INFLUENCE OF INTUITION AND ANALYTICAL THINKING ON GRAPHIC REPRESENTATION OF PROBLEM SITUATIONS

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Abstract

This work aims to report the results of a research carried out with beginners from the Bachelor's degree (BA) course in Mathematics to verify the most common errors in the graphic representation of problem situations. Different activities were applied to 23 students and each of them involved: 1) graphic representations and intuitive thinking; 2) graphic representation, exploring intuitive and analytical thinking; 3) graphic representation, exploring intuitive and analytical thinking through more complex situations.

In general, intuition and analytical thinking were observed for this study. 33.09% of correct answers were obtained from the overall average, in percentage, among the activities. Such figure is considered very low given the fact it refers to real problem situations. It can be noticed that most learners have difficulty in perceiving what is being proposed and, many times, they perceive it, their intuitive thinking is equivocated, preventing them from getting to a correct analytical thinking.

The kinds of errors made by learners are common and a result of high school background knowledge, in which most courses approach the teaching of functions through contents restricted to the classification of functions - linear, quadratic, polynomial, trigonometrical, etc. – a group of properties related to operational aspects. Not always is emphasis given to the applicability of functions towards the solution of problems.

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From my experience as a teacher of beginners, in subjects related to Math, I have noticed that learners keep stuck to formal standards, to the use of tables to build up functions graphics.

The concept of functions is one of the most important ones in Math and is highlighted in many topics, a swell as in other areas of knowledge. Nowadays it is common and convenient to express physical, biological and social phenomena, for instance, through graphic representations. For this reason, studying functions in High School is very important.

Several investigations about the concept of functions have demonstrated the importance of specific research regarding learners' difficulties to get from one concept representation to another. Many learners find it difficult to graphically represent a problem situation, without writing the formation law – formalization of this function.

Based on this premise, learners have lost the general ideas about function behavior, prevented from having a graphic visualization of a problem situation.

In an ideal world, learners should be able to create a mental image of a concept, of a situation according to the mental transformations of acquired concepts, and should be able to externalize the mental images of a concept, so that it is observable, verbal or about a graphic representation, tables, etc. (Espinosa, 1995).

The initial intention was to show the most common errors made by beginners from the Mathematics undergraduate course in the graphic representation of problem situations. For this study, intuition and analytical thinking were observed.

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Activities involving graphic representations of several problem situations (from common to complex ones) were applied to this study. Intuitive and analytical thinking was used for the graphic representations.

When it comes to intuitive thinking versus analytical thinking, Science has prioritized the mental work based on the principles of analytical thinking, emphasizing the objective analyses of data, aiming to organize explanation to registered and controlled facts, disciplined method leading the organization and explanation , disciplined method leading to appropriate answers in the previous supposed directions, according to the existing knowledge.

Intuitive thinking works from a subjective perspective to capture what is going on. For this reason, it implies thinking through data, but not necessarily thinking about them, generating configurations, ideas, knowledge and new mental constructions, not derived from current data, although mostly related to the studied subject.

According to Piaget's[1] view, intuition is a representation constructed by interiorized and fixed and is not at operation level. Or it is an imagined thinking, affecting group configurations and no more syncretic collections symbolized by outstanding types.

To Espinosa (1995), intuitive thinking has a global systematic view, opposite to analytical thinking.

Bruner (1998) investigated the issue of intuitive thinking in mathematical sciences, showing the difficulty to accept this type of mental work, although it is remarkable in more productive research areas, especially in exact sciences. Some scientists considered themselves capable of working out intuition. According to Bruner, intuitive thinking is a subjective intimate feeling that a given situation is being better understood, the impression of being capturing a type of knowledge that, in spite of being related to the available data for study, seems to go beyond such data. At the moment a person is able to understand and express this "feeling" in a logical way, he starts to analyze it and, from this moment, he replaces intuitive thinking with analytical thinking. It is in such moment of undescribable subjective internal feeling and impression that the action of creation takes place.

Many times intuition happens with or without the knowledge about the subject. Believing that knowledge has to be exhausted before creativity takes place is an attitude that can inhibit the creative thinking, and transform curious and gifted children and adolescents in "knowledge holders" or in "data analyzers", uncapable of allowing themselves to think beyond the provided data; children and adolescents who are afraid of dealing with the "new", afraid of "not knowing it", of "making mistakes" and afraid of taking risks through unknown mental ways (Bruner, 1998).

It is possible to state that intuitive thinking is a mental process that is based on creativity, inventivity and imagination; it is as a proficous and ligitimate way to deal with life situations as

[1] http://www.pedagogiaemfoco.pro.br/per09a.htm

thinking analytically and critically. In some situations this way of thinking might be more appropriate and more useful, taking the new ideas and unexpected configurations.

As educators we have a great challenge: motivate our learners to think, develop intuition, free and exercise the creative imagination, find time, space and appropriate ways to enale their learners to effectively think! Think for himself, in a more productive and satisfactory way, about aspects of his physical , psychological and social world which promote interest and curiosity.

