Challenges with international collaboration regarding teaching of mathematical modeling

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Introduction

There is no doubt that there is a strong relation between different teachers' teaching styles and their students' actual learning in mathematics. Teachers are of course influenced by their theoretical knowledge from a didactical and subject content perspective, but also by quite specific cultural and geographical matters. It is for instance quite obvious that teaching and learning can not be conducted or organized in exactly the same way around the globe. In fact there are huge differences also between countries in Europe. In November 2007, I and other project members in the DQME II project had the opportunity to experience mathematics teaching on an ordinary day in a typical gymnasium in Bratislava, Slovakia. We, the observers, were all seated in the back of the classroom. About 15 students came in and sat down at their desks, silent small talk could be heard. When the teacher arrived, they all raised into an almost military "attention", standing silently beside their desk, all staring straight forward. The teacher walked up to her desk and called out: "God morning, students". The students, who were about 18 years old and in a natural science program, answered in chorus: "God morning, teacher!"

After that the lesson started with carefully, well prepared exercises -- the content were geometrical series, both finite and infinite. The teacher set the agenda and the students immediately reacted when the teachers gave them a sign to step up front and do some calculations at the black board. No loud discussions among students were heard. The students were allowed to borrow fairly modern calculators from a mutual cabinet, but as far I as could notice no student actually used the calculator. The students seemed rather contented in the silent classroom experience we took part of.

I like to compare this experience with a so called "normal" lesson at a Swedish gymnasium, which I visited two weeks later with students in the same age and also in a natural science program. The teacher went in first into the classroom to make sure that everything was in order. Then she opened the door to the hallway outside the classroom, allowed the students to come in, looking them in their eyes and exchanging some small talk to each and every one when they passed her. When the class was in order after some few minutes, the teacher and the students started a negotiating discussion. What should we discuss today? What did we do the last time we met? Are you all satisfied about what they know about geometrical series? (The content was more or less the same as in the school in Bratislava). After some discussions, the teacher clarified some basic concepts within the theory of geometrical series on the black board before the students started to work on a small project task, organized in small groups. The teacher walked around in the classroom, assisting the groups when they asked for help, but not really teaching. She seemed to be listening more than talking. All students had their own calculator, which they used frequently together with different paper and pencil techniques. The level of discussion was sometimes very high.

Is it really self evident that I saw the facilitation and growth of the same mathematical knowledge taking place in these two different classrooms? Is it possible to use exactly the same tasks in these two different classroom situations? Would it be possible to just switch the two teachers? Would it be possible to neglect the social behavior of students and teachers when comparing results? Do the students in these two different classrooms have about the same mathematical platform to stand on when they are challenged with a new problem? No student acquires exactly the same toolbox of mathematical tolls as any other student, but it is of course important to know that students who are expected to tackle the same modeling problem also have about the same preparation.

Classroom practice

During the last ten years or so, a rather intensive debate has been going on regarding the relation between teacher's teaching and the learning among students that eventually becomes the result of that teaching. TIMSS video studies has among other things clearly illustrated that teaching traditions are heavily depended on cultural and national background (Straesser, 1998).

Lipings (1999) comparisons of teaching practice in China and USA together with Stiglers & Hieberts book The Teaching Gap (2000) has caused a rather extensive public discussion in the USA regarding teaching practice. The Learner's Perspective Study is examining the patterns of participation in competently-taught eighth grade mathematics classrooms in fifteen countries spread over Europe, Asia and in Australia and in the USA.

There is also a growing concern that of all the research that is done regarding teaching practice, not much actually reaches the classroom floor and the practicing teachers. Therefore it is of utterly importance that collaboration regarding the practice of teaching mathematical modeling involves teachers from the very start and at all levels of the intended collaboration. The classroom is a very complex milieu, almost impossible to interpret, analyze and understand from mainly a distant researcher's perspective. See Figure 1.

