The ways of teaching mathematics to visually impaired students

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Abstract
In this paper we present overview of actual situation of teaching mathematics to visually impaired students at each level of education (primary, secondary, university) in Slovakia. Problem of accessibility of mathematics to visually impaired students has the solution in linear notation. One of the possibilities to access math is the Lambda editor, which besides the linear notation in Lambda code offers also the graphic visualization for sighted people. Hence in the second part we briefly describe the effort to integrate Lambda editor in to the teaching and study of mathematics, which is an initiative of Support Centre for Visually Impaired Students.The Lambda editor is considered as tool within the material milieu of didactical situation.

1. Overview of the actual situation

1.1. Primary schools

Most of the Slovak visually impaired children (blind and partially sighted) attend special primary schools. At the lessons of mathematics they are using Braille books with tactile pictures, to make notes they use electronic notebooks and for calculations mechanical typewriter. The disadvantage of typewriter is in the first place that the way to get result of calculus takes too long, so pupils try to calculate in their minds and secondly: notation of the calculations is too verbiage, so after a while the pupil is lost.

Teaching mathematics on the elementary level means first of all helping children to use and organize their experiences, which they gain from actions and interactions with the world around them. In opinion of some authors (Csocsan et al., 2002) the main goal of mathematical education is to develop an awareness of numbers and coping with different relations and dimensions. The most frequent mathematical problems of blind pupils are as follows:

- generalizing – finding the similarities in different activities in everyday life,
- translating activities and actions into mathematical language,
- lacking flexibility in problem solving and in calculations,
- translating and transferring three-dimensional objects into two-dimensional iconic forms. [Example: The blind pupil cannot understand a geometrical drawing of a cube from a perspective view because of his/her lack of visual experiences. S/he also has difficulties in enlarging and minimizing two-dimensional forms.]

1.2. Secondary schools

There are special high schools for visually impaired students in Slovakia, but mostly oriented on music, some handicrafts, etc. If a student wants to come into contact with mathematics then s/he needs to attend "normal" high school. As we know, mathematics is a subject which is important for studying not only natural sciences as physics, chemistry, informatics or biology are but it also begins to be popular at humane sciences as psychology, philology, sociology, etc. The direct consequence of this mathematical requirement almost everywhere causes that also more and more blind students today start their education in mainstream schools, which is place, where they can study maths.

Because teachers of these schools are not special educated in this field they often have to use the ”trial and error” method to find out the best way of teaching their blind students who are only integrated among sighted students. Visually impaired students encounter also with lack of textbooks and study material and limited Braille notation for maths.

On the other side, visually impaired students of this level of education mostly do not have problems with calculations; they already know all basic mathematical operations. However, scale of mathematical knowledge increases here very sharp in all fields: algebra, analysis, and geometry. Hence, they will have to overcome a lot of other new challenges, especially with Braille notation of all new symbols. After study of system of Braille notation (Kohanová, 2003) in several European countries we can state that more or less each of the mentioned norms suffers from lack of the rules for notation of mathematical text. Therefore, the major part of visually impaired students creates their own particular mathematical language that is adapted to their conditions and requirements. But this forms new problems, because these languages do not have to be comprehensible for people who are visually impaired students communicating with.

1.2.1. Communication with visually impaired student

The integration of visually impaired student among sighted ones in the common schools causes that in these specific conditions we find different (new) relations in the classroom. One has to distinguish between communication between teacher and sighted students at the lesson and between teacher and non-sighted student, in addition, between non-sighted and sighted students. The first case is more dominant it runs in various forms. Oral communication is irrecoverable at education; however, this fact does not valid in mathematics. In addition, mathematics requires exactness, definiteness, totality and comprehensibility of presentation. It is very arduous only by oral communication (e.g. when modifying expression or by geometrical construction) and so it is supported by graphical way - text or picture. This connection is typical for mathematics; because of insufficient style of expression some students rather prefer notation or picture. If we are talking about graphical communication in the frame of communication between teacher and non-sighted student, we mean communication supported by for example relief's picture, typhlographic images and plane or space models (construction kit, cubes, skewers, paper). The other case is communication supported by notebook; the non-sighted student takes notes or computes in electronic form, mainly linearized.