This work was based on studies carried out by Espinosa (1995) and Gravina (1992), which used the expression of mathematical visualization as an introduction to the abstraction of concepts.

23 learners, from the Mathematics undergraduate course, participated in this investigation. 8 of them were from the 2nd semester, whereas 15 were from the 3rd semester level, in 2007. Three activities, involving graphic representations, were applied:

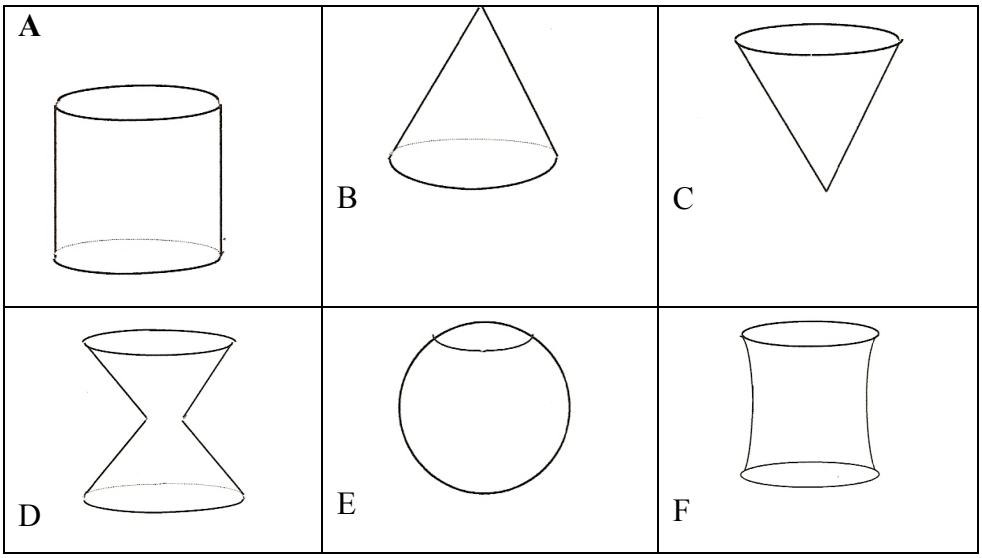
1st activity – The graphic representation and the intuitive and the intuitive thinking, where learners graphically draw six common daily problems.

A) The growth of a person in relation to age.
B) Water temperature set for boiling from its natural state in relation to time.
C) Water behavior on a typical winter day (Tendency).
D) Quantity of electricity of a battery turned on continuously, in relation to time.
E) Square perimeter in relation to the length of its sides.
F) Square area in relation to the length of its sides.

As regards the 23 learners' overall performance, 47.83% of them answered the situations correctly; two of them got 100% of correct answers in the six situations proposed; 5 learners got 83.33% of correct answers, 1 got 66.67%, 3 got 50%, 6 got 33.33%, 4 got 16.67% and 2 learners got 0%.

When it comes to the rate of correct answers, in each problem situation in the first activity, the highest index was in situation A, which corresponds to 73.91% of correct answers in relation to participants, followed by 56.52% of situation B, 47.83% of B and C and 13.04% of D.

2nd activity – Graphic representation – it is composed of six situations which had reservoirs of the same size and height. All of them are filled up by constant faucets and equally for all of them, as shown in picture 1.



Picture 1 – Reservoirs for liquid

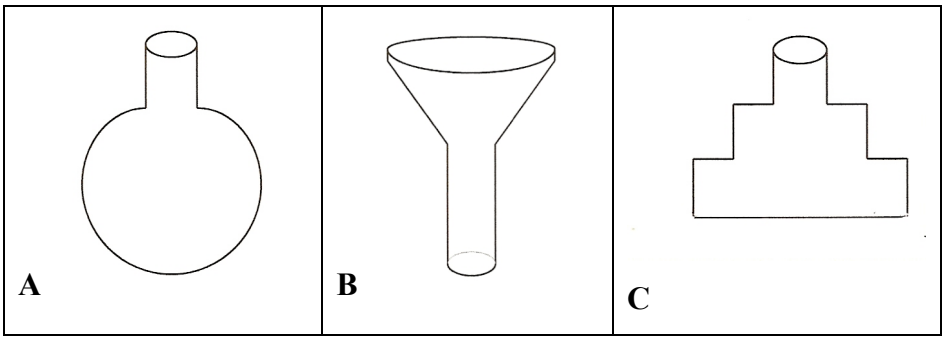
In this activity, the general average result of correct answers for problem situations given by the 23 participants was 40.58%. Only one student answered all the proposed situations correctly, 2 students scored 83.33%, 3 scored 66.67%, 4 scored 50%, 5 scored 33.33%, 7 scored 16.67% and 1 student did not get any answers right.

Considering the rate of correct answers, in each problem situation, the highest result was 95.65% for situation A, followed by 39.13% for E, 34.78% for F, 30.43% for B, 26.09% for D and 21.74% for C.

