

OPERATIONS WITH WHOLE NUMBERS: TEACHERS' UNDERSTANDING

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Abstract

This paper analyzes school teachers' interpretation of the students' procedures while working out whole number calculations. The students are from the 2nd grade (6-7 year-olds) of a private school in the City of São Paulo in Brazil. The teachers teach 3rd to 5th graders (7-10 year-old pupils) at the same school. The paper offers an overview of the work done in class with the students to contextualize their production, which is the object of the interpretation by the investigated teachers. In the analysis, it is explained that, even though the teachers are aware of both the use of negative numbers by the children while working out their tasks and the context of their productions, these facts are not always realized by the teachers. Therefore, the teachers have a lack of understanding of the operations with whole numbers. This study has the potentiality to contribute to a debate concerning the Mathematics syllabus proposals for the initial grades and the programs developed in courses for High School Teaching Certification in Education, both in Pedagogy courses and in other formation processes.

Key words: whole number operations; negative numbers; initial grades; teacher's formation

The knowledge of whole numbers in Arithmetic is essential, because it is required in several activities, being intimately interrelated to the knowledge of Algebra. In teaching mathematics, this aspect is important because the analysis of the procedures used by students can reveal algebraic properties and concepts *implicitly* used by them. Because they are implicit, it is possible that teachers do not realize their use. In Machado (2003), researchers studied the possibility of work expansion in the field of whole positive numbers, to the additive group of whole numbers, among 8-9 year-old students from the 4th grade of the Elementary School. It was thought that, in case we considered such an expansion convenient, a wide range of study would

be opened for the teaching of mathematics. This expansion is unheard of among 8-9-year-old students from the 4th grade of the Elementary School in Brazil. This is because the Mathematics National Curricular Parameters (Parâmetros curriculares nacionais: matemática, 1998) revealed, in its consideration of previous proposals that such an extension had never been suggested before for the Elementary School. Teaching is still restricted to working with natural numbers until the 5th grade of the same teaching segment (6 -11 year olds). The previously mentioned document refers to a study of the additive conceptual field based on Vergnaud (1994). The study does not focus the use of negative numbers by children in their initial grades, but focuses it on the Middle School students (11 - 14 year olds). More recently, the spontaneous use of negative numbers by children from the initial grades has been identified in the cognitive psychology perspective research by authors like Borba & Nunes (2004), in a publication of international recognition. Among the research that focuses on teachers, either from didactical or from their formation points of view, we only find some generated by our own research group, which deals with such themes in the initial grades. Nevertheless, it is equally important to diagnose teachers' interpretation of the children's work that shows this handling of negative numbers. It is also necessary to mention in a detailed way the conditions and the context in which these phenomena take place. Bearing in mind the former premises, we carried out the present investigation, which analyzes eight 3rd grade Fundamental School teachers' interpretations, in regard to the solution of a calculation involving whole numbers by 2nd Grade Elementary School students. This research aims to support the understanding of the knowledge formation process about operations with whole numbers, which contribute to the debate concerning High School Teaching Certification in Education, both in Pedagogy courses and in other formation processes.

Methods

Because of the queries involved in the present study, the description of the practices in the investigated institution is relevant, focusing on signification and the significant attributed to concepts by the group which participated in the research. A wide range of data was used and it was carried out by a researcher who has been working in the investigated school for over ten years. Because of these features, this work constitutes a research which is tightly connected to the ethnographic type (André, 1998).