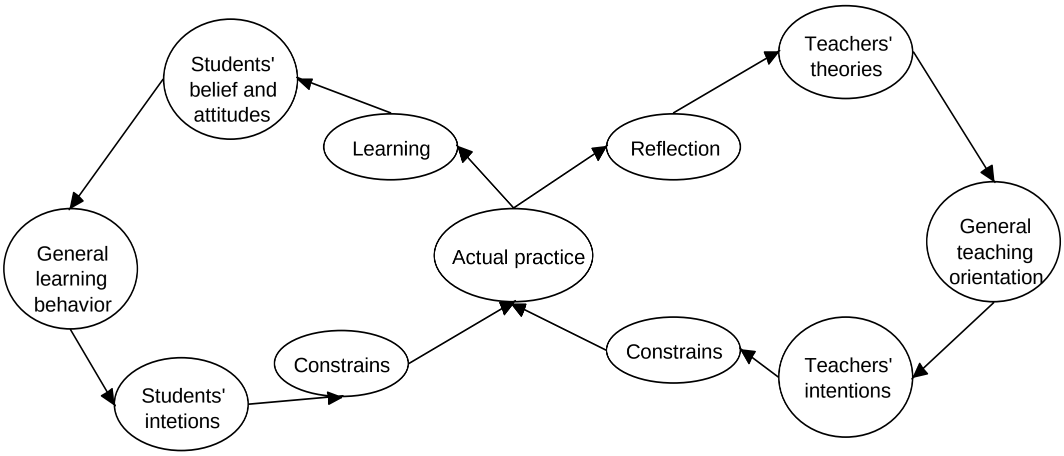

Figure 1: An attempt to illustrate how the actual practice in the classroom is affected by the teacher's and the students' circumstances.

The model in Figure 1 is developed from a model by Bennet (1999). It demonstrates in a simplified manner how the complex interplay that always exists in some sense between teachers teaching and students learning may be seen. Learning always take place in a complex web of contexts – any student is affected by many different factors, depending on internal as well as external circumstances.

> With this theoretical insight in hand, the field is now able to grasp the idea that interaction in any mathematical classroom is not merely the projection of what an individual teacher, individual students, or the discipline of mathematics itself brings to the encounter, but rather the joint construction of a viable response to the conditions and constraints that make possible as well as affect their encounter (Herbst & Silver 2007, p. 63).

Regardless of how we for instance interpret and understand the statement, so very ambiguous and full of nuances, *learning of mathematics in a classroom*, we probably would agree that learning itself has not only a cognitive dimension, but obvious also emotional and social dimensions (Illeris, 2004). The social dimension could further be divided into two sub-dimensions: one focused on the spontaneous social interactions, and the other focused on interactions that are structured by an organization. Hopefully we could also agree that there is a distinct difference between spontaneous and often instant learning in the so-called "everyday life" and the more well-defined goal oriented long-term learning in school. Quite different subjects or concepts are learned and different ways of thinking are developed. Learning mathematics in a mathematics course may also be quite different from learning mathematics in the workplace; in financial or engineering institutions, or in technical positions in factories or offices, where it is mostly tacit knowledge that is necessary. Each of these domains has their individual and very specific learning discourses. So any theoretical framework connected to any of these various discourses, would have different aims.

> Theories generated by research *in* mathematics education have various functions. They can have a *descriptive* function, providing a language to frame a way of seeing, and in this sense they affect an ideology. They may offer an explanation of how or why something happened, thus relating what has been observed to the past, whether through statistical correlation, cause-and-effect-analysis, influence, or co-evolving mutuality. They may attempt to *predict* what will happen in similar situations through stating necessary and appropriate conditions (and for this they need to specify what constitutes 'similar' and 'situation'). They may serve to *inform* practice by sharpening or heightening sensitivity to notice and act in future.
> Which ever of these functions a theory contributes to, it comes from, belongs to, even constitutes, a *weltenschauung*, and communication between different world-views is at best problematic. (Mason and Waywood, 1996, p. 1060, italics in original)

Any classroom in mathematics has its boundaries and constraints, in terms of curriculum, content in text books and number of mathematics hours per week, teacher attitudes, beliefs, and competencies, outer resources, technology, and so forth. The list of frame factors may easily be longer. The fact that some frame factors actually also influence for instance the implemented curriculum makes the situation even more complex. In a project where we are sharing materials across geographical and cultural borders, it is important to acknowledge that learning always take place in a social and cultural framework.

Sharing teaching materials

To share educational materials, courses or even full programs across national or cultural boundaries is becoming more and more common. One may even say that it is a clear trend at present time to share educational experiences in a global world. A number of factors are related to the growth in institutions offering courses to distant students in other countries, whether to one other institution half way around the world, or to dispersed students in many distant countries, or to their own citizens that are resident in other countries. Some factors are financial, other factors could be technological, and some might be mainly practical. Sometimes researchers and/or teachers enjoy the idea to share educational resources across borders for a variety of reasons. In the European project Developing Quality in Mathematics Educations II we have decided to do it because the project has several common goals across national and cultural borders. To share mathematical modeling projects is also an excellent opportunity to compare and learn more about the teaching of mathematical modeling.

