Learning the experimental approach by a discrete mathematics problem

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In this paper, we discuss the learning of the experimental approach in mathematics by using discrete mathematics problems.

Generally, the experimental approach is associated with physics or biology, sciences that are called "experimental sciences". Therefore, mathematics are not free from experiments. Indeed, when we draw a geometrical figure to try to answer a geometrical problem, we do an experiment. It is also the case when we calculate some terms of a sequence to have an idea of its evolution. When we do mathematics, we can do experiments. Today, we have more and more experimental instruments, we have calculators and computers with a lot of software that can do lot of mathematical work (Maple, Scilab, Mathematica, Xcas...). Our point of view is that experiment is at the heart of a more general process, the experimental approach. The experimental approach is a way that we can use to discover, it is not only experiment, it is also conjecturing, proving, questioning, defining...

In France, experimenting in mathematics made its appearance in mathematics classrooms with school curricula. Very soon, we will evaluate the experimental capacities of our students with a test of experimental practice at baccalaureate (high school diploma), as we already have in physics or biology. Experimenting in mathematics is entering French mathematical classrooms.

Our proposal to learn the experimental approach and so, experimenting, conjecturing, questioning, defining, proving... uses problem from discrete mathematics. More generally, we use what is known as a SiRC. A SiRC (Situation de Recherche en Classe) is a problem which is close to a research one. You can find a complete description of a SiRC in [GP03]. The SiRC situation, which we propose here, is a problem from discrete mathematics. It is given with an "easy understandable question" : the question only uses common language terms. The situation is given with some material, with which students can experiment. We think these situations could develop capacities used in an experimental approach.

Here, the problem "the game of obstruction" that we propose is foregoing by a didactic study in which we use Brousseau's theory of situations, the notion of research variable of Grenier and Payan. We also use a description of the experimental approach to ripen our studies. This didactic study allowed us to see, a priori, which learnings are in game in these situations and which experiments, conjectures, proofs students can make.

We experimented this situation with high school students. So, we also present the results of our experiments.

We made a study of the place of the "experimental approach" in French high school, by studying curricula and books. We present results of our analysis.

I) What is an experimental approach in mathematics ?

The experimental approach in experimental sciences :

In experimental sciences, a lot of work has been done about the experimental approach since the seventies. We are going to recall briefly some results on it. It started with the OHERIC model of the experimental approach. This model emerged from Claude Bernard's book "*Introduction à la médecine expérimentale"* and the initials mean : Observation, Hypothesis, Experiment, Result, Interpretation and Conclusion. Today, this model is criticized because observation is not, generally, the starting point of an experimental approach. Indeed, an experimental approach is generally motivated by a question.

Moreover, this model does not acknowledge the knowledge that is necessary to deduce a result from an observation. The last point criticized is the linearity of the model, where the stages follow one after the other without feedbacks.

So, Giordan characterized the essential elements of an experimental approach as : question, hypothesis and arguing. Between these elements, there are multiple interactions and feedback.

We have recalled, briefly, some work about the experimental approach for experimental sciences. We will use it to try to characterize the experimental approach in mathematics.

The experimental approach in mathematics :

Question :

First of all, as in experimental sciences, the starting point of an experimental approach in mathematics is a question. We enter an experimental approach to try to answer a question. However in an experimental approach, the initial question is not the only question. New questions can be asked that will lead to new problems. The fact that in an experimental approach we are led to ask new questions is important because it allows pupils to ask themselves their own questions and try to answer them. This is common for a mathematician but it can be assumed that it is not the case for a pupil.

Observation/Experimentation :

In this part, we will see the difference between observation and experimentation and their roles for answering a mathematical question.

Observation can be defined as noticing and supervising a phenomenon. Experimentation as the whole checking means and procedures to check an hypothesis or a theory.

The major difference between observation and experimentation is that observation is a passive act while experimentation is something that we create by making choices. We can illustrate this with the question : Can an even number superior to 2 be written as the sum of two prime numbers ? (this question is the Goldblach conjecture). To answer this question we can try to carry out an experiment on the first terms 2, 4, 6, 8, 10 and observe that is it true for them. After that, we can continue with 12, 14 etc. However, we can also choose to do our experiment on the numbers of the form $2*3^n$ or on all the power of 2. Experimentation, is making choices. Here the choices are on the numbers, but more generally it is a set of choices. When we use a computer or a calculator we first have to choose the software with which we want to do our experimentation. It is an important part of our experiment because the results that we get will be different if, for example, we use a formal arithmetic software instead of a numerical one. So, the observations that we will make will probably be different according to the software we choose. We have also to set the software, when we want to observe a curve, following the zoom and where it is placed, we can observe different things. We can say that observation is a part of the experimentation and that experimentation is a set of choices that we make to answer a question.

