2%	41.6%
Global and Local behavior	53.0%	55.4%	58.7%
Transformations	67.0%	32.0%	38%
Relative growth	38%	12.8%	28%
Optimization	41.5%	25.0%	34%
Inequalities	42.0%	45.8%	60%
Average of all topics		44.5%	46.2%

The in-service test was administered during the summer of 2007, while the pre-service test was given in the spring of 2008. Each individual's score in every concept area was calculated as a percentage. For this comparison, some textbook criteria, itemized above, were combined (Table III). Only in the areas of local and global behavior and inequalities the teachers scores are better; this may be the result of further reinforcement received by the teachers while taking the calculus sequence. In the remaining areas, the teachers' performance was lower or within 1% of the textbooks results. These results may indicate the existence of some connection between the amount of attention paid to these topics in precalculus texts, and the conceptual knowledge of the same topics by both in-service and pre-service teachers. High textbook scores for transformations and sketching, as contrasted to test scores, may possibly be due to an imbalance between text and homework problems in the textbooks: students may simply not be getting all of the

fundamentals from the text (with sketching and optimization) or not getting enough homework problems with which to practice (in the case of transformations.) Although the data used for pre-service teachers was obtained from only one college, the group of in-service teachers received their degrees from a large number of different colleges, so it is surprising that we found a correlation $r = 0.75$ between pre- and in-service teachers' scores. The data suggests that further investigation, with larger groups of students, may help refine these connections.

Summary

As one surveys today's textbooks, it is clear that the recommendations of learned societies (such as MAA and NCTM) and changes in technology have precipitated shifts in precalculus instruction. However, many remnants of the pre-technology curriculum remain. The result is a "continuous extension of the previous edition" approach; hence new, relevant mathematical ideas are juxtaposed with an assortment of topics of dubious worth. This results in many textbooks nearing 800 pages, the conflict continues between adding new relevant ideas to the curriculum and the need for eliminating some of the traditional material. Clearly, as evidenced from our inventory of school texts, a need to reevaluate the essential components of precalculus in a technological world currently exists. We need to ask ourselves the following question as we begin to modernize the precalculus curriculum: Do the topics we teach respond to a conscious decision based on their relevance and accessibility, or are we still teaching topics because traditionally they have been taught at this level? Symbolic calculators have been in the market for ten years, and they bring their own set of curricular and pedagogical questions: Should we not agree on most of the questions raised here before moving forward? In addition, our preliminary data on in-service and pre-service teachers seems to indicate that further research is warranted to further illuminate the effects of textbook inconsistencies on students' and teachers' understanding of these relevant concepts.

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