It is in the nature of mankind to compare things and if we share we also start to compare. There has been a huge expansion of major comparisons with focus on the learning of mathematics during the last decade or so; TIMSS, PISA, and so on. In general these studies may be seen as rather high scaled studies with main focus of the learning outcome among the students and not so much on the actual teachers work. One nice example of a relatively high scaled but nevertheless more direct and personal study is the study reported by Ludwig and Xu in this TSG21. It seems that it is easier to relate to one or at least just a few problems that the students are supposed to solve and thereby in some sense understand the comparison. How one peels a pineapple in the most economical way is an excellent example of how mathematics is used all around us in everyday life. The comparison between Chinese and German students' accomplishments also raise questions about how school students actually develop their mathematical competences over time.

Nevertheless, teachers are often omitted and unseen in international comparisons. There are good reasons for this and one major problem is to compare the teaching as such, what instruments can possibly be used to do the comparisons between different schools in different countries? A regular international comparison is defined by standardized tests at a certain time and date; school coordinators; test administrators, and national and international monitors. A modeling project is stretched out over time; it includes extensive group work and the use of advanced technology. Our project has no school coordinators, no test administrators, and no national or international monitors. Nevertheless, as pointed out in the beginning in this article, there are many good reasons why one should also observe and document what teachers do with the students in their class and reality enables us to use what we can; video cameras and notebooks. Of course a small scale comparison study creates reasons for other differences. The teachers in different countries and different schools are involved in both the teaching and the documentation of the event. The researchers in different countries and different schools are involved in both the documentation and in some events also in the teaching. In any such low scaled and low budget endeavor, there are major difficulties when trying to make sure that the different classrooms activities in different countries will be videotaped in about the same way. That is naturally connected to the fact that the researchers and teachers in the different countries do not have exactly the same idea about what it is that is most important to document.

A collaboration takes place

Already in Bratislava in November 2007, some of the researchers and teachers from Germany and I decided that we should collaborate around a teaching experiment in a German school. The class was a class in grade 8, which means that the students are around 15 years old. For a Swedish point of view, it is impossible to be a student in the gymnasium when you are 15, since the Swedish gymnasium is for grades 10 – 12. Before that we have a compulsory school with the same curricula across Sweden. Germany contains at least 16 different curricula and modes of school culture. In addition to that, Germany has three different secondary school forms called Hauptschule, Realschule and Gymnasium, while Sweden in present time has one unified Gymnasium with all different programs covering all sorts of vocational and theoretical programs. In Sweden the pupils enter their 3-year Gymnasium program after 9 years in compulsory school, in Germany pupils are divided into different school forms at the age of ten or eleven. The Gymnasium in Germany is organized over 8 years, and each state has its own mandate to organize it, which has led to a great variation.

Nevertheless, we decided that I should communicate a “typical” task regarding the concept of volume from a Swedish textbook and that the German teacher would see if it was useful for his intentions. In fact the concept of volume is taught in grade 9 in Sweden so I translated a text from a grade 9 book and mailed it to Germany in early December. See Figure 2.

> Despite the rather cool weather during winter, small outdoor swimming pools are popular among private house owners in Sweden. Imagine a swimming pool that is circular with a radius of 2.75 meters and a depth of 1.18 meters. The distance between the water surface and the pool edge is 0.06 meters. Every spring the pool is filled through two water pipes, each of them delivering 20 liters of water per minute. The water cost 2 Euro per cubic meter.
>
> Questions
>
> a. How much water is there in the swimming pool? Answer in the unit cubic meters.
>
> b. How much does it cost to fill the swimming pool?
>
> c. How long does it take to fill the swimming pool?
>
> d. How many humans should be in the swimming pool at the same time in order for the water to pour over the edge? Find out the average volume for an average person yourself.
>
> From Matte Direkt /Mathematics Directly (Grade 9, 2003, p. 53). Translation: Thomas Lingefjärd

Figure 2: A mathematical task regarding volume from a Swedish textbook.

