

Conceptualizing and Measuring Mathematical Knowledge for Teaching: Issues from TEDS-M, an IEA Cross-National Study¹

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ABSTRACT

This paper describes the conceptual framework for mathematical knowledge for teaching used in the Teacher Education and Development Study in Mathematics [TEDS-M 2008] and discusses challenges associated with attempting to define and measure this construct cross-nationally. TEDS-M 2008 is a study of the preparation of future mathematics teachers at primary and lower secondary levels and related policies, practices and outcomes. It is being conducted in 18 countries under the auspices of the International Association for the Evaluation of Educational Achievement (IEA). Because data entry will not be complete by July 2008, no results from TEDS-M can be shared with ICME-11 participants. However, the discussion of issues and challenges associated with defining and operationalizing mathematical knowledge for teaching in an international context will inform the participants of Topic Study Group 27 and others doing research in this area.

¹ Dozens of people have contributed to the work described in this paper, including numerous researchers, graduate students, and experts from MSU, ACER, the countries participating in TEDS-M, and other countries. We thank all who contributed to the conceptualization of the framework for Mathematical Knowledge for Teaching, and to the development and review of items and scoring guides for items.

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Interest in teachers' knowledge of the content of school subjects has a long history. Government officials and policy makers in many countries share a common concern that too many teachers are ill-equipped to teach mathematics well. Ever since the seminal work of Shulman (1986, 1987), researchers and those who prepare mathematics teachers have wondered about the nature of subject matter knowledge needed for teaching, and to what extent ideas such as pedagogical content knowledge interact with subject matter knowledge in the work of teaching. Many researchers from around the world, e.g., Adler and Davis (in press), Ball and Bass (2000), Blum and Krause (2006), Even (1993), Even and Tirosh (2002), Fan and Cheong (2002), Hill and Ball (2004), Hill, Rowan and Ball (2005), Kunter et al. (in press), and Ma (1999), have been studying various aspects of mathematical knowledge for teaching. As reviews by Ball, Lubienski, and Mewborn (2001) and Hill, Sleep, Lewis and Ball (2007) indicate, these studies address matters of definition (e.g., What comprises mathematical knowledge for teaching?), measurement (e.g., How can we quantify mathematical knowledge for teaching?), and effects (e.g., How is mathematical knowledge for teaching related to pupil's achievement?).

Recently, researchers have begun to study mathematical knowledge for teaching cross-nationally. The Mathematics Teaching for the 21st Century (MT21) project (Schmidt et al., 2008) is a cross-national study that investigated the preparation of middle school mathematics teachers in 34 institutions in six countries. The frameworks and item development work from MT21 have informed the work described in this paper. MT21 also provides preliminary evidence about the extent to which teacher preparation practices and achievement of future teachers vary in Bulgaria, Chinese Taipei, Germany, Korea, Mexico, and the USA.

For the past several years the Teacher Education and Development Study [TEDS-M 2008], a large-scale cross-national research project to study the preparation of teachers of mathematics at the primary and secondary level has been building upon this earlier work. Among other things, TEDS-M is examining the nature and extent of knowledge for teaching among future teachers enrolled in their last year of teacher preparation programs, and factors such as opportunities to learn, nature of route, and length of program that are hypothesized to be associated with that knowledge. TEDS-M 2008 is sponsored by the International Association for the Evaluation of Educational Achievement [IEA] with leadership provided

by researchers at Michigan State University [MSU] and the Australian Council for Educational Research [ACER].² Researchers at MSU were responsible for developing items for future lower secondary teachers; researchers at ACER were responsible for leading the development of items for primary teachers.

TEDS-M is the first cross-national research on mathematics teacher education to collect data on the policies, curriculum, programs, processes and outcomes of teacher education from nationally representative samples of institutions, teaching staff and students in these institutions. In addition to reporting descriptive data, the TEDS-M research team will also test hypotheses at the international level about relations between future teachers' knowledge and beliefs and characteristics of teacher preparation routes and programs.

In each country the target population consists of all students who will eventually teach mathematics at either primary or lower secondary level, and who are enrolled in their last year of a teacher preparation program in an institution of higher education. As is the case in all IEA studies, TEDS-M 2008 uses national probability samples to estimate characteristics of the intended population.

