ICME 11 - Monterrey, Mexique, july, 4-11, 2008
TSG 15 - Research and development in the teaching and learning of discrete mathematics

Some specific concepts and tools of Discrete Mathematics

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Introduction

Discrete Mathematics deals with finite or countable sets, and thus, in particular, with natural numbers. This mathematical field bring into play several overlapping domains, e.g. number theory (arithmetic and combinatorics), graph theory, and combinatorial geometry. As a consequence of the peculiarities of discreteness versus continuum, interesting specific reasonings can be developed (Batanero and co 1997), and new tools can be constructed, such as coloring, proof by exhaustion of cases, proof by induction, use of the Pigeonhole principle (Grenier et Payan 1999 and 2001). Furthermore, several concepts involved in other mathematical domains are also used in this field, in a particular manner, e.g. the Bijection Principle[2], optimization techniques and the notions of « generating set » or « minimal set ».
In this paper, I wish to develop two of these specific tools, the *Pigeonhole Principle* and the *Finite Induction Principle*. I frequently use these tools in my courses, to introduce students to discrete mathematics and initiate counting, along with modelling and proof-elaboration activities.

1. The Pigeonholes principle

The Pigeonhole Principle[3] plays an important role in numerous reasonings of discrete mathematics, especially those involving integer numbers. Its generalization capability allows « existence-problems » to be solved (for example, existence of injective functions on finite sets, of integer solutions of a system of linear equations). In its elementary version, this principle simply asserts that :

Given m pigeons to put in n<m holes, at least one hole must contain more than one pigeon.

In France, this principle is not taught, and in fact almost never discussed[4]. However, we consider that there are several reasons, from the standpoint of epistemology or didactics, which would justify the explicit introduction of the pigeonhole principle at some point in pupils'curriculum.

The first reason is the peculiarity of this principle, in that it can prove the existence of objects belonging to a class of objects defined by one or several properties. There are few

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[2] see Michel Spira's paper for this TSG
[3] Also called « Dirichlet box principle ».
[4] This principle is taught for example in Hungary to 15 year-olds students, and a lot of problems solved with this tool, are given in schoolbooks at that level.

such tools in standard teaching activities[5]. A second reason is its unusual effectiveness, either because it simplifies the exposure of a proof or a solution, or because it may appear as the only possible way of solving the problem. We analyze a few examples below. Of course, this is not pure magic. If the PP allows a complex problem to be solved, it is because the initial problem has been rephrased and turned into a model.

In French teaching, modelling to pupils has been the constant focus of curricula for years, at all levels. However, teachers tend to avoid it, as they face difficulties which they do not know how to handle. They get a real incentive when problems are made available to them, in which the modelling is particularly effective, even though it can still be rather tough to find : this is especially true when the model leads to an easier, accessible and convincing solution.

The general model well suited to an application of the Pigeonhole Principle turns out to be rather simple : pigeons and holes (or objects and boxes), and a rule specifying how to put the pigeons into the holes. The difficulty is rather to recognize the model, i.e. to find, in the initial data, the « holes », the « pigeons » and the rule. It will then be necessary to translate the solution expressed in these terms into a solution of the initial problem.
However, it appears that the Pigeonhole Principle does not carry in itself these difficulties, since they are inherent to any genuine modelling activity, i.e. one which is not a trivial rephrasing of the given problem.

Teaching the Pigeonhole Principle is thus in our view a very good opportunity to demonstrate the interest of certain mathematical modelling procedures, in which the Pigeonhole Principle's effectiveness is apparent for solving problems whose proofs are not immediately accessible.
I illustrate the strength of this "obvious" principle here with a few problems, from arithmetic or combinatorial geometry. For each of them, I will give the mathematical and didactic background.