Within one week I received an answer from the German teacher that he would be happy use this task, but that he would not use the questions a – d (see Figure 2). Instead he would prefer to organize his students in groups and ask the group members to invent their own questions. See there for a cultural change! The classroom experience during one lesson was filmed and three minutes of the film was cut out and subtitles in English were added, and then this three minute cut was sent to me. At the end of January I was presenting the project and our cross national collaboration at a mathematics education conference in Stockholm where it was very convenient for me to actually show the film as a genuine proof of our collaboration. That short film, and the description of how our collaboration changed the task, seemed to draw much more attention than I had expected. Overall, it tuned out that the challenges and opportunities within this collaborative project were larger than I thought at first hand, but the outcome turned out to be more spectacular than I could imagine. See Stefanie Meier's article in this TSG21 for an extensive description of the Swimming pool problem.

The Asthma project

The next endeavor in the DQME II project was that the Danish and Swedish members of the DQME II research group decided to use a mathematical modeling problem from medicine (see appendix) as their next mutual project. It is a fact that many students seem to appreciate modeling problems from the area of medicine as more real and more interesting than others (Lingefjärd 2006, p. 111). After some negotiating, also the Hungarian and Romanian colleagues in the DQME II project agreed to use the same problem in their schools. One main concern was what the gymnasium (upper secondary) teachers in respectively country would think about the problem? Would they approve to the level of mathematical thinking? Would they approve to the directions of inquiry? Mutual teaching experiments across cultural and national borders require a lot of work and concern some very delicate issues. It is a fact that no one in the research group knows much about the actual teaching that takes place in Danish, Hungarian, Romanian or Swedish classrooms as existing apart from their own countries. The research group agreed about the intention to film the experiments in the classrooms. Would that become an obstacle or an opportunity to run the project even better? What about the actual teaching and learning that will take place? In what way will that effect the facilitation of mathematical thinking in the classrooms?

Research questions:

- What can pass as a good mathematical modeling project in four different countries?
- How will mathematics teachers in upper secondary schools in four different countries actually teach the same mathematical modeling project?
- What will the students in these four countries learn and how will it be assessed?
- What will be different in learning outcomes between the four countries?

At present time the problem has been introduced in Denmark, Romania and Sweden. Students in these three countries have been working on the Asthma problem at different levels and events in different classes in these three countries has also been video documented. When sharing a common project over national borders, it is likely that one apprehend also different schools in one own country or in this case one own city as one whole. In contradiction it seems as if the difference between the two Swedish teachers way to handle the Asthma problem was maybe larger than any cross cultural difference.

One major surprise was the message I got from Hungary just a couple of days ago. Within the mathematics community of Europe, I dare to allege that Hungary is viewed as a true mathematical country. When I was a student, there were rumors that in Hungary all families went for small mathematics contests during the weekends, walking around in the hometown, looking at various mathematical tasks that someone put up on public places, and solving them together family wise. Well, that was a beautiful but probably mostly false picture. Nevertheless, in Hungary some students are taught advanced mathematics about functions already in grade 8. The very same content Sweden teaches in courses at the gymnasium, approximately two years later. So I was surprised of the content in the message from the project member in Hungary.

> *Dear Thomas*
>
> *I have very bad news. I have heard form the three colleagues who I asked about the Asthma project. Unfortunately two of the teachers have stepped back from the Asthma project. They told me after long hesitation that this project is too hard for their students. The third teacher could make it only in September. I have heard from the teacher in Cluj-Napoca in Romania that they are working together with university students. Is it possible to use some similar solution in Budapest as well? If so, maybe in September the third teacher could bring a video about this asthma project.*
>
> *I am very sorry that it is so slowly ongoing but the Asthma problem is very alien for Hungarian pupils since there is nothing about so deep statistics and calculus in Hungarian curriculum. I thought it would be suitable for special mathematics classes but they are currently working hard on more elementary problems and preparing for international olympiads.*
>
> *I am looking forward to your opinion about collaboration with university students?*
>
> *Best wishes*

Conclusions

So far we have video documentation of students solving the Asthma problem in one gymnasium in Denmark and in two gymnasiums in Sweden (Gothenburg). I have been promised a film from the project members in Cluj-Napoca (Romania) who lucky enough have carried through the problem, but I haven't seen it yet. I will be very interesting to compare the way the teacher and students in Cluj-Napoca handled the Asthma problem with how the teachers in Denmark and Sweden did. It is interesting that the two Swedish gymnasiums only are about 5 kilometers from each other in a geographical sense, but the way the two Swedish teachers handled the Asthma project were quite different.