1.2.2. Didactical system in specific condition

Following the Theory of didactical situation introduced by Brousseau (1997)[1], when acquiring new knowledge ("connaissance") in the frame of **didactical situation**[2], the teaching of a new notion consists of setting up its situations and carrying out interactions in which the learner can take part. It is itself, an interaction. This interaction is also largely specific to the knowledge being taught but it takes a form of a-didactical situation[3], necessarily different from the non-didactical forms in which knowledge ("savoir") is used. This result changes the entire approach to mathematics education and the education of teachers.

Consequently, we can represent the **didactical system** (triangle) as a system of relationships between three subsystems: educator (E → teacher: TE, tutor: TU), learner (L → sighted student: SS, non-sighted student: NSS) and knowledge (K), where the non-sighted student has particular position in the frame of didactical situation (DS), not only because of the way that s/he obtains the knowledge.

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[1] Theory of Didactical Situation, which fundamental methodological principle is built upon: "a piece of mathematical knowledge is represented by a "situation" that involves problems that can be solved in an optimal manner using this knowledge". The characteristic situations for pieces of mathematical knowledge can be studied or even modelled within the framework of mathematics itself, which sometimes makes it possible to use computation to predict their evolution.

[2] Situation that enables to obtain new knowledge.

[3] The part of didactical situation that enables to the student to acquire new knowledge own and consequently s/he is able to put it to use in situations, which s/he will come across outside any teaching context, and in the absence of any intentional directions.

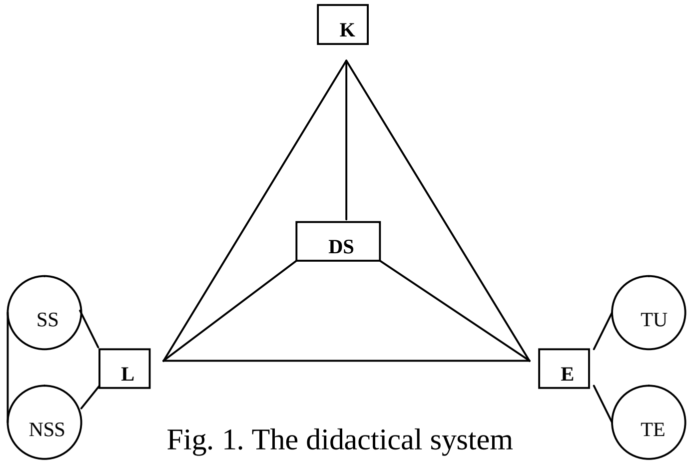

Fig. 1. The didactical system

To obtain new knowledge in the point of view of visually impaired student it is needed also to clearly specify the **didactic contract**[4] in sense what kind of compensating tools it is allow to use (notebook, Braille typewriter, etc.), which are actually becoming the part of **didactic milieu** (DM) and its levels. The mentioned tools are part of **material milieu** (MM) that activates non-sighted student to the activity that leads to acquisition of new knowledge. Concerning the cognitive element of material milieu of visually impaired student we may predict that it is different than the cognitive element of sighted student.

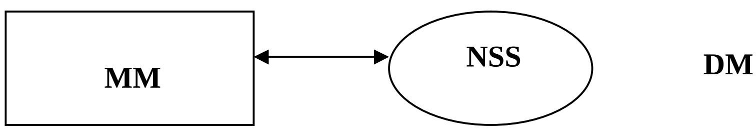

Fig. 2: Didactical milieu

In general, we think the process of acquisition the knowledge is different than the same process of sighted students. In follows, we describe the didactical triangle in form of sides (Sbaragli, 2004), which are stated by three main vertices, focusing on non-sighted student, taking into account personal experience of interviewed people – teacher of mathematics and her non-sighted student:

§ *teacher (TE) – knowledge(K)*

This side is characterized by the verb "*to transpose*", where the main activity is the