Data for the Main Study are being collected between October 2007 (in the Southern Hemisphere) and June 2008 (in the Northern Hemisphere) in the following countries: Botswana, Canada, Chile, Chinese Taipei, Georgia, Germany, Malaysia, Mexico, Norway, Oman, Philippines, Poland, Russia, Singapore, Spain, Switzerland, Thailand, USA. Samples per country vary from about 140 future teachers in 7 institutions in Botswana where a census of future primary and secondary teachers and institutions has been taken to a sample of about 3600 future teachers from more than 60 institutions in the USA. We expect the entire sample participating in TEDS-M to include about 24,000 future teachers in the 18 countries. Final results of TEDS-M will be available for release in early 2010.

TEDS-M addresses several research questions. In this paper we focus on two research questions related to the theme of Topic Group 27 at ICME 11:

- What are the level and depth of the mathematics and related teaching knowledge attained by prospective primary and lower secondary teachers that enables them to teach the kind of demanding curriculum currently found in the higher achieving countries and required by the higher standards that many states have adopted?

² TEDS-M is funded by the participating countries through the IEA organization, with major funding from the US National Science Foundation (NSF) (First International Mathematics Teacher Education Study (TEDS-M); Maria Teresa Tatto [PI]; and John Schwille and Sharon Senk [Co-PIs], NSF REC 0514431 9/15-2005 to 9/15/2008).

- How does this knowledge vary across countries?

In the following sections of this paper, we focus on describing the way that TEDS-M conceptualizes mathematical knowledge for teaching, and on discussing challenges encountered when trying to define and measure this construct in a cross-national study.

A FRAMEWORK FOR MATHEMATICAL KNOWLEDGE FOR TEACHING

In the TEDS-M 2008 study, *mathematical knowledge for teaching* is assumed to have two dimensions: *mathematics content knowledge* and *mathematics pedagogical content knowledge*. Each of these dimensions is hypothesized to be composed of various sub-domains, which we will test empirically when final data are available.

Because one part of the TEDS-M study (not discussed in this paper) examines relations between teachers' mathematics content knowledge and the content of the curriculum they will be expected to teach, TEDS-M uses the mathematics content framework that has been used in the TIMSS studies of primary and lower secondary students. In that framework, mathematics content knowledge is reported in four content domains: number, algebra, geometry, and data. TEDS-M also uses the same framework for levels of cognitive demand (knowing, applying, and reasoning) used by TIMSS. (See Mullis et al. (2007) and Garden et al. (2006) for details on the TIMSS frameworks.)

In TEDS-M some mathematics content assessed is at the level that the future teacher might teach, i.e. primary or lower secondary levels. However, other questions require knowledge of mathematics several years beyond that at which the future teachers will teach. For instance, items assessing the mathematics content knowledge of future teachers of lower secondary mathematics include some questions about calculus and linear algebra.

Due to psychometric constraints of reporting by sub-domain (number of items required to report reliably per sub-domain) and practical constraints on available survey time, in the International Report for TEDS-M, sub-scores are expected to be reported for only three content domains: number, algebra, and geometry. Some items about data have been included on both primary and secondary instruments to increase the overall validity and reliability of the measures, but there are not enough questions to report a sub-score about data.

Based upon our reading of the literature and recommendations from various expert reviewers, the framework for mathematics pedagogical content knowledge is hypothesized to have three sub-domains called *mathematics curricular knowledge*, *knowledge of planning mathematics*, and *knowledge of enacting mathematics*, which are elaborated in Table 1. This

framework pays attention to the temporal dimension of teaching, moving from knowing what mathematics to teach, to planning to teach it, to carrying out instruction.

Readers may be interested in comparing the TED-M framework with the frameworks developed for the COACTIV and MT21 projects, which each divide mathematical pedagogical content knowledge into three theoretical sub-domains as follows. COACTIV uses tasks and multiple solutions; misconceptions and difficulties; and explanations and representations (Blum and Krauss, 2006); whereas MT21 distinguishes curricular knowledge, instructional planning, and student learning (Schmidt et al., 2008).

Table 1: Sub-domains and aspects of the sub-domain of mathematics pedagogical content knowledge used in TEDS-M.