One of the Swedish teachers gave her students the problem and allotted them a couple of weeks to work on the various tasks before I came and filmed the presentations of the different groups in the class. Her students were organized in groups of five or six students in every group and almost all students in her class showed up. The whole event was taking place within one lesson in April 2008 (45 minutes) and was regarded as an important mathematical application within the course the students were studying: Mathematics D. The students also gave me group wise written solutions afterward, and it was clear both from the presentations and from their written solutions that they had worked carefully and enthusiastically on the problem.

The other Swedish teacher organized a workshop between 08.30 and 12 on a day in June 2008 when school actually was over. His students were studying mathematics D over two semesters, spring and fall, and were asked to come in to school for an extra enterprise they would benefit from. The students did not know anything about the problem when they came and had no chance to really select groups. The teacher organized them in groups of two and two (and one group with three students) from how they were seated in class. Altogether seven students showed up at 08.30, several other students came around 10.00 but they had no chance to really accomplish anything with such a short time to work on the task and refused to present anything. For me it was quite a surprise that the students who came in at 08.30 actually could accomplish anything, with such a short notice and when being forced to work under the presence of a camera. One group actually gave me a report of the accomplishment they had done during that morning.

The Danish team decided to video film one class during three different occasions, namely January 3, January 7 and January 14 (2008). There are altogether ten different video cuts from these three occasions each covering a double lesion of 90 minutes. I addition the class worked with the project for one more double lesson of which there is no video documentation. Although I not can follow the Danish easily, it seems to me that the teacher in Denmark carefully introduces and tries to problematize the Asthma project for the students during the session at January 3. Among other things, he introduces the concept of compartment models in relation to the Asthma project. The students were also encouraged to ask questions during that session. The students are mainly scattered around the computers in what seems to be a computer laboratory, although the film cuts sometimes shows an interview with just a few students where they can explain how they think about the Asthma project.

The next session, at January 7, starts with the teacher who asks the students if there are any new questions since the last time. The teacher emphasizes that the task at hand is to select the suitable model out of several different models. Then the researcher also becomes more and more involved in the students work, asking probing questions and giving support to several groups. The fact that the researcher takes active part in the modeling process the students are going through unfortunately also affects the quality of the filming in a negative way. It is hard to be a film producer, a researcher, and an involved teacher at the same time.

The final session, January 14, is a session totally devoted to group wise presentations of solutions the different tasks within the Asthma project. The students show over head images and write on the black board. It is hard to read the text on the images and the writings the students do on the black board for some of the presentations. Nevertheless, it seems as if the students are presenting throughout solutions. Other students are encouraged to ask questions and some interesting discussions seem to take place. But my Danish is poor and at this part of the session, a sub title in English really would be wonderful.

A look back at the research question I put forward in the beginning of this paper might be useful.

- It seems as if the Asthma project will pass as a good mathematical modeling project in at least three out of four different countries.
- Even without the Romanian contribution we could conclude that the two Swedish teachers and the Danish teacher all taught the Asthma project quite differently.
- It is hard to exactly define what the students in Denmark and Sweden learned, but at the same time it is evident from the video films that they did learn. Hopefully the teachers will bear that in mind when grading her or his students.
- When we receive the film from Romania, it might be feasible to answer to what was different in learning outcomes between the four countries. It is also necessary to get the Danish film sub titled in English.

Here are the comments from the different teachers regarding the Asthma project in their teaching:

Mikael: The students were generally positive working with the Asthma. They seemed to appreciate the fact that this was “a real” problem, a problem they could actually see the use of trying to solve. Examples of negative aspects that were brought up was that they were given to little time to work on the problem and that they didn't understand that this was actually a part of the mathematics course (which would have made them take the task more seriously). It seemed that the few negative comments were usually related to lack of proper communication regarding the importance and the seriousness of the problem than the actual problem itself.

One difficulty for me, as a teacher, when working with this kind of problems, is to know how much help I should to give to my students. A certain amount of help is of course necessary to deepen their understanding of the problem. But I also believe it is very easy to "give away to much help" which results in that it is more the teacher than the student that solves the problem. I've experienced that I need to decide for myself if I want the students to arrive at a "mathematically correct solution" or if I am more focused towards trying to develop their problem solving capabilities. These two perspectives open up for two very different types of methodologies when working with the cases.