Problem 1 « Handshakes » problem
Proposition. *If there are n persons (at least two) who can arbitrarily shake hands with one another, there is always a pair of people who shake hands with the same number of people.*

It is rather easy to do the experiment with pencil and paper and to convince oneself that the proposition is true. Now the question is, how can be it proved ? Something similar to graph representations may emerge rather quickly. Otherwise, such representations could be suggested. The statement involves in a natural way the evaluation of the number p of hands that a person may shake. The answer can be given in two steps : p is a number between 0 and n-1, but these extreme values are not simultaneously possible. Therefore, there are only n-1 possible values : either 0, 1, ..., n-2 (in case one person at least does not shake any hand); or 1, 2, ..., n-1 (in case everybody shakes the hand of somebody else).
The two cases can be studied separately. The « holes » are the admissible values of p, there are n-1 such values. « Pigeons » are the persons. Every person is sent to the « hole »

[5] Actually, existence questions are rare in the french secondary education, to the point that a statement such as « In some cases, an example may be enough to provide a proof » is considered to be wrong by a huge majority of students. Fact is, that they have been told for years that « An example is not enough to produce a proof ».

corresponding to the number of hands he or she has shake. Since there are n persons and n-1 holes, there will be at least two persons in the same hole, QED.

This property can be expressed in graph theory terminology as follows:
Theorem. *In every graph with simple edges, there are at least two vertices which have the same degree ».*[6]

Problem 2.
Proposition. *In an arbitrary sequence of m+1 natural integers, at least two of them have the same remainder in the Euclidean division by m.*

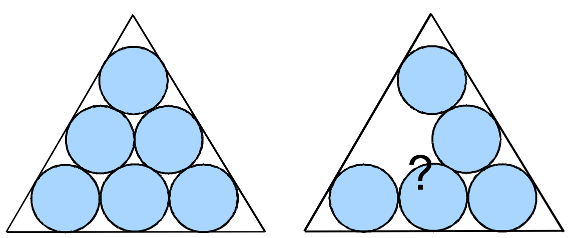
Proof. The m possible remainders of a division by m are 0, 1, ..., m-1. To each remainder we associate a « hole ». Then we put each of the m+1 given natural integers into the hole which corresponds to the remainder of its division by m. There are m holes and m+1 numbers, so at least two of these numbers must lie in the same hole. QED.

This proof is short, convincing and accessible already from the French « collège » level (11 -15 year-old students). The model can become apparent from the fact that one tries to classify numbers according to their remainders by m. Classes are subsets of N, the analogy with holes or boxes is not too far.

The efficiency of the Pigeonhole Principle lies here in the way the proof is expressed, since the model avoids working algebraically with numbers of the form mk+r, where k is an integer and $0 \leq r \leq m-1$. However, the « algebraic proof » is still relevant from the point of view of epistemology. This problem can be an opportunity to discuss, with students, the interest and the limitations of each of these approaches.

Problem 3.
What is the smallest equilateral triangle in which k disks of diameter 1 can fit ?

Let us mention that this problem is solved only for small values of k and for triangular numbers, namely whole integers of the form $k=q(q+1)/2$. This problem is a genuine contemporary research question and it has been well popularized (Steward, 1994).

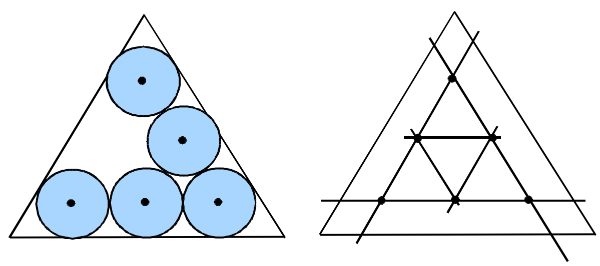


For example, on the above left figure, 6 disks of diameter 1 fit in an equilateral triangle of edge length $2+\sqrt{3}$. Can one put 5 disks (or maybe even 6) in a smaller equilateral triangle ?
One can easily anticipate that it would be difficult to solve this question in analytical geometry, because of the large number of parameters of the function to be optimized.