[4] The relationship which determines implicitly what each partner, the teacher and the student, will have the responsibility for managing and be responsible to the other person for, specifically target on mathematical knowledge.

first part of the **didactical transposition**[5] from the *scholarly knowledge* to the *knowledge to be taught* (Chevallard, 1985). The question is whether the teacher is conscious of necessity of different approach that s/he has to use when transposing the knowledge to the non-sighted student. It consists in extra preparation of a-didactical situation, so the non-sighted student can also participate in activity. When making analysis a-priori, the teacher has to take into account possible difficulties of understanding and questions that may appear at the lesson and so be able to react dynamically and clear, but not at the expense of other sighted students.

§ *tutor (TU) – knowledge (K)*

This side is specific by various ways of transposing the knowledge, which are not used when teaching sighted students. Sometimes the teacher is not conscious of all that might be a problem to understand for non-sighted student, s/he finds is out at the lesson and in analysis a-posteriori. So as a tutor, s/he has to select different view on given subject and try to transpose the scholarly knowledge to the knowledge to be taught in appropriate and customized way, sometimes by using models and tactile pictures.

§ *student (NSS) – knowledge (K)*

This side is expressed by the verb "*to learn*", where the predominant activity is the involvement of the non-sighted student that is characterized by nouns "*motivation – interest - volition*". S/he accepts the devolution and takes personal care of her/his proper knowledge. The knowledge is so constructed by the student and finally institutionalized by the teacher/tutor.

§ *teacher (TE) – student (NSS)*

This side can be represented by the verbs "*to facilitate* **-** *to advice* **-** *to guide*". When the non-sighted student faces with a-didactical situation, s/he produces her/his knowledge as a personal response to the didactical milieu by keeping the didactical contract. The teacher should allow a harmonization of the different phases of the learning process and intervene only in cases of possible misconceptions or false beliefs of the student.

§ *tutor (TU) –student (NSS)*

"*Individual consultation*" – it is the name of this side. Consultation between non-sighted student and tutor is held apart from lesson, depending on needs that have occurred at

[5] The process that isolates notions and properties, takes them away from the network of activities which provide their origin, meaning, motivation and use. It transposes them into a classroom context.

the lesson. Thanks to individual approach there is time and space for deeper understanding of the piece of knowledge and its consistent arrangement into the general knowledge structure.

- *student(NSS) – student (SS)*

This side can be described by the verbs "*to help – to correct*" and by the noun "*cooperation*" of the non-sighted and sighted student at the lesson. In case of any doubt, incertitude or misunderstanding the non-sighted student can ask for help of her/his sighted neighbour. On the other side, the sighted student (who is good at mathematics) can correct some errors or mistakes if s/he sees them on the screen of her/his neighbour's computer.

Since there is no time for the teacher to engage fully in work with visual impaired student at the lesson (in sense not at the expense of sighted students), it is needed to accent that mentioned model of didactical system works on voluntariness and interest of the teacher/tutor; especially tutor works in her/his free time. So it seems like the integration of visual impaired student into common school is fictive, concerning the point of view of studying mathematics. The integration by itself has rather social aspect. Consequently, the question is: Who can be integrated and who not? Sometimes the integration might harm, on the contrary, it might help to some of non-sighted students to express them. The other remarkable thing is the question of limit. Since in Slovakia there is no standard for teaching mathematics of integrated visual impaired students on secondary level, the teacher has to determine requirements on these students by her/his own, on her/his subjective opinion.

1.3. University level

Comparing to secondary school, there is quite a different situation for a blind student at University in math. The student is supposed to have skills necessary to study - make notes during lectures, read scientific text, perform complex calculations, communicate with teachers and other students in written form, etc. There is much more independent work required.

If s/he graduated at special school/class and used only Braille notation and spoken language for before mentioned purposes, he will have to overcome a lot of new challenges.

There are very limited sources of scientific literature in accessible form for a blind student. Therefore he should be able to read different mathematical notations.

Another way of delivering mathematical expressions in accessible, written form is electronic text document on personal/portable computer or special note taker for the blind users. This sort of document usually contains linear mathematical notation with expressions built up of ASCII characters.