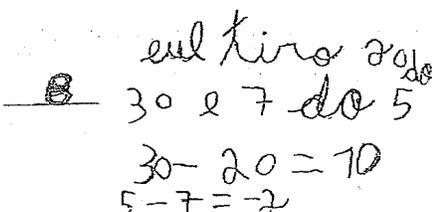
The investigated school has as a usual procedure to record all of the conversations of all meetings. These are dated and copies are filed together with copies of the texts which had been read or studied. When an individual's or a group's work is required about a specific subject, copies of such work are attached to the previous material, in addition to the work done by school students that are related to the subject of the meeting. This procedure allows recovery of data for research reports like this one, for which I selected data covering the period from October, 2004 to October, 2006, that would permit detection of teachers' concepts on the theme, revealing certain knowledge necessary to the comprehension of students' ideas that operate with negative numbers. The teachers of the Elementary School and the Mathematics advisor of this teaching segment, who participated in this research, were general Pedagogy graduates. This advisor has been working for about 20 years with these teachers. According to the Director of the school, she was chosen for this position due to her didactic and mathematical knowledge. Because I have an M.S. in Mathematics and a PhD in Mathematical Education, and because I am a researcher in this field, I have been working as a consultant in Mathematics for this teaching segment in the investigated institution. The school conducts about 4-hour monthly meetings with me and the Mathematics advisors for Kindergarten and Elementary School for the development and implementation of the Mathematic Integrated Project based on the School Pedagogical Project. These meetings aim to unify the principles, goals and educational values of Mathematics and the deep education of such teachers in this kind of knowledge, their mathematical content and didactics of this content in the different levels. It is relevant to mention that, before this research, all the school pedagogical team had studied the *tool-object dialectics* and the interactions among domain theoretical frameworks by Douady (1984) and the categories of additive problems by Vergnaud (1996). They had also studied the children's hypothesis about reading and writing numbers; and the numerical decomposition for calculations that the children performed, based on the work by Parra & Saiz (1996). The texts from these works were read; classes were prepared and analyzed with students' protocol, based on these authors. At many of the meetings with the advisor and the teachers who worked directly with me (Kindergarten and Middle School teachers), many times, aspects of Mathematics, Mathematics Didactics and Methodology of Research were studied. In order to do that I used my previous practice of teaching those disciplines in Mathematics Education Post-Graduate courses. We also used numerous investigations of the theme in the process of the study and the research in which we were

involved in. Moreover, those teachers were always in communication with several researchers from PUC-SP, who observed their classes in order to investigate topics together with me or tutored by me. It is important to tell that the 3rd to 5th Elementary teachers did not have the same opportunity of contact with the research until we started this one reported here. It is still important to mention in this work that all of the school's pedagogical team participated in an internal event at the school in October, 2004, when several teachers presented collaborative research that were carried out with me and other team members. This work was later published by these teachers in a work organized by me together with Mercadante (Maranhão & Mercadante, 2006). It is especially important to highlight here that the Mathematic coordinating-teacher of the 2nd. grade approved different work performed by her students involving whole negative numbers as a consequence of the collaborative research on activities, which aimed at the meaning of operations on the set of whole numbers.

Soon after the internal event, as a part of the research, we began with the 3rd. grade, the Mathematics advisor and I selected two protocols for the 2nd. grade children, for a diagnosis regarding the knowledge of the 3rd grade teachers concerning the Arithmetic of whole numbers. We agreed to require interpretations from the eight 3rd. grade teachers about the calculations, reminding them of the experience children had had and, following it, we would require discussion of such interpretations. At the area meeting before the last one, at the end of 2004, the 3rd. grade teachers and both the educational and the Mathematics advisors were present. The Mathematics advisor proposed the beginning of the research, which had already received the approval of the teachers and the technical-pedagogical school team. According to the records of the meeting, using copies of the original texts by Maranhão & Mercadante (2006) which were still being prepared, the advisor read a part of the publication in which there were several situations faced by the 2nd. grade students. Games in which they had a negative score, which should be passed to the next step, then discounted; role-playing shopping situations, where students purchased and sold goods, exhibiting records of debt (involving interrelations among: geometrical, arithmetical and financial playing domains). In this part, it was also recorded that the children had solved additive problems in Vergnaud (1994)'s categories based on the games and the role-play in the stores, and the activities with front and back cards inspired on ERMEL (1991). The fact is that several calculations revealed the use of negative numbers (sometimes implicit) by the children. But, I highlight that although the protocols used in the present research

were included in the mentioned publication, they had not been examined at this meeting. Neither have the interpretations by the 2nd. grade teacher concerning the protocols been read. I selected for this research report, only the interpretations regarding the protocol 1, related to the calculation 35-27. This protocol shows *I take 20 out of 30 and 7 out of 5 in child's handwriting*:

Protocol 1

$$35 - 27 =$$


The image shows a student's handwritten work for the subtraction problem 35 - 27. The student has written 'eul tiro do' above the numbers. The numbers are written as '30 e 7 do 5'. Below this, the student has written '30 - 20 = 10' and '5 - 7 = -2'.

Source: Maranhão & Mercadante (2006, p. 34)

The teachers started to write their interpretations on the copy of the children's protocol.

Results

All the teachers admitted from the beginning, that first the child decomposed the numbers involved in the subtraction, and two of them referred to the numerical decomposition in decimal order. The study demonstrated its usefulness as well as did some research carried out in previous years concerning numerical composition and decomposition according to Parra & Saiz (1996).