Mathematical curricular knowledge	<ul style="list-style-type: none"> • Establishing appropriate learning goals • Knowing different assessment formats • Selecting possible pathways and seeing connections within the curriculum • Identifying the key ideas in learning programs • Knowledge of mathematics curriculum
Knowledge of planning for mathematics teaching and learning	<ul style="list-style-type: none"> • Planning or selecting appropriate activities • Choosing assessment formats • Predicting typical students' responses, including misconceptions • Planning appropriate methods for representing mathematical ideas • Linking didactical methods and instructional designs • Identifying different approaches for solving mathematical problems • Planning mathematical lessons
Enacting mathematics for teaching and learning	<ul style="list-style-type: none"> • Analyzing or evaluating students' mathematical solutions or arguments • Analyzing the content of students' questions • Diagnosing typical students' responses, including misconceptions • Explaining or representing mathematical concepts or procedures • Generating fruitful questions • Responding to unexpected mathematical issues • Providing appropriate feedback

CHALLENGES OF MEASURING MATHEMATICAL KNOWLEDGE FOR TEACHING

Like others, TEDS-M researchers have faced a number of challenges related to measuring mathematical knowledge for teaching. Four that we discuss in this paper are: motivating future teachers to participate in the research, content validity of the items, reliability of scoring of the items; and identification and delineation of the two hypothesized dimensions forming mathematical knowledge for teaching.

Motivating Future Teachers to Participate in Research

Because TEDS-M assesses adults who are university students, the TEDS-M research team was worried that future teachers might not be as keen as primary or secondary school students about completing an achievement test. There was a concern that a large proportion of “no-responses” would appear in the test items if the test looked like an “ordinary” achievement test. Something needed to be done to attract and motivate future teachers to respond to the items we wished to administer.

TEDS-M has responded in two ways. First, researchers in several participating countries have found that offering financial enticements to future teachers who complete all parts of the survey increases the survey response rate.

Second, as has been done in other recent studies of teachers’ knowledge, e.g. the Learning Mathematics for Teaching project (Hill & Ball, 2004) and the Mathematics Teaching for the 21st Century study (Schmidt et al, 2008), researchers developed items testing mathematics content knowledge that are set into classroom contexts. (see examples in section on Content Validity below). Similarly, mathematics pedagogical content knowledge, as described in Table 1, is assessed using references to mathematics problems appropriate to the level at which the future teacher hopes to teach. The assumption is that such items are not only appropriate for measuring both content knowledge and pedagogical content knowledge, but they are also interesting for the future teachers, because the items are connected with their future profession.

Item formats

As Hill, Sleep, Lewis and Ball (2007) note, there are both advantages and disadvantages of using different item formats to assess mathematical knowledge for teaching. Multiple choice items are easy to score reliably, and have a history of use in testing across the world. However, “even when the content of the multiple choice questions is focused on robust expressions of what teachers know how to teach,” the format may inadvertently

support the “misconception that mathematical competence is demonstrated by quick solutions to routine mathematical problems.” (p. 150). In contrast, constructed response items may have greater face validity, but responses are often harder to interpret reliably.

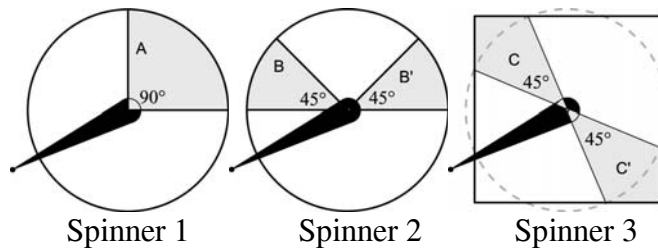
TEDS-M uses three item formats to assess both mathematics content knowledge and mathematics pedagogical content knowledge. Two formats, known as multiple choice (MC) and complex multiple choice (CMC) items, request the respondent to select among two or more choices. Constructed response (CR) items elicit open responses. Table 2 shows the number of items of each format used to measure mathematical knowledge for teaching in TEDS-M.

Table 2 Distribution of item formats by domain in both primary and secondary TEDS-M instruments.

Level	Primary			Secondary		
	MCK	PCK	Total	MCK	PCK	Total
MC	23	6	29	10	4	14
CMC	56	20	76	64	23	87
CR	5	14	19	13	2	15
Total	84	40	124	87	29	116

Figure 1 shows a complex multiple choice item administered to future teachers of lower secondary schools to assess mathematical content knowledge. Figure 2 shows an item for future primary teachers requiring constructed responses. Part (a) is a short constructed response testing mathematics content knowledge; whereas part (b) asks for a longer constructed response about mathematics pedagogical content knowledge, specifically for the sub-domain of enacting mathematics teaching.

