MATHEMATICAL TASKS IMPLEMENTATION IN THE U.S. AND CHINESE CLASSROOMS

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***Abstract**. This paper investigates the features of classroom instruction in the U.S. and China through examining the cognitive demands of mathematical tasks and the strategies used to implement the tasks. Based on find-grained analysis of 10 consecutive lessons in each of the two countries, we came to the following conclusions: the U.S. and Chinese teachers tried their best to implement high level cognitive demands in the classrooms through effectively demonstrating high level performance, appropriate soliciting and use of students' answers, and appropriately organized exploratory activities; the Chinees teacher seems to be successful at maintaining a high level when implementing the tasks while the U.S. teacher had some difficulties in doing so.*

BACKGROUND

The efforts to pursue high quality classroom instruction in the U.S. have led to an increased interest in exploring instructional practices in high achieving education systems in East Asia, including China. It has often been observed that classroom instruction in China is exam oriented，lecture dominated and strictly controlled by the teachers, all of which seem to be associated with a passive learning environment (Biggs & Watkins, 2001). However, some studies found that there were some active and student-centred learning features in Chinese classrooms (Huang & Leung, 2004; Mok, 2006; Stevenson & Lee, 1995). On the other hand, comparative studies indicated that the teaching observed in U.S. eight-grade mathematics classes did not reflect the innovative pedagogical practices advocated in reform documents and recent research (Jacobs, et al., 2006; Wood, Shin & Doan, 2006). Based on a detailed analysis of three U.S. classrooms taught by competent teachers, Wood et al. (2006) were struck by the finding that "the presentations are divested not only of reasons, but are also completely devoid of any richness of thought that allows the learner to reason and gain insight into what one is doing mathematically when using the procedure" (p.83). These unexpected descriptions of mathematics classroom instruction in the United States and China encourage us to compare the classes in the two countries in order to get a deeper understanding and possibly shed insight on the improvement of classroom instruction in both countries. Through an analysis of pedagogical representations in U.S. and Chinese classrooms in which linear equations were extensively explored, Huang and Cai (2007) found that the U.S. teachers seemed to develop multiple representations simultaneously over subsequent lessons through different activities, while the Chinese teachers tried to develop symbolic representations and graphic representations through solving problems by making use of tabular and numerical representations. These findings help us to understand why U.S. students preferred to choose concrete strategies and drawing representations both for fostering understanding of a concept and also for applying knowledge (Cai & Lester, 2005). This paper investigates the features of classroom instruction through examining ways of implementing mathematical tasks.

THEORETICAL CONSIDERATIONS

Mathematical task and quality of student learning

The roles of mathematical tasks to engage students in thinking and reasoning about important mathematics ideas has been recognized and explored by many researchers (Doyle, 1988; Stein et al., 2000). Although mathematical tasks are given different terms such as “instructional tasks” (Hiebert & Wearne, 1993) and “academic tasks” (Doyle, 1983), generally tasks refer to projects, questions, constructions, applications, and exercises in which students engage. Mathematical tasks are central to students’ learning because “tasks convey messages about what mathematics is and what doing mathematics entails” (NCTM, 1991, p.24). Mathematical tasks provide an intellectual environment for students to learn and develop mathematical thinking. The nature of mathematical tasks can lay potential influence and structure the way students think and can serve to limit or to broaden their views of the subject matter (Henningsen &Stein, 1997). A number of studies have showed there are connections between mathematical tasks and student cognition and learning (Stein, Grover& Henningsen, 1996).

Factors associated with the maintenance and decline of mathematical task levels

Mathematical tasks often lend themselves to certain types of conversations that may stimulate rich discourse, which foster higher order thinking (Silver & Smith, 1996). The difficulties of maintaining higher order mathematical thinking and reasoning throughout tasks implementation has already been noted by researchers (Doyle, 1988; Henningsen & Stien, 1997). Hennigsen and Stein (1997) identified and illustrated several classroom-based factors that support and inhibit high level mathematical thinking and reasoning. According to Stein et al. (2000), mathematical tasks can be examined in terms of their cognitive demands set up by the teacher and implemented by the students in classroom. Further, mathematical tasks are categorized into four different levels of cognitive demand:

memorization, procedure without connection, procedure with connection and *doing mathematics.* Stein and Smith (1998) identified several factors associated with maintaining and declining high-level demands. In this study, we intended to identify the factors associated with the implementation of mathematical task in these U.S. and Chinese classroom.