Blind student can access this type of notation in two ways. S/he can use refreshable Braille display and read line by line corresponding Braille cells (groups of 6 or 8 raised dots) by touch or listen to synthetic voice, which reads each written ASCII symbol for him/her. The second method is more difficult for reading complex mathematical expressions, however could be quicker for longer text with simple mathematical expressions. The ideal is combination of both methods, when student can choose appropriate method depending on current situation (what is s/he reading, writing or calculating).

Some solutions, originally dedicated to electronic publishing of scientific text documents (TeX, LaTeX, AmSTeX, HrTeX, MathML), could be red and written by blind student.

Computer Algebra Systems (CAL Systems) are dedicated at the first place for algebraic calculations, e.g. differentiation or integration; solving of equations. They are also able to perform numerical calculations; visual graphs of functions, curves and 3-dimensional objects. Such a softer is Derive, MuPAD , MAPLE , MathCad or Mathematica . All of them contain as well a lot of functions of analysis, linear algebra, statistics, numerical analysis, number theory, graphics, etc.

They are also useful for visually impaired students, especially by calculations. It has no sense to urge them to act calculations that are often just very tedious and mechanical. That is why CAL Systems are helpful. If the commands we put into command-line are linear, it means they are fully textual and therefore suitable for visually impaired students. The other advantage is that screen-reader does not have any problem to read linear text on the screen and so, access it to the student. Thus, blind students of Computer Science use CAL Systems for example for calculation during Algebra seminars. It is useful tool for calculations with matrixes, which are time consuming and quite complicated. If they understood a principle, such a tool can save a time and a lot of manual work.

2. Utilization of information technologies - The Lambda system

It has been mentioned above the information technologies might be very helpful for visually impaired students who are studying mathematics, since they have largely improved the educational opportunities. IT have began to be very important part of material milieu of didactic situations. The most important requirement for secondary school students is to handle mathematical expressions quickly and efficiently in the same way as their sighted classmates. Teachers who do not have any knowledge of Braille (usually those in integrated schools), are asking for most suitable tools which are enabling the communication between the teacher and visually impaired student. Later on, at university, it is important to have a mathematical writing system that is powerful, flexible, and compatible with most common format standards, to enable independent scientific and mathematical work to be distributed digitally. The fact that has to be considered is accessibility.

In the last period LAMBDA[6] - **L**inear **A**ccess to **M**athematic for **B**raille **D**evice and **A**udio-synthesis appears to supply all needed requirements. The LAMBDA project makes the provision for an integrated system based on linear code and a software management system (the editor). The code (Lambda Math Code) is a direct derivation of MathML[7], which is an XML[8] based language. The editor allows the writing and manipulation of mathematical expressions in a linear way. Few facts about Lambda (according to Fogarolo, 2006):

- LAMBDA is intended to be used the most by young people who are learning mathematics, especially visually impaired students.
- LAMBDA is above all, even if not only, a didactics instrument. It is the functional component which implements the strategies devised in order to make easy to read, write and manipulate text and mathematical expressions by means of vocal output and Braille display, in an educational setting.

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[6] The purpose of the project was to produce an efficient system for non-sighted middle school, high school and university students to manage mathematics documents. More information at: www.lambdaproject.org.

[7] The Mathematical Markup Language is language used for displaying mathematical notation and content, especially on the web. It is a World Wide Web Consortium (W3C) recommended standard, and has been receiving increasing support by mathematical software vendors.

[8] Extensible Markup Language is a W3C initiative that allows information and services to be encoded with meaningful structure and semantics that computers and humans can understand.

- It is important to define the didactics requirements needed for a software writing system compared with Braille traditional ones. The change towards mechanic writing systems (as typewriter and PC) requires skills relative to management of textual documents using PC. If they are missing, traditional tools are preferable and the passage to new technologies has probably to be put off.
- All most frequent operations such as, for instance, opening and saving files, selecting a portion of text, deleting, correcting, copying, pasting are performed according to Windows standard modalities and do not present any training or adaptation-related problems.