Your students are examining the three spinners shown below. They are discussing the probability that the spinner stops over a shaded region.



Please indicate whether the following statements of four students are Completely True, Partly True or Completely False. If one sentence is true, but the other is false, check Partly True.

		<i>Check <u>one</u> box in each row.</i>		
		Completely True	Partly True	Completely False
A.	Sherry says, "The probability is twice as large for spinners 2 and 3 compared to spinner 1 because they have two regions to stop on and spinner 1 has only one region."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
B.	George says, "Spinners 1 and 2 have the same probability since the shaded regions have the same area. Spinner 3, however, has a higher probability than spinner 2 because the shaded region is a larger area."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
C.	Paul says, "Spinners 1, 2 and 3 have the same probability because the angles of the shaded regions are the same size."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
D.	Rainey says, "The probabilities for spinners 1 and 2 are the same because those areas are the same proportion of the whole circle. For spinners 2 and 3, however, the probabilities are different because the shaded areas for spinner 3 are a bigger proportion of the whole square than they are of the circle."	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Figure 1. Complex multiple choice item testing mathematics content knowledge administered to future teachers of lower secondary schools in the TEDS-M Field Trial. Idea for item was provided by Prof. Walter Whitely, York University, Toronto, Ontario, Canada. Key: A, Completely false; B, Partly true; C, Completely true; D, Partly true.

A teacher gave the following problem to her class.

The numbers in the sequence 7, 11, 15, 19, 23, ... increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, ... increase by nine.

The number 19 is in both sequences.

If the two sequences are continued, what is the next number that is in BOTH the first and the second sequence?

(a) What is the correct answer to this problem? _____

(b) A student gives the response 27 and 46 to the question above. What is the most likely reason for this response?

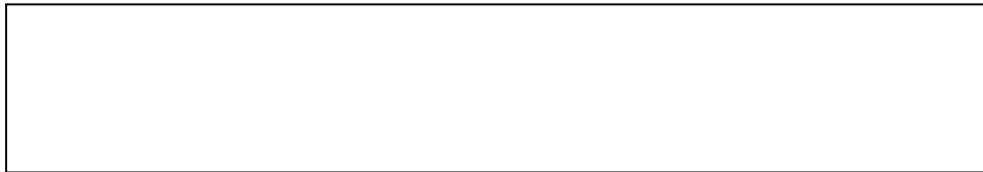


Figure 2. Two constructed response items administered to future primary teachers in the TEDS-M Field Trial. Item TD1P060A, which tests mathematics content knowledge, is a released item from TIMSS Grade 8 2003. Item TD1P060B, which tests mathematics pedagogical content knowledge, was developed by ACER.

Content Validity

To ensure content validity of mathematical knowledge for teaching cross-nationally, potential items were solicited from researchers around the world, tested in the field, and reviewed by panels of international experts. For example, as mentioned earlier, one key request was that, where possible, items measuring mathematics content knowledge are set in a classroom context to improve face validity and make the instrument look less like a traditional pen-and-paper mathematics test.

Some potential items were obtained from members of the National Research Centers and their colleagues in participating countries. Also, researchers who had developed items for other studies gave permission to TEDS-M to use their items testing mathematical knowledge for teaching, e.g. the MT21 project (Schmidt, et al., 2008) at MSU and the Learning Mathematics for Teaching Project at the University of Michigan (Hill & Ball, 2004; Hill, Rowan & Ball, 2005).

Potential items were piloted in six countries in 2006 and the best were administered in a Field Trial conducted in 12 countries in early 2007. Expert panels of mathematicians and mathematics educators were convened in Australia and the USA to review items after both the item pilot and the field trial to improve their clarity, to check categorization of the items into sub-domains, to comment on cultural relevance, and after the field trial, to help select those items with the best mix of content validity and psychometric properties.

Reliability of Scoring

To ensure reliability of scoring of constructed response items, detailed scoring rubrics have been developed for each item. All guides use a two digit scoring scheme with the first digit indicating whether the item is completely correct, partially correct, or incorrect, and the second digit indicating the approach the student took to the problem or a particular incorrect solution. No constructed response item will contribute more than two points to a total score for each sub-domain.