6 This theorem is now explicitly taught in the final year of high school, for students taking the « specialization courses ».

If we look differently at the problem and at its input, the following questions arise : how can one express the fact that the disks are identical, enclosed by circles, that they cannot overlap and must lie entirely in the triangle ? In fact, the point is to realize that positions of the centers completely determine the positions of disks, so that the non overlapping condition is equivalent to a suitable lower bound for the distance between centers of two disks and, finally, that the disks lie inside the given triangle if and only if their centers lie in a certain smaller triangle, as depicted below.

Therefore, we are led to make a modelling which is internal to mathematics, which maps in a one-to-one manner a disk to a point, the non overlapping condition to a lower bound between two points, and the initial triangle to a smaller one.
By a translation of 1/2 unit toward the interior of the triangle, in a perpendicular direction to the edges, one obtains an equilateral triangle of length 2 which contains all circle centres. The question is thus reduced to the following one : find the smallest equilateral triangle containing 5 points with the property that any two of them are separated by at least 1 unit of length.



Let us show that such a triangle cannot have an edge length less than 2. If not, divide the triangle in four homothetic ones in the ratio 1/2, by joining the edge middle points. One of these triangles would contain at least two of the points. The distance between these two points would be less than the smaller triangle's edge length, thus less than 1, which is a contradiction.

Experiments at various levels have shown that this problem is interesting in view of the modelling activity, and is helpful in order to assess the power of some models and tools. In particular, there are meaningful questions pertaining to the equivalence between the problem posed in terms of disks and the problem posed in terms of points. Finally, we observe that the solution provided by the Pigeonhole Principle is accessible and convincing. However, the modelling part may be difficult to settle.

The general problem of finding an equilateral triangle of minimal size containing n disks is solved only for certain specific values of n, and quite often they are values for which one can apply the Pigeonhole Principle model in the way described above.

2. Induction, a specific tool for constructing and proving solutions

Induction can be viewed as an axiom which lies at the foundation of the definition and structure of natural numbers. It is most often presented in the form of a « principle » which allows properties of integer numbers to be proven, or more generally, of well ordered sets.

2.1. Students' conceptions about induction

In the French curricula, induction appears for the first time when *sequences* are studied (sequences defined by induction, students aged 16-17), and is later used essentially while proving formulas or identities depending on an integer n.

My research over the past years (Grenier, 2002 & 2003) has shown that the knowledge of French students studying mathematics at the undergraduate level concerning induction is very limited and very often inaccurate. I give here a synthesis of these results.

First, ***induction is neither taught as a concept*** nor as a feature of N, but only as a « principle », or a « technique » for proving a « property » P(n), of which the only explicitly stated formulation is composed of two steps : *initialization* (selecting n_0 for which $P(n_0)$ true) and *heredity* (for any $n \geq n_0$, $P(n) \Rightarrow P(n+1)$ is true).

Second, ***this principle is almost always used when P is an algebraic relation between functions of n*** (polynomials, or general terms of recurrent algebraic series, or an algebraic inequality), practically never when P is a *property of sets of cardinality n*[7]. I will develop further some examples of this type of application.

Furthermore, while ***exercising the technique***, the initial value n_0 is the first step and, either its value is given, or it is « obvious » (0 or 1!). Students, therefore, typically cannot understand where this value comes from!! For example, in a standard illustration such as the problem « Prove that for $n \geq 6$, $2^n \geq (n+1)^2$ », it is generally unclear to students why one takes $n_0 = 6$ (or why this is permitted).