METHOD

Data resource

The schools and teachers. In this study, we selected one school's data set from China and one from the United States from LPS data (Clarke et al., 2006). The reason for this selection is the similarity of the content taught and comparability of the schools' background (Huang&Cai, 2007 for detail).

The contents and lessons. The Chinese teacher started with an introduction to the concept of linear equations and the solution and introduced the concept of rectangle coordinate planes to graph linear equations, and the concept of the system of linear equations with two unknowns and its solution. After that, several methods to solve a system of linear equations with two unknowns were introduced and consolidated. The U.S teacher started by introducing the concept of linear and non-linear relations in general, and then the teacher discussed extensively the features of linear relations and focused on the transformation of multiple representations of linear and non-linear relations through group activities. Finally she applied this knowledge to solving word problems. The teacher intended to develop the concepts (linear and non-linear relations) and foster understanding of the features of linear and non-linear relations, through multiple representations and students' group work.

Data analysis

The data analyses mainly focus on the mathematical tasks in first four Chinese lessons (CH1- CH4) and the middle six U.S. lessons (US3 - US8) because these lessons included extensive coverage of linear relations. Stein and Smith (1998) identified several factors associated with maintaining and declining high-level demands. Based on Stein and Smith's framework and using a pilot check with the data in this study, we developed the coding system as showed in Table 1.

Table 1. Factors associated with implementing mathematical tasks

Category	*Decline Factors*	*Maintenance factors*
Student thinking and reasoning (SR)	The teacher takes over the thinking and reasoning and tells students how to do the task (SRD)	Scaffolding of student thinking and reasoning (SRM)
Demonstrating problem-solving behaviour （DM)	The teacher emphasizes correctness of completeness of the answer (DMD)	Teacher or capable students model high-level performance (DMM)
Questioning style (QS)	Closed or yes or no questions (QSD)	Open, probing questions (QSM)
Feedback of student answer (FB)	Students are not held accountable for high-order product or process (FBD)	Emphasizing justifications, explanation through comments or feedback (FBM)
Concepts connection (CC)	Demonstrates answer without emphasizing relevant conceptual connection (CCD)	Teacher draws frequent concept connections (CCM)
Exploratory activity (EX)	Inappropriate task or inappropriate exploratory time (EXD)	Appropriate task or sufficient exploratory time (EXM)

Some of the above codes are illustrated in the following episodes. For example, regarding open questioning (QSM), the teacher in US2-L07, questioned her students as follows:

T Every group. Either look back at your work if you were able to do these, or if you had difficulty with these pairs in your homework.

T Get an idea now. And I want every group ready to report a similarity. One minute. Find a similarity, a similarity between these two equations.

With regard to teacher feedback (FB), the teacher in SH3-L03M commented on and built on the students' answers as follows:

T: Okay, let's exchange our ideas, for this point on the x-axis. What are the characteristics of the coordinate? On the y-axis, what are the characteristics for such a coordinate? Fino.

Fino: That is. . . the numbers on the x-axis, the former, um, the latter one must be zero.

T: The latter one. . . must be zero, so what coordinate is the latter one called?

Fino: Vertical coordinates.

T: Right, vertical coordinate is zero, on the x-axis.

Fino: The coordinate on the y-axis is zero.

T: The coordinate on the y-axis is zero. Are your discussion results the same as his?

E: The same.

With regarding the coding system, the first author and a research assistant carefully developed a code using specific video analysis software, *studio code*. After coding several lessons, it was found that there were some periods when the teacher did not organize the students who were working with mathematics task. Then, we added one more code: unrelated mathematical task.

RESULTS

Factors associated with maintenance and decline of cognitive demand

The percentage of time spent in different activities when implementing mathematical tasks are shown in table 2.