Moreover, experimentation can lead us to make conjectures, enunciate some hypotheses. It is a way to generate facts. It can also be useful to check a conjecture or a hypothesis. We can refer to the thesis of Dahan for more information about different types of experimentation [Dah05].

We can also say, that the observations that we make depend on our knowledge. Here, if we do not know any prime numbers it can be very difficult to make "good" observations. Without knowledge, making "good" observations can be impossible.

Hypothesis/Conjecture :

Giordan use the term hypothesis for experimental sciences but in mathematics we have also the term conjecture. What is the difference between a conjecture and an hypothesis ? Hypothesis is a proposition that we enunciate without opinion. We do not affirm or contradict it, whereas a conjecture is a proposition that we enunciate as true but without proof. Hypothesis and conjecture play a great role in mathematical discovery and in an experimental mathematical approach because, as Polya said in [Pol54] :

You have to guess a mathematical theorem, before you prove it.

A conjecture or an hypothesis is a proposal that replies to a question. It can be a reply to the initial question or to a subquestion. When we try by using experimentation to answer the question : "Can an even number superior to 2 be write as the sum of two prime number ?" at some point, when we will observe that many even numbers superior to 2 can be written as the sum of two prime numbers without any counter-example, we can be convinced that the answer is yes. We can then say that yes is our conjecture.

To conclude this part, we can also say, that if we have a conjecture and a counter-example, we must not forget all the conjecture. Sometimes, it is enough to slightly modify the conjecture to make it still plausible, as we can see in [Pol54], when Polya shows how he found Euler's formula for convex polyhedron. In the beginning, he thought that the formula was true for all polyhedrons and then he found a counter-example. So, his conjecture was false. Therefore, his experimentation shows that for a lot of polyhedron the formula is true and these polyhedrons have a common property which the counter example did not have : convexity. So he modified his conjecture : the formula is true for all convex polyhedrons. With this example, it appears that there is a link between a conjecture and an experimentation. A conjecture can be deduced from experimental results. But there is also another link : a conjecture can generate experiments with a validated role. It is the case when an experiment is made with the aim of testing a conjecture or a hypothesis.

Arguing/Proving :

In the characterization of the essential elements of an experimental approach for empirical sciences, Giordan talks about arguing. Arguing is also present in an experimental approach for mathematics. Indeed, when we conjecture something, it is because we have arguments which allows us to think that our proposition is true. However in mathematics, we have something that goes “farther” than arguing : proving. Today, for the question : Can an even number superior to 2 be written as the sum of two prime numbers ?, we know that it is true for all even number higher than 2 and lower than $3*10^{17}$. By using a computer, we have more than a billion cases where the property is true without any counter-example, but we can not say that the property is true until we have a proof. It is the major distinction between mathematics and the empirical sciences. In the experimental sciences, an experimental proof can be enough. : this is not the case in mathematics.

A question we can ask is : what is the link between experimenting, arguing and proving ? We do not have an answer. It is one of our current research topics.

If you wish to have more information about the link between arguing and proving you can refer to the article of Tanguay [Tan06].

Characterization of an experimental approach :

Now, we have all the elements to provide one characterization of an experimental approach :

For us, an experimental approach in mathematics is a set of actions and feed-backs between : Questions, Experimentations, Hypothesis, Conjectures, Proofs.

This characterization is certainly not unique but we think that the five elements given here are essential.

II) <u>The experimental approach in French high school :</u>

Our studies only concern a equivalent of tenth and eleventh grade in France.

School curricula :

We are setting forth results of our research into the school curricula (2001) of a French “seconde” and “première scientifique” classes (tenth and eleventh grade).

Tenth grade : The term experimental approach is not present but it seems that the mathematical activity of searching and asking questions is important.

The term experiment is present. This term is exclusively associated in the curricula with the use of computer software in the arithmetical, geometrical and statistical field.

Eleventh grade : Here, it is the school curricula of the “première scientifique” which is the object of our studies. A “première scientifique” class is the more scientific class equivalent to an eleventh grade in the French education system.

Here, the experiment is at the “heart” of a mathematical approach. The mathematical approach appears to be essential in all scientific courses for the curricula. There are four elements which are emblematic for a mathematical approach : observation, abstraction, experimentation and proving. It also says that these elements have dialectic links between them. So, this tallies with our characterization of an experimental approach.

Moreover, there are some aims of an experiment : find counter examples, understand how to solve the particular case of a problem and generalize it, make conjectures.