Despite the fact that there was a balance in the average between the first and the second activity, there were learners who had obtained lower results in the first activity than in the second one and vice versa, suggesting there should be deeper studies observing the degree of difficulty mostly found in the ways of presentation of the proposed problem situations. Future research should be carried out to verify a learner degree of difficulty to take a current language to mathematical language; a situation shown in picture and translate it verbally so that it can be later transferred to mathematical language, and finally graphically represent them.

3rd activity – Graphic representation, where intuitive and analytical thinking were explored. In this activity, all the three recipients have the same capacity and different shapes, and are filled up by constant faucets and equally for all of them.

One raised question: What happens to the liquid surface area during the filling up process in relation to the height reached by the water.



Picture 2 – Recipients with the same capacity and different shapes.

In this activity, the average rate of correct answers was 10.14%; 8.7% referred to situation A, 4.35% to B and 17.39% to C. These figures were much lower, considered surprising in relation to the results of the previous activities. Only one learner scored 66.67%, 3 scored 33.33% and the rest of them (18 learners) did not perform any of the proposed situations correctly. Learners' performance in this activity was very low. The overall rate of correct answers provided by the 23 learners for the problem situations was 10.14%. One learner got 100% of correct answers, 2 learners got 83.33% of correct answers, 3 learners got 66.67%, 4 got 50%, 5 got 33.33%, 7 got 16.67% and 1 got 0%.

This result is worrying, since 82.6% of the learners were not able to graphically represent the proposed situations, showing the necessity for more intense research and pedagogic work.

This activity is slightly more difficulty than the second one. Its proposed situations comprise more than one type of behavior. In the second activity, the situations which involved more than behavior were inverse situations, not exactly different. This probably shows why performance in this activity was so unsatisfactory. This issue needs to be seriously studied later.

Considering the overall average rate between the three applied activities, there were 33.09% of right answers, which is a very low index since it refers to real problems. It can be noticed that most learners have difficulty in realizing what is being proposed and many times when they realize it, their intuitive thinking is wrong, preventing them to get to a correct analytical thinking.

For every sample, considering the average performance rate of all the activities, only 4 learners scored higher than 50%. In the first and second activities, 8 learners scored higher than 50%. In the third activity, 4 learners scored higher than 50%.

The four learners who scored higher than 50% kept their average in the three activities. These learners are enrolled in the second grade. Based on the proposed results, I intend to use these data to thoroughly investigate the existence of correlation with other factors such as: year of high school graduation, age, type of course in high school, job, average scores, etc.

The original proposal of this work also aimed to verify the most common errors when involved in situations linked to a representation in a physical context. In fact, when most learners make use of intuitive thinking, they do it very differently from one another and, most of the time, wrongly.

Se partirmos que isso faz parte de uma situação atual encontrada nas séries iniciais do ensino superior, temos uma grande missão: suprir as dificuldades e conceitos errôneos e tentar mos determinar as possíveis causas e procedimentos educacionais alternativos que possam ajudar a corrigir.

It is fundamental learners get familiarized with the applications of functions and problem situations in general in other scientific domains, not only due to the usefulness of mathematical concepts they are learning but also as motivation to be able to appreciate the value of these concepts.

From the results presented, it can be noticed that there are concepts taught at university which could be worked out in high school, as long as they are introduced adequately. To illustrate, real situations such as population growth, bacteria or plant growth, or the speed and the acceleration performance of an automobile.

The capacity to represent, analyze and interpret graphics is very important, in any scientific domain. It is necessary to restore certain concepts in our learners, from understanding instructions to conceptualization, so that they can be able to mathematically formulate the proposed situation.

Bibliography

BICUDO, J. C. (1941). *O ensino secundário no Brasil e sua atual legislação: 1931-1941,* 2ª edição, São Paulo.

BOYER, C. B. (1974). *História da Matemática.* Tradução: Elza F. Gomide, Editora Edgar Blücher, São Paulo.

BRUNER, J. *O.* (1998) *Processo da Educação.*:Cap. 5, Editora Nacional, São Paulo Cap.

CARAÇA, B. J. (1951). *Conceitos Fundamentais da Matemática,* Lisboa.

CARNEIRO, V. C. (1992). A Matemática aponta pontos críticos de outra Ciência. *Revista do professor de Matemática.* **22**: 32-37*,* Sociedade Brasileira de Matemática.

ESPINOSA, F. H. (1995). Intuicion primera versus pensamiento analítico: Dificultades en el paso de una representación gráfica a un contexto real y viceversa. *Educacion Matemática.* 7(1):63-74, México: CINVESTAV-IPN..

GRAVINA, M. A. (1990). O quanto precisamos de tabelas na construção de gráficos de funções? *Revista do professor de Matemática.* **17**: 27-34*,* Sociedade Brasileira de Matemática.

GUELLI, O. (1992). *Contando a História da Matemática - A invenção dos números.Editora*: Ática, São Paulo.

HOGBEN, L. (s/d). *Maravilha da Matemática.* Editora Globo.

SMOOTHEY, M. (1997). *Atividade e jogos com gráficos, Editira* Scipione, São Paulo.