Last but not least, in many schoolbooks, ***the most widespread formulation of the heredity property of induction has the following characteristics*** which end up weakening the pupils' understanding :

- The « if...then » is implicit, the implication is done in two steps (« first assume that, for any particular $n \geq n_0$, P(n) is true, and then one proves that P(n+1) is true »).
- It is sometimes difficult for students to distinguish between the two statements « P(n) is true for a given n », and « P(n) is true for all n »
- The value n_0 can be confused with the first value for which « $P(n) \Rightarrow P(n+1)$ » is true. This is false. For example, the property P(n) : « $10^n + 1$ can be divided by 9 » is wrong for every n, however the heredity $P(n) \Rightarrow P(n+1)$ is true for every n ! ($10^{n+1} + 1 = 10.10^n + 1 = (9+1) . 10^n + 1 = 9.10^n + (10^n + 1)$.

One of the consequences of these didactic challenges is that misconceptions can persist in the minds of a great number of students. For the sake of illustration, here are a few 20 year old mathematic students' answers to the question « Do you think the induction principle is a tool for proving mathematical statements ?».

« Yes, because it leads to correct results »
« I don't know, because it is obvious that if one assumes that P(n) is true for any n, one concludes that P(n) is true for all n ! So this proves nothing ! »
« Yes, because otherwise, we would not use it for proving ! »
« No, but it seems to be efficient when it is impossible to examine every value of n ».

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[7] It seems that in some countries, one of the first uses of the Finite Induction Principle is to prove that « A set of n elements has 2^n subsets ». This is usually not the case in France.

Quite often, the induction principle appears merely as a « stereotype » which students have to learn and use in a correct formal manner and order. Of course we do not object to a presentation of some normalized form of the principle, but the fact that only poor examples and realizations of it are studied. As a consequence, many students use induction without ever getting a grip on why it really works.
Finally, the induction principle is sometimes used in which the properties to be proved do not epistemologically refer necessarily to induction, such as for instance « Prove by induction that the derivative of $f(x)=x^n$ is $f'(x)=nx^{n-1}$», whereas the concept of differentiability does not make sense on discrete sets.[8]

Now, I will propose a few particular problems, the solution of which is well-suited to improving student understandings of the mechanisms of induction. These problems have been worked out over a long time at various school levels.

2.2. Study of the « standard » statement of the induction principle

In France, a very common way of stating problems related to induction is the following « Prove that for every $n \geq n0$, P(n) is true », the value n0 is given and P(n) is stated as true.
It would be very easy to make variations in the following correct way.

Problem 4.
Here is a generic way of stating problems :
Given a proposition P(n) depending on an integer n, answer the following questions :
(a) For which values of n, is P(n)=> P(n+1) true ?
(b) For which values of n, is P(n) true ?
(c) What is the obvious initial value of n which can be used to start the induction ?

Let us take an example. Consider, for any integer n, the inequality P(n) : $2^n \geq (n+1)^2$.
Then the answers to questions (a), (b), (c) above are respectively : $n \geq 1$; n=0 or $n \geq 6$; n=6.
In texbooks, the problem would usually have been stated in the form : « Prove that for every $n \geq 6$, $2^n \geq (n+1)^2$ ». This unfortunately does not explain to pupils why 6 was chosen as the initial value.
For the variant : Q(n) : $3^n > n^3$, the answers would be : $n \neq 2$; $n \neq 3$; n=4.

The reader will easily find other suitable examples, in which the goal is to distinguish questions (a), (b) et (c) so as to acquire a better understanding of heredity and/or of the reasons of the choice for the initial value n0.

2.3. Some examples where « P(n) is a property of sets of cardinality n »

In France, the induction principle is most often used to prove a property P(n) which is right away given as being true, and where P(n) merely involves algebraic identities or inequalities. Below, I propose two examples of problems where P(n) is a property of sets of

8 We will not discuss here the relevance of this type of use of induction, and the confusion made between tools used for « economy of work » and tools used to promote « meaningful perception ».

objects of cardinality n. In this setting, the property P(n) is necessary written with a quantifier indicating that P(n) is considered « for *every* object » of the set of cardinality n.

Problem 5.

Here is a proposition and its proof. What do you think about them?