It appears that the curricula gives the mathematical approach a great place among notions that students must learn.

Handbook :

Now, we give the results of our research on tenth and eleventh grade French handbook *"Déclic maths seconde"*[MCG$^+$01] and *"Déclic maths première S"* [BBF$^+$01]. In these books, we tried to analyze exercises which are able to bring experiments into play. We made this choice because understanding what the manual's point of view on experiment is, is necessary to understand its position with regard to the experimental approach.

We drew the following conclusion :

- Only a few exercises an experimental approach bring into play.
- Experiment in exercises is not always motivated by a question.
- The experimental approach is linear : stages follow one after the other without possible feed-backs. There is no interaction between the proof and the experimentation.
- Sometimes, the stages of experimenting and conjecturing are useless because students can prove the conjecture without knowing it, only by following the question of the proof stage. Handbooks do not take into account the fact that a student can conjecture a false result.
- Exercises only put students in a role of observer and do not allow them to be an experimenter. Students do not have the choice to do or not to do an experiment and when they "do" an experiment, it is the experiment requested by the book.

Conclusion : It seems in the two handbooks that the experimental approach (as we characterize it) is present in a very small quantity. This matches the "seconde" maths syllabus but not the "première scientifique" one, where it is written that students must learn the mathematical approach, which has experimentation at its "heart".

In the last part of this article, we present a SiRC [GP03], the game of obstruction, which we think is able to put students in a "true" experimental approach.

III) <u>The game of obstruction :</u>

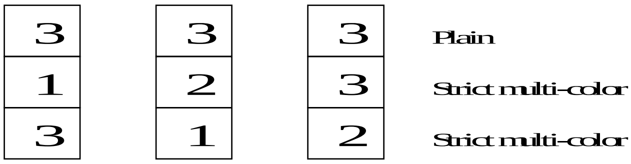
This situation was suggested by Sylvain Gravier.

In order to present the problem we will need some useful definitions. A *(n, c)-card game* (or for short *card game*) is a set of cards having *n* lines, each of which contains a color in *{1, ..., c}*.

1	3	3	2
2	3	2	2
1	3	1	3

A (3, 3)-game.

Given a (n, c)-card game, the color of the ith line of a card C will be denoted by C_i. A *bad line* in a set of 3 cards C, C' and C" is a line i for which either ($C_i = C'_i = C''_i$) or ($C_i \neq C'_i \neq C''_i$ and $C_i \neq C''_i$). An *obstruction* is a set of 3 cards such that all lines are bad.



An obstruction.

Now the problem can be stated as follows:

Given two integers n and c, find the largest (n, c)-card game which does not contain an obstruction. *(P1)*

First observe that:

Lemma 1 : *one may consider a card game with all distinct cards.*

Proof. Indeed, given an obstruction-free card game of cardinality m with all distinct cards by duplicating each card, we obtain an obstruction free card game of cardinality 2m. Conversely, there are no 3 copies of the same card in an obstruction-free card game. □

According to Lemma 1, we will now consider only card games with all distinct cards. The cardinality of a largest (n, c)-card game with no duplicated cards will be denoted by *max(n, c)*.

1. Mathematical analysis

It is worth noticing that (P1) is still an open problem so before trying to solve it one may study a weaker version:

(P2) How can we build a set without obstruction ?

(P2) problem suggests determining an efficient method (algorithm) to check if a given set of cards contains an obstruction. We will denote this problem by (P3).
Another way of simplification will be to fix *n* and/or *c*. To work on optimization problems, we need to consider the following problem:

(P4) How can an upper bound be found?

(P2) and (P4) split (P1) into the two aspects of an optimization problem: lower and upper bounds.

Unfortunately, since (P1) is still an open problem, we do not have yet a general strategy to solve (P4) efficiently. Mainly, a strategy (SP4) to answer (P4) is based on *enumerating all possible obstruction-free card games*. For a low value of n, an easy enumerating argument shows:

Theorem 2 [Gir07]
For any integer $c \geq 2$, we have
$Max(1, c) = 2$ and $Max(2, c) = 4$. □

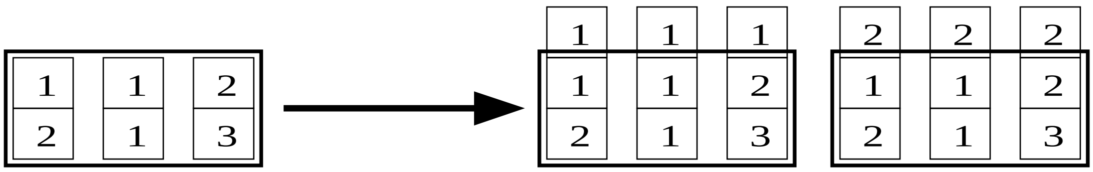
We will present some strategies to solve our problems. First, concerning (P3), a "naïve" way would be to check all sets of 3 cards among a given card game. Nevertheless this strategy fails when the number of cards m is large since it requires $O(m^3)$ cases to be explored. Therefore, we propose a strategy based on the structure of the given card game. For i in {1, …, c}, the *i-block* of a card game G is the subset $C^1, \ldots, C^t$ of G such that $C^1_1 = \ldots = C^t_1 = i$.