Tatiana: It feels rather important to choose the right time for working with new problems or in a new manner. To solve the Asthma problem was not easy at all for my students. It was exiting to see the several different suggestions to solutions and the many different constraints which the students acknowledged such as: not wakening the patient at night, not expose the patient against to large fluctuations in medication, and so forth. The students we surprisingly enthusiastic and they appreciated that data was authentic; they asked me a lot about measuring methods and about the accuracy of the measuring instruments that were used. It is my opinion that they worked much more seriously with the Asthma problem than with nay of the problems they find in the text book. And when the students finally got a good grip around the large amount of mathematical concepts that actually play a part in the Asthma problem, then they really acknowledge deep Aha feelings.

I, as teacher, consider the lessons within the DQME II project as rewarding. To solve the Asthma problem has been difficult for many students, but it has also given many students the possibility to surpass their own expectations. That is perhaps the most beneficial with using this kind of working method.

Peter: The translation from differential equations to spreadsheet layout caused the students some trouble, which we found was due to conceptual difficulties concerning the concept “rate of change” rather than technical problems with the spreadsheet “language”. The oral presentations by the groups from the last session of the project were repeated during a seminar open to the public (“The Day of Research”), and were followed by a good discussion between the students and the audience (upper secondary teachers, university teachers, upper secondary students). The discussion showed that the students really had considered details of the situation where two different models can fit the same data. The authentic data did in fact enhance the motivation, which was notably higher than normal. This project did not train the early part of the modelling process (translation from “the real situation” to “mathematical model”), nor did it focus upon the use of modelling as a means of learning mathematical concepts, which therefore might be worthwhile to dwell more upon in a future project.

Final remarks

It is extremely beneficial as a researcher to have this opportunity to actually be part of what takes place in the class room where I do my video documentation. It is also exceptionally beneficial to be allowed to see how different teachers handle the same problem quite differently and in harmony with what they consider possible to achieve in their class. In one of the Swedish classes the students used graphing calculators as the main technology but mainly worked with paper and pencil techniques, nevertheless they got involved in quite advanced mathematical discussions. In the other Swedish class they used a computer based tool by name Graphmatica and although restricted by a tight time frame they also did a nice mathematical work. The Danish teaching experiment were much more ambitious with six video recorded lesson over the occasions, the class was also more mathematically matured and could therefore use more sophisticated tools in their modeling process and reach more throughout results. The main technological tool in the Danish class room was Excel. For me as a researcher it is a diligently conclusion that all students in these three classes learned a lot of mathematics in connection to their work with the medication problems although not necessary exactly the same mathematics. That is something we must accept and be happy with when working with comparisons of the teaching and learning of mathematical modeling over national and cultural borders. I am convinced regarding the fact that we all learn a lot from our fellow mathematics educators around Europe, including both researchers and teachers, is something most beneficial for both researcher and teachers in each participating country.

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Appendix

Humans that suffer from Asthma are often treated with the medicine theophylline. Theophylline, also known as dimethylxanthine, is a methylxanthine drug used in therapy for respiratory diseases such as asthma under a variety of brand names. Patients are often treated with an equally large dose, D mg, over equally large time intervals, T hours. A doctor has measured how the concentration of theophylline in the blood of one patient varies after the patient has being injected with a dose of 60 mg. The results from this measurement are the basic data for the project. See table 1.

Table 1: Accumulation of theophylline in the blood

Time (hours)	Concentration mg/liter (mg/L)
0	10.0
2	7.0
4	5.0
6	3.5
8	2.5
10	1.9
12	1.3
14	0.9
16	0.6
18	0.5

The students will be asked to construct a mathematical model for the whole situation and to write a report to the doctor that addresses the following questions:

1. How will the concentration of theophylline in the blood decrease over time?
2. How can we plan a continuously medication schema with a fixed dose D over a fixed time interval T, so that the concentration after a couple of injections is in the interval 5-15 mg/L?
3. How can we plan a continuously medication schema with a start dose and thereafter a fixed dose D over fixed time interval T, so that the concentration **directly** will be within the interval 5-15 mg/L?
4. What considerations must be taken into account before one use this medication plan for a patient?