The teachers' interpretations regarding the meaning attributed by the child to the negative number and to the operation in the set of whole numbers were organized into categories. These categories were ordered according to the frequency of correlated interpretations. In Category A, there are five teachers. *They all admit that the child attributes meaning to the negative number and to the concerned operations in its production in the mathematical domain.* That happened because all these teachers explained that they consider that the child worked with the "negative." However, they did not mention the context faced by the child before working out the problem. Because of the assertiveness of their interpretations, that do not consider the possibility of such meaning concerning the child's experience in several domains, I consider them to belong to the mathematical domain. Thus, especially taking into consideration what those teachers had studied, I reflected that they reveal *restricted concepts of the negative numbers and of the concerned*

operations in the child's work. Besides, I point out that, in this category, *all the teachers show failure in mathematical knowledge*, because: (1) *All of these five teachers admit that for the child -2 corresponds to take out or to subtract.* Because they did not explain or make any comments considering that this would be inappropriate in mathematics, I reflected that *the teachers themselves show a concept of negative number as corresponding to the operation.* Such correspondence was also identified by Borba & Nunes (2004) among children from the initial grades; besides that (2) *Three of these five teachers show failure in the expression involving the natural language of the child's calculation.* Two of them admitted that the child “subtracted -2 ”, *which corresponds to adding 2 in Mathematics.* Another one wrote “while subtracting a smaller number from the bigger one”, when the child subtracted a number from the other one smaller than it, obtaining a negative result; and in addition (3) *One of these five teachers presents failure in the arithmetical expression of the carried out calculation*, because she wrote that the expression $35 - 27$ equals $(30 - 20) - (5 - 7)$, which corresponds neither to the calculation worked out by the child, nor to the proposed calculation. Category B includes the interpretation of three other teachers. *They admit that the child attributed meaning to the negative number and to the concerned operations in its production, concerning the experiences domains in class.* They possibly took into consideration the games and role-play experienced by the child (involving interrelations among: arithmetical, geometrical and financial playing domains). Although this type of interpretation is coherent to a great extent with what the teachers had studied and, therefore, it demonstrates certain didactic knowledge of the content, because of the assertiveness of their interpretations, which does not consider the possibility that the child is operating in the mathematical domain; it reveals restricted conceptions concerning the negative number and the concerned operations in the child's work. In this category, only one of the teachers shows *failure in mathematical knowledge*, probably generated by failure of the understanding of the negative number representation and of the concerned operation in several domains. She interprets that the child realized that the signal “-”, is the same as “I owe something”. From her statement, I concluded that she *attributed meaning to a mere signal, instead of attributing meaning to the negative number -2 , or to the operation $5-7$, expressed in the child's calculation.* This is also found at Borba & Nunes (2004).

Conclusion

Analyzing the interpretations, it is obvious that this strategy of formulating interpretations originating from a student's protocol, having previously reported his/her experiences, was rich as a diagnosis, because it revealed the urgency of the exchange of ideas.

As it could be seen, the teachers' interpretations presented failures in the knowledge of numbers and of operations in whole numbers set. They also reveal failures in the didactic knowledge of meanings corresponding to those concepts in several domains according to Douady (1984). These failures prevented them from understanding the child's ideas.

At a meeting with the advisor, we outlined that the meanings that the students could attribute to a negative number and to the operations, considering their experiences, would be debated taking into account what the teachers had written in their interpretations, and based on Maranhão & Mercadante (2006). Some possibilities of work with 2nd. graders, who would become 3rd. graders, would be considered, too. We would like to know "what in fact was going on in students' minds," therefore we devised interviews with students, which should be given during the children's activities. This part of the project happened in 2007 and will be object of a future publication. It was clear that this was subject for many meetings, and that these should be based on the problems seen in the teachers' practices, mingled with study activities and research concerning the next grade. These could also be presented in other publications. Even having overcome several failures, an intervention as a result of the interpretations concerning operations in the whole numbers set could be useful for the teachers to reach *algebraic knowledge and algebraic didactics knowledge*. This can demand participation by the consultant to explore properties of the whole numbers set including the addition operation. The intention is to lead teachers to understand that a whole number b has an opposite $-b$, and that the subtraction $(a - b)$ of two whole numbers is the addition of a and the opposite of b (approaching features of the additive commutative group of whole numbers set structure). Such knowledge could eliminate failures revealed in the work.

I believe that the present research has reached its aims, by supporting the comprehension of the process of knowledge formation on operations with whole numbers, by contributing to the debate concerning High School Teaching Certification in Education, both in Pedagogy courses and in other courses or other formation processes and the Mathematics syllabus proposals for the initial grades.

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