«Let P(n) be the following proposition: « Every set of n ghosts including at least one Scottish ghost only holds Scottish ghosts ».[9]

Proof.

P(1) is evidently true.

Let G be a set of n +1 ghosts with at least one Scottish ghost, labelled x, and G={x, g_1, ..., g_n}. Let us consider two subsets of G : H={x, g_1, ..., g_{n-1}} and K={g_1, ..., g_n}. H and K are two sets with n elements, H ≠ K, and H contains the Scottish ghost x.

If P(n) true, then all the ghosts in H are Scottish, in particular g_1 is Scottish, and then all the ghosts in K are also Scottish, so all the ghosts in G are Scottish, i.e. P(n+1) is true.

This proves that P(n)=> P(n+1).

Conclusion : P(n) is true for every n≥1. »

Let us analyse this problem. The proposition is clearly false. It should be concluded that the proof is false, and the next question is then : where is the error ? Students often give wrong arguments, e.g. as follows :

« A set with 1 element is not a set » !

« The proof is false because one cannot argue with two different sets of n elements, H and K »

« We cannot use P(n) twice, in P(n)=> P(n+1) »

« The proposition is false, but it seems to me that the proof is right » !

Our concern is to understand why students consistently have such difficulties. The major reason is probably that n is the cardinality of a set of objects. In the set G of n+1 elements, there are n+1 different subsets of cardinal n (such as H or K). So the question is, for which values of n is it possible to follow the previous argumentation. G has to contain at least x, and also another different element y, in order that two different subsets of G (H and K) can be considered, thus necessarily n ≥2. So, P(n)=>P(n+1) is true for every n≥2, however P(2) is false.

In this example, we can see first the fact that P(1) is true is misleading and not useful, and second, that proving heredity is not just an arithmetical manipulation !

Problem 6. Tiling polyminos (combinatorial geometry)

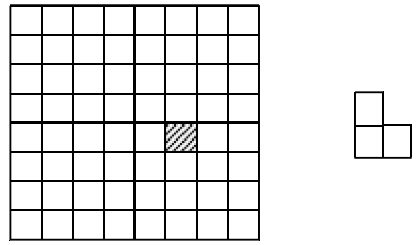
A polymino is a finite union of elementary squares from a square grid, which form a connected set (even after removing the grid points, so they are connected by their sides).
The general problem of tiling a given polymino of any shape with a given smaller polymino is a genuine research problem in discrete mathematics. Here, we examine a particular case.

Let us study the following property P(n) :

Any square of side length 2^n with a deleted square in any position, considered as a polymino, can be tiled by L-shaped triminos.

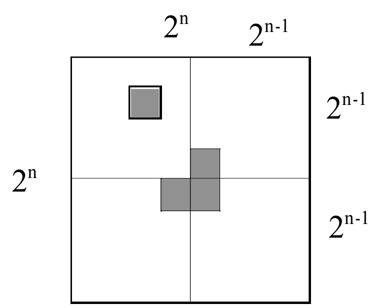
[9] There are a lot of equivalent contexts for this problem.

Here is an example for n=3 and a particular position of the deleted square.



In this problem, for every n, P(n) is a property of the set E_n of all polyminos consisting of a 2^n-side square with a deleted square in any position. (One can show that there are $2^{n-2}(2^{n-1}+1)$ different elements in E_n modulo the group of symmetries of the square).

A elementary and convincing inductive proof exists and can be produced. Consider a 2^n-size polymino with an arbitrary deleted square. Splitting it in four parts, and putting a trimino in its center, as in the figure above, we obtain four 2^{n-1}-size polyminos, each with a deleted square in different positions. If these four 2^{n-1}-size polyminos can be tiled, then the 2^n-size polymino can be tiled. This reasoning and the decomposition in four parts is possible for every n≥1. Conclusion : P(n)=> P(n+1) is true for every n≥1.
It remains to verify that P(1) is true (obvious).
By induction, this proves that P(n) is true for every n≥1.