2.1. The experience of working with Lambda system

First we get in contact with Lambda system in December 2005 at international conference in Rome: „I don't see The problem: new prospects to access Mathematics and Scientific studies for Blind students", where the software was presented by its athours and by italian visually impaired pupils who are using and testing it. Later on, by initiative of Support Centre for Visually Impaired Students (CEZAP), Comenius University, Bratislava we offered to run the mathematical club for visually impaired pupils. We started to give lessons in April 2006 together with 2 university teacher-students (specialization: mathematics-informatics) at Elementary school for Visually Impaired in Bratislava. Together there were 5 meetings and 4 pupils who were attending the course. All 4 pupils were students of 9th grade, 2 of them were short sighted, 1 virtually nonsighted and 1 non-sighted.

The main aim of the course was to test the utilization and efficiency of Lambda system at Slovak elementary and secondary schools. In the case that Lambda would prove to be a significant tool (component of material milieu of didactical situations) for study of visually impaired students, the Support Centre for Visually Impaired Students planed to provide the Slovak version of Lambda (at the length of the courses we used English version) and find the ways of its practical application at Slovak schools.

During the course the participants became familiar with the working environment and following important and useful features of the program:

- Possibilities to input characters and mathematical symbols (Bernareggi, 2006):

ü *input from the menu*

(The tag is gathered within the groups that are shown in a pull-down menu, so the user can choose the proper tag browsing the menu items. It is not a quick input strategy, but it is very easy for those users who are not acquainted with the system.)

ü *input from a list*

(All the tag names are sorted in lexicographic order in the list. One just digits only few characters of the element name and the list is reduced to few items, easily manageable both through Braille display and voice.)

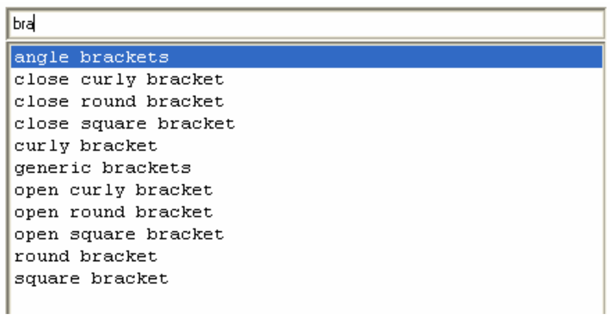

Fig.3: Input from a list

ü *input by short-cut keys*

(This input is available by default for some dozens of common elements, but the list can be increased and customized according to the user needs.)

ü *input through the mouse*

(The selection of mathematical tags by mouse from icon graphic menu is implemented as well, since the mathematical editor is can be used by sighted assistants too.)

ü *input according to the context*

(In order to adhere the structure of the expression, separator or a closing tag can be automatically inferred by the system. Input technique exploits two short-cut keys to input the corresponding separator or the corresponding closing tag.)

§ On many situations, it is important to perceive global information, related to the structure or the relation, to define every single time the most suitable paths and methods to face different issues. For example, having following expression:

$$\frac{x^2-4}{x^2+4x+4}+\frac{4}{x+2}$$

Its linear representation in Lambda is:

//x^2−4ø x^2+4x+4\\+//4ø x+2\\

It is evident that in reading the linear representation it is more difficult to find specific parts and to quickly understand the relations among the structures making the expression that is an immediate operation for sighted people who use global and bidimensional exploration. So we presented to the students the effective exploration strategies. Lambda offers exploration through movement operations, in sense that one can move to the next nominator, denominator or corresponding separator or tag. The second possibility is tag structure of an expression that enables to understand overall structure of expression and to find specific parts. The most compressed structure of expression mentioned above is: //ø\\+//ø\\ , the other depth level seems as follows: // (   ) ø (   ) \\+//4ø x+2\\ .

This visualization modality that hides the content of the block by maintaining blank spaces is useful as well to get some information about the size of hidden blocks.