(SP3) First check that each block does not contain an obstruction (you can apply this strategy recursively). Secondly, search obstructions that have at most one card per block.

In general, this strategy is no more efficient than the “naïve” way. Nevertheless, it appears that for large obstruction free card game G, the colors are recursively and equitably distributed on each block, therefore (SP3) checks in $O(Log_c(m)^3)$ steps that G has no obstruction.

Another interest for using (SP3) is that it allows first results on max(n, c) to be obtained. Indeed, consider an obstruction-free (n, c)-card game, then each block is at most max(n-1, c) in size. Therefore max(n, c) $\leq$ c.max(n-1, c), which gives an answer to (P4).

Moreover, from an obstruction free (n-1, c)-card game G of cardinality t, one can build an obstruction free (n, c)-card game of cardinality 2t. Indeed, for i = 1, 2, consider the obstruction free (n, c)-card games G_i obtained from G by adding a line to each card and assigning color i to this new line. The set $G' = G_1 \cup G_2$ is an obstruction-free (n, c)-card game of cardinality 2t, which gives an answer to (P2).



obstruction-free (2,3)-card game — obstruction-free (3,3)-card game

These two remarks lead to:

Theorem 3
Given integers n and c $\geq$ 2, we have that 2.max(n-1, c) $\leq$ max(n, c) $\leq$ c.max(n-1, c). ☐

Observe that for c = 2, we get:

Corollary 4
Given an integer n, we have that max (n, 2) = 2^n. ☐

Nevertheless, when c $\geq$ 3, one can find obstruction-free card game of larger cardinality than 2.max(n-1, c). To find such obstruction-free card game one can apply “greedy” strategies:

(S1P2) Start from an obstruction free card game G (it can be empty) and add a card C such that $G \cup C$ is still obstruction-free until there is no such card.

Or equivalently:

(S2P2) Start from a card game G and while there is an obstruction in G, remove a card from this obstruction.

Observe that these two strategies give Corollary 4 since there is no obstruction in a (n, 2)-card game. In general, an obstruction-free maximal card game G is built (i.e. for every card $C \cup G$, $G \cup C$ contains an obstruction). It is worth noticing that (SP3) produces also obstruction-free maximal card game G, but this requires additional argument. If we choose an appropriate order for eliminating cards one can find an optimum of (P1) using (S1P2) or (S2P2). Of course, finding such an order remains an open problem. Nevertheless, when n is ‘large’, one may use a suitable order which ensures that we consider all possible cards ; for instance the **lexicographic ordering**. Unfortunately, even when n = 3, the lexicographic ordering gives a maximal obstruction free (3, 3)-card game of cardinality 8. However, by applying (S1P2) or (S2P2) with other orderings, one can find an obstruction free (3, 3)-

card game of cardinality 9 (> 2.max(2, 3)). Similarly, one can exhibit an obstruction free (4, 3)-card game of cardinality 20.

Moreover, by applying a (SP4) strategy one can prove:

Theorem 5 [Gir07]
Max(3, 3) = 9 and Max(4,3) = 20. □

2. Didactical analysis

In (P1), it appears that to define a card game we need two variables:

- n, number of lines,
- c, number of colors.

We decided do use these variables as *research variables*. A *research variable* is a didactic variable associated with the triple (question, conjecture, proof). A research variable specifies the interest of a question, its opening to new questions, the expansion of research strategy and the possibility of transforming the problem. For a more detailed definition we refer to [GP03].

It should be noted that there is a more general problem than (P1) where the number of cards involved in an obstruction could be variable. We decided to propose only (P1) where the size of obstruction is only a didactic variable (we did not let students use this variable). We chose a size of 3 because for 1 or 2, the situation is very easy. It becomes complex from 3.