Let us note two particularities of this problem. First, the initial rank n_0 is given by the proof of the heredity. Second, to prove heredity, it is necessary to consider four different elements of the set E_n, e.g. to use four times the premise « if P(n) true ... ».

2.4. Recurrence axiom and infinite descent (Fermat)

The induction axiom can be written in this way :
No infinite strictly decreasing sequence in N exists.

It leads to the following principle of induction :
If « for every n such that P(n) is true, there exists m<n such that P(m) is true », then « for every n, P(n) is false ».
On the set N, this statement is equivalent to the previous one.

In discrete mathematics research, proofs by induction are very often constructed through the following « reasoning by contradiction » (« reductio ad absurdum ») : suppose there is a counter-example at some step n ; take a minimal counter-example (step n_0) ; n_0 exists because of the Peano axiom : a non empty subset of N has a smallest element ; from this minimal configuration, the proof proceeds by producing a smaller one : we then have a contradiction. Often, the smaller element is obtained by splitting the minimal one. This scheme of proof gives, at the same time, an algorithm to construct the objects and an initialization of the induction.

Problem 7. « √3 is irrational »

A good and classic example of the « infinite descent » is the proof of the property that « √3 is irrational ». Let us assume that there exists two positive integers a and b, such that $\sqrt{3}=a/b$ (E). Then, $a^2=3b^2$. As 3 divides a^2 and 3 is a prime number, then 3 divides a. So a=3a' and (E) is equivalent to $3a'^2=b^2$. The same reasoning leads to « 3 divides b ».
We have proved that « if there exists a and b such as $\sqrt{3}=a/b$, then there exists a' and b', with a'<a and b'<b such as $\sqrt{3}=a'/b'$». According to the axiom of induction given above, this is impossible, therefore a and b do not exist.

The foundation of this proof is the fifth Peano axiom (No infinite strictly decreasing sequence in N exists). This foundation is never recognized by students. For many years, I have regularly asked mathematics students (3rd level undergraduate) the same questions, as follows :
« Give a proof of the property « √3 is irrational ». Do you see a proof by induction ? What properties or theorems underly this proof ? »

Students' answers are essentially always along the same themes, here are a few samples.

E1. I do not see the connection, because induction is used with integer numbers and to prove a property depending on an integer n. (Property suggested : if a=bc and b is prime with a, then c divides a).
E2. It is not a proof by induction, because it is a proof by contradiction. I used properties on divisibility of integer numbers.
E3. I do not see what is the matter, and I do not understand the question. My proof is a proof by contradiction. It is just supported by properties of numbers.

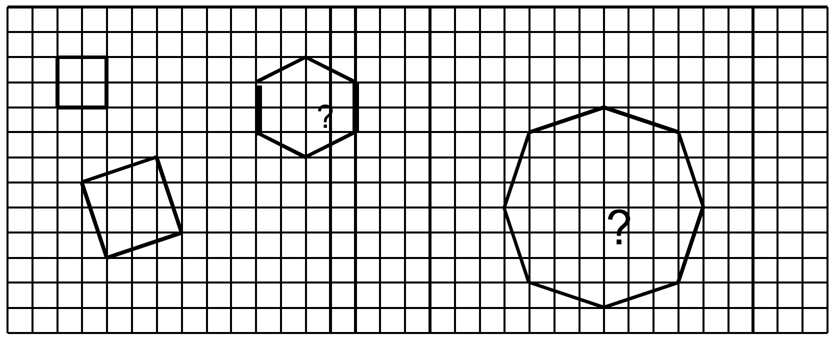
In the usual reasoning by contradiction, when it is assumed that the fraction a/b is irreductible (the g.c.d. of a and b is 1), the induction is hidden. That can explain the confusion between the form of the reasoning « by contradiction » and the foundation of the reasoning « induction ».

Problem 8. Regular polygons with integer vertices

Here is another problem which comes with combinatorial geometry. This solution was published 10 years ago (Payan, 1992).