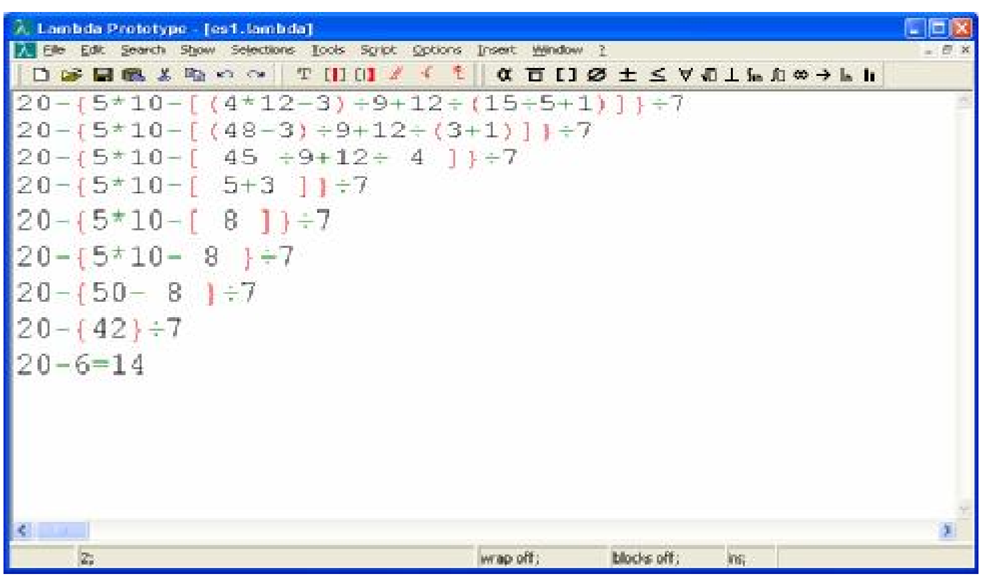

Fig. 4: The main window of LAMBDA

- The other beneficial tool is scientific calculator that is able to calculate numerical expression and paste the result.
- LAMBDA enables automatic double copy of selected row, so we get "checking" and "working" line. Here we have the possibility of checking the steps against a previous, unchanged "checking" line.
- Even the linear notation in LAMBDA is more or less intuitive; the graphic visualization might be helpful for the teachers, parents or sighted schoolmates in order to be able help or correct by doing calculations.