Table 2. Time distribution in different activities in the U.S. and Chinese classrooms

	U.S. lessons				*Chinese lessons*				
Category	*L34T(%)*	*L56T(%)*	*L78T(%)*	*Total(%)*	L01	L02	L03	L04	Total
SRM	4.2	0	3	2.6	4.9	0	0	4.4	2.4
SRD	5.7	10.5	7.5	7.7	0.9	0	0	0	0.2
DMM	1.5	0	0	0.6	12.1	23.2	5.2	11	12.8
DMD	3.7	3.2	8.3	5	0	0	13	0	3.1
QSM	2.7	0	1.1	1.4	0	0	6.4	2.2	2.1
QSD	8.8	0.4	0	3.5	0	0	0	0	0
FBM	8.5	0	0	3.3	9.8	27	20.6	15.3	17.9
FBD	6.2	0	18.2	8.3	0	1.1	0	0	0.3
CCM	9.3	12.7	4.8	8.8	3.1	10	0	2.8	3.9
CCD	0	0	0	0	0	0	0	0	0
EXM	13.4	23.4	22.4	19.2	23	28.5	27.1	50	32.5
EXD	26.9	0	0	10.3	0	0	0	0	0
Unrelated	6.4	0	16.4	7.8	0	0	0	0	0
Duration(min)	104.6	78.5	89.5	272.6	44	40.3	40.5	45.5	170.3

The above table indicates that the U.S. teacher was apt to emphasize the connection of concepts (CCM, 8.8%), and organize exploratory activities (EXM, 19.2%). However, sometimes, the task demands and exploring time are inappropriate when organizing exploratory activities (EXD, 10.3%). In addition, the students were not properly encouraged to justification and explanation (SRD, 7.7%). Moreover, inappropriate model of high level behaviors (DMD, 5%), more closed questioning, and occasional "taking over" students' answers (QSD,3.5%), or inappropriate feedback to students' answers (FBD,8.3%) may cause the decline of high level of cognitive demands. It is also a problem that the teachers wasted time (7.8%) doing unrelated mathematical tasks.

As can be seen in the above Table, the most popular activity was the effective organization of exploratory activity (EXM, 32.5%). And the appropriate response to students' answers such as praise and using students' answers for further development of new ideas etc. (FBM, 17.9%) were also common strategies that maintained high levels of thinking. In addition, effectively demonstrating high levels of performance by the teacher or students (DMM, 12.8%) was used frequently to achieve a high level of implementation of mathematical tasks. It was found that sometimes, the teacher was not able to demonstrate a high level of performance (DMD, 3.1%), and it is hard to identify the activities which inhibited high level cognitive demands.

Comparing the time distributions in different activities in the U.S. and Chinese classrooms, we found that (a) the Chinese teacher fully used classroom time focusing on mathematical task implementation while the U.S. teacher spent some time doing unrelated mathematics tasks; (b) the Chinese teacher was more skillful in maintaining a high level of cognitive demands than the U.S. teacher was. In addition, the Chinese teacher was more apt to organize an exploratory activity than the U.S. teacher was.

CONCLUSION AND DISCUSSION

Based on the above analysis, we came to the following conclusions: the U.S. and Chinese teachers tried their best to implement high level cognitive demands in the classrooms through effectively demonstrating high level performance, appropriate soliciting and use of students' answers, and appropriately organized exploratory activities; the Chinees teacher seems to be successful at maintaining a high level when implementing the tasks while the U.S. teacher had some difficulties in doing so.

U.S. and Chinese teachers are attempting to provide students with high level cognitive mathematics tasks. These attempts may reflect the positive influence of math curriculum reforms in the two countries in the past decade. The finding that the Chinese teacher is able to implement mathematical tasks that maintain at a high level through different teaching strategies is echoed by Ma's (1999) description of elementary mathematics teachers in China who are apt to adopt multiple teaching strategies. The difficulties in implementing and maintaining mathematical tasks with high level cognitive demands have been recognized by many researchers (Doyle, 1988; Stein, & Smith, 1998). The challenges faced by the U.S. teacher enrich the details about which factors are associated with the decline of the cognitive level of mathematical tasks. Thus, it should be meaningful to determine why the Chinese teacher is able to maintain a high level while the U.S. teacher was not as successful at doing so. Considering that these two teachers are competent in terms of educational background and teaching experience by local standards, the contrast in performances in classroom instruction may be related to other factors such as the teacher's subject matter knowledge or the teacher's belief about teaching and learning, or the teacher's underling culture values. It is worthwhile to study this further.

In addition, since the data analysis mainly was done by a Chinese researcher who has never been in U.S. classrooms, there may be a disparity in the understanding of the U.S. classroom. Moreover, due to the limitations of the data, only examining consecutive lessons in two classrooms, researchers should be cautious in interpreting the findings of this study.

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