One can construct infinitely many squares whose vertices are located at the nodes of a square grid. On the other hand, it is easy to show, through suitable geometric arguments, that no equilateral triangles whose vertices are (Gauss) integers can exist. A similar question can be raised for all « regular » polygons.

For example, below, one has drawn two squares, one hexagon and one octagon. Are they actually regular?

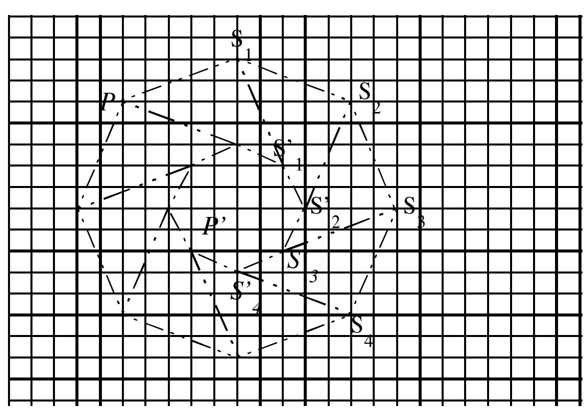


It may appear that the hexagon is not « quite regular » (but then perhaps some regular one does exist on another position in the grid). The octagon seems to be pretty regular. Can one prove or disprove these facts ?

Let us consider the following question:

Do regular octagons with integer vertices exist ?

For an arbitrary polygon P with vertices on the integer grid, let us call « area » the number n of grid nodes which are located in the interior or on the boundary of the polygon P (the convex envelope).
Pick 8 points on the grid, assuming that they form a regular octagon ***P***. Let S_1, S_2, ..., S_8 be its integer vertices, and let n be its area. We apply to ***P*** the following transform : to each vertex S_i of ***P*** is associated a point S_i' such that S_i' is the image of S_i by the rotation of center S_{i+1} and angle pi/2 ($S_9=S_1$). We obtain in this way another regular octagon ***P'*** whose vertices are nodes of the grid located in the interior of ***P*** (see figure below).



The area n' of ***P'*** is strictly smaller than the area n of ***P***. Therefore, we have proved that « for any regular octagon with integer vertices of area n, there exists another regular octagon with integer vertices and strictly smaller area n'». We would then produce an infinite strictly

decreasing sequence of positive integers, associated to an infinite strictly decreasing sequence of regular octagons. This is impossible.
So, in conclusion, there are no regular octagons with integer vertices.

In fact, the general statement is that the only existing regular polygon with integer vertices are squares. We do not know of any other « elementary proof » of this theorem which is likewise valid for every n-gon, $n \geq 5$ (notice that the interior angles of the polygon are then larger than pi/2, which guarantees that the polygon obtained through the construction is contained in the interior of the given one [we exclude here the other proof derived from the theory of cyclotomic fields $Q[\zeta]$, which immediately gives the result for every n different from 4, but requires more advanced knowledge in mathematics ...]

What can be learnt from this problem ? Besides the non standard use of an interesting geometric transform, one directly uses the induction axiom in its conceptual form that there are no infinite strictly decreasing sequences in N. Of course, the procedure for constructing the derived polygon may be suggested to students as an intermediate step.

Conclusion

The examples which I presented here show the strength of two basic principes related to arithmetic and more generally discrete mathematics, the « pigeonhole principe » and the « induction principle ». These tools are sometimes the only ones which practically allow a problem to be solved. For example, the problem of the piling of n disks in a triangle (problem 2), is actually solved only for values of n for which the pigeonhole principle can be used. For most other values, the problem is open. For problems 6 and 7 (irrationality of $\sqrt{3}$ and investigation of regular polygons with integer vertices), the only « accessible » proof which is known uses the induction axiom of the « infinite descent ». For the other problems, the reader can easily convince himself that these tools frequently lead to nice proofs and shortcuts.

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