Sample scoring guides for the two constructed response items shown in Figure 2 are provided in Tables 3 and 4. Item TD1P060A is assigned a maximum of 1 score point, whereas Item TD1P060B is assigned a maximum of two score points.

The scoring rubrics were discussed at scoring workshops held in the USA and in Switzerland, where those in attendance scored sample responses from participants in the item pilot or field trial to check their understanding of the scoring rubrics. Those who attended the scoring workshops were also trained in conducting reliability scoring and bias training. Each country representative was asked to replicate these training sessions in their own countries in order to score the responses with high reliability.

Factors that may affect reliability of scoring include poor handwriting making it difficult to decipher what the future teacher meant, and rubrics that do not anticipate all possible solution strategies or errors. During the main study some papers in each country will be double scored to check for reliability of scoring, with the goal being 100% agreement on each item; however, an agreement of 70% will be considered acceptable.

Table 3: Scoring Guide for Item TD1P060A

Code	Response	Item: TD1P060A
	Correct Response	
10	55	
	Incorrect Response	
70	Either of the responses '27 and 46' or '19' indicating a misreading of the question.	
71	Any other incorrect value, probably indicating a calculation error.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

Table 4: Scoring Guide for Item TD1P060B

Code	Response	Item: TD1P060B
	Correct Response	
20	<p>A response that recognizes that the student has misread, misinterpreted or misunderstood the question and explains the likely misinterpretation.</p> <p><i>Examples: The student misinterpreted the question to mean, "What is the next number in each sequence/both sequences?"</i></p> <p><i>The student interpreted 'BOTH' as meaning give 'two' answers.</i></p>	
	Partially Correct Response	
10	<p>A limited response that recognizes that the student has misread, misinterpreted or misunderstood the question but does not explain the likely misinterpretation.</p> <p><i>Example: The student misread/misunderstood the question.</i></p>	
11	<p>A response that simply explains that 27 and 46 are the next numbers in each sequence (but does not give a reason <u>why</u> the student has answered this way.)</p> <p><i>Examples: They are the next numbers in each sequence.</i></p> <p><i>The student gave the next numbers in each sequence rather than the '<u>same</u>' number in each.</i></p>	
	Incorrect Response	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	No response	
99	Blank	

Relation between Content Knowledge and Pedagogical Content Knowledge

To examine the question of whether mathematics content knowledge and mathematics pedagogical knowledge are different constructs or whether they are simply testing the same latent trait, factor analysis and Item Response Theory were used on data from the Field Trial. TEDS-M researchers were encouraged by the results. The mathematics content knowledge scale and the mathematics pedagogical content scales appeared to be different dimensions of mathematical knowledge for teaching. This preliminary finding is supported by the work of Hill, Schilling and Ball (2004) who found similar results. However, data from the field trial did not allow TEDS-M researchers to distinguish among the three hypothesized components of mathematics pedagogical content knowledge. That is, we did not have a sufficient number or the right type of items to distinguish curricular knowledge from knowledge of planning or knowledge of enacting teaching. It remains to be seen whether similar results will be attained from the main study; but for now we are encouraged that mathematical pedagogical knowledge in the TEDS-M instruments appears to be distinguishable from mathematics content knowledge.

SUMMARY

The TEDS-M study is an innovative and unique cross-national research project investigating the policies, programs and outcomes of teacher education in mathematics in 18 countries. After a successful Field Trail, the project is now in the data collection phase of the Main Study with first reports due for publication in early 2010.

TEDS-M has been most fortunate to have the goodwill and cooperation of many scholars and colleagues in the participating countries that have given so generously of their expertise and knowledge in this field. The pioneering work of others cited above has built a solid foundation for work in this field and greatly informed TEDS-M.

It is vital, however, that the pitfalls and criticisms leveled at other comparative international studies which are sometimes based upon Western conceptions of curriculum and pedagogical practices are avoided. In Clarke's words,

“International comparative research must be undertaken on the basis of mutual benefit to all participants. ... We must guard against the cultural imperialism of a global curriculum and instead stress the centrality of local interpretation of all findings.” (Clarke, 2003, p.180)

In the case of TEDS-M we also want to avoid promoting a ‘global pedagogy.’ We hope that the extensive collaboration of the countries participating in the TEDS-M study to date and

their rich contributions to the framework design and item pool strongly suggest that findings from this study will be for the mutual benefit of all.

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