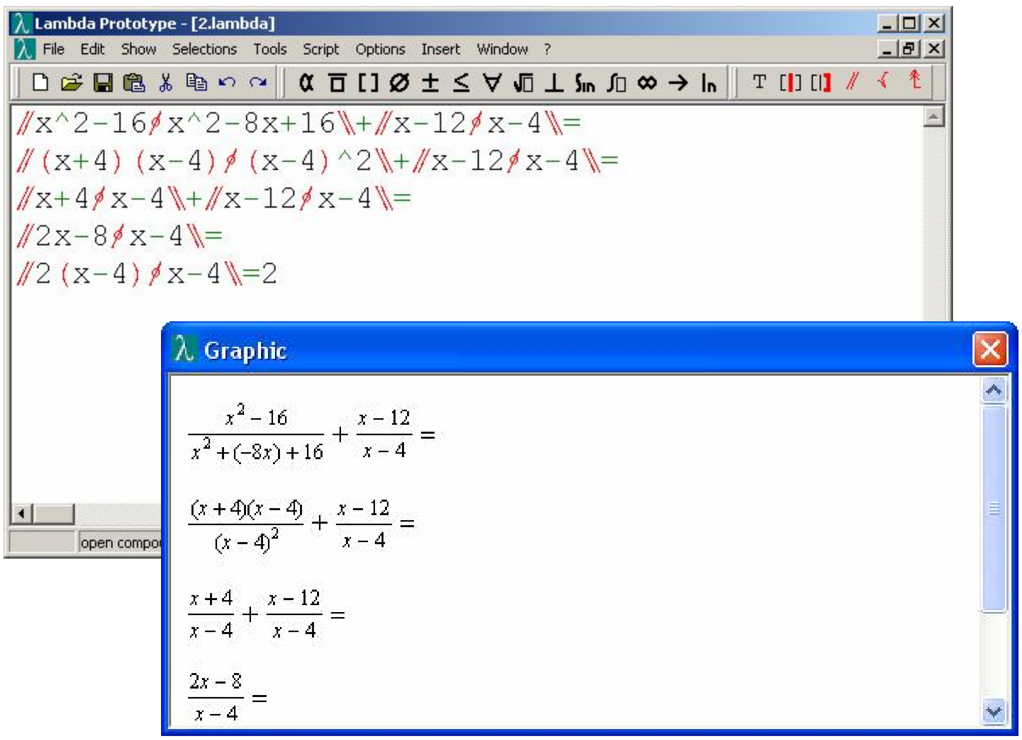

Fig. 5: Graphic visualization of the calculation

During these 5 lessons all 4 pupils had learnt to work with Lambda and they considered it as userfriendly and as helping tool to do mathematics. Hence, we decided to translate Lambda menu into slovak language and in academic year 2006/2007 to continue with courses.

In November 2006 there was two days lasting intensive course with non-sighted students coming from different parts of Slovakia. The aim of the course was beside the familiarization with the software also the testing the prototype of slovak language version. Afterwards in 2007 there have been other two meetings concerning the slovak version and learning some new functions added by authors. In February 2008 The Support Centre (CEZAP) have prepared the meeting with teachers from elementary and secondary schools who are teaching mathematics to visually impaired pupils/students in order to introduce them LAMBDA as the educational tool. One of the outputs of realized courses will be also the methodical guide of using LAMBDA intended for teachers and parents of visually impaired students. It is actually subject of master thesis (dissertation) of one of the above mentioned teacher student, the thesis will be submited in summer 2009.

3. Conclusion

In the last years the solution of problem of accessibility mathematics to visually impaired students seems to have an electronic form. There exist softwares that are blind friendly and so the visually impaired students can do (calculate, read, write) mathematics the way that is also accessible for their sighted schoolmates and teachers. In Slovakia we have started to use the LAMBDA system and adapt it to the local conditions. The time will show whether LAMBDA proves as the effective and helpful tool for study and teaching mathematics at slovak elementary and secondary schools and universities, in other words, whether LAMBDA will be essential element of material milieu within the education of visually impaired students.

References

Bernareggi C. (2006): *A framework for vocal and tactile access to science in an educational environment*, Universitŕ degli studi di Milano, Facoltŕ di scienze matematiche, fisiche e naturali, Doctoral thesis, Milano

Brousseau G.(1997): *Theory of didactical situations in mathematics.* Edited and translated by Balacheff, Kluwer academic publishers

Csocsan E., Klingenberg O., Koskinen K. L., Sjostadt S. (2002): *MATHS "seen" with other eyes: a blind child in the classroom: teacher's guide to mathematics.* Ekenas Tryckeri Aktiebolag

Chevallard Y. (1985): *La transposition didactique. Du savoir savant au savoir enseigné*. Grenoble: La Pensée Sauvage

Fogarolo F. (2006): *LAMBDA system and mathematics teaching method.* CSA Vicenza

Kobolková M., Lecký P. (2002): *Experience with access to mathematics for the blind students in Slovakia.* International Conference on Computers Helping People (ICCHP), Springer Verlag, pp. 510-511

Kohanová I. (2003): *Možnosti sprístupňovania matematických textov v elektronickej forme nevidiacim.* Diploma thesis. FMFI UK, Bratislava

Miesenberger K., Klaus J., Zagler W. (2002): *Computers helping people with special needs*, 8th International Conference, Proceedings of ICCHP, Springer

Sbaragli S. (2004): *Teachers' convictions on mathematical infinity.* Doctoral thesis, FMFI UK, Bratislava

Schweikhardt W. (2000): Requirements on a mathematical notation for the blind, in Vollmar, R. und Wagner, R. (eds.) Computers Helping People with Special Needs ICCHP 2000, Wien: Österreichische Computer Gesellschaft, 661-670