Through mathematical analysis one can determine the following knowledge involved in solving (P1):

- The definition of an obstruction requires the understanding of logic quantifiers "**for all**" and "**there exists**" and of connectors "**either … or**".
- (S1P2) and (S2P2) suggest using an **algorithmic** approach to solving (P2) using **eliminating ordering** (for example **lexicographic ordering**). Moreover, since these strategies build a maximal obstruction-free card game, one can discuss **local /global maximum**. Therefore, these strategies will produce solutions which could be **conjectured** as optimal.
- (SP3) proposes a **structural analysis** of a card game in order to solve (P3). It allows a card game to be **modeled** which can be reinvested to (partially) solve (P2) and (P4) as shown in proof of Theorem 3. Observe that (SP3) applied on (P2) gives an **inductive construction** of obstruction-free (n, c)-card game based on two copies of an obstruction-free (n-1, c)-card game.
- (SP4) is a **enumerating** approach for solving (P4). To reduce the number of cases to be considered it will be convenient to use **variables** for the enumerating.
- The distinction between problems (P2) and (P4) is related to **lower and upper bounds** on an optimization problem (P1) which is closely related to **necessary and sufficient conditions**.
- Solving (P1) with c = 2 (Corollary 4) provides all possible 2^n cards in a card game on n lines to be counted.

We will define the conditions of experimentation to facilitate the access to **mathematics experiment**. So we will present (P1) as a material game.

3. Description of experiments

Two experiments were carried out, one with a "seconde" (tenth grade) class, E1, and another with a "première scientifique" (eleventh grade) class, E2. Pupils worked in groups of 3-4. In each class, we let them search for 2 hours. The E1 experiment was carried out before the E2 one. We filmed one group on each experiment.

In the two experiments, we presented the problem orally with examples on the black board.

In E1, the only material we gave was plain circles of 4 different colors and we left to the pupils the choice of n. Whereas in E2 we also gave plain circles of 4 different colors but we added to that n-line stands with n = 1, 2, 3 and 4.

But in both experiments the problem is posed generally as (P1), we did not ask students only to use the number of colors or the number of lines that is given materially.

We have limited the number of colors to 4. Since an obstruction uses 3 cards, if we use fewer than 3 colors then 3 copies of a same card will form the only obstruction. In that case the problem does not lead to problems (P3) and (P4).

Likewise the situation becomes more complex with 3 colors. We proposed 4 colors in order to allow the following questions: is there a link between a 3-color game and a 4-color game? Or do we have the same result for a 3-colors game and a 4-colors game? Answering these questions may arise to Theorem 3.

Moreover, using 4 colors leaves also more choice for experiments. We limited ourselves to 4 because if we suppose that pupils use all the given colors, it will be more difficult for the pupils to obtain results from experiments involving more than 4 colors. With more sequences, we should try to give more colors. Then note that is it "easier" to solve the problem with fewer colors.

In E2, we added card stands going from 1 line to 4 lines without saying that they had to play only with these values.

Nevertheless, we assumed that pupils experiment only with the given size of card game. With a up to 3-line card game the observations are easier to make. We propose also a 4-line card game to permit the generalization of results obtained with 2 or 3 lines and approach (SP3) strategy.

4. <u>Results of experiments</u>

Common to both Experiments

The first difficulties came from the definition of an obstruction. As we mentioned, it involves logical quantifiers and connectors *"for all lines there exists either ... or ..."*. In the beginning, there was confusion with quantifiers such as *"there exists a line which is either ... or ..."* and with connectors *"all lines are identical"*.

We took some time (about 10 minutes) to ensure that pupils understood (P1) well.

When this first difficulty passed, pupils started to manipulate. Quickly they saw that it was necessary to find some 'routine' to check if a card game contains an obstruction. Here they approach problem (P3) through **experiments**.

At this stage, to avoid doing the same check several times, some pupil started to write their solutions. The **representation** used by pupils are close to the material game (they used colors or symbols). Some groups, with no coloring pencils, decided to use the first letters of the color name (W for white). They did not need more specific representation since they used at most 4 colors and did not try to generalize with more colors. Here we see a consequence of the didactic contract: they did not use the research variable 'number of color' outside the range of given value.

Even if the experiments lead to problem (P3), we observed that strategy (SP3) involving blocks did not appear after a sequence of 2 hours. So to solve (P3) they used a "naïve" strategy but it was difficult for them to certify that they had checked all cases. Sometimes they proposed a card game as solution for which we exhibited an obstruction contradicting their assertion.

Nevertheless, strategy (S1P2) occurred in all groups and provided some solutions. For two colors, at some step of (S1P2), when pupils tried to add a new card, they observed that it already belongs to their card game. So they **conjectured** that (S1P2) gives an optimal solution for two colors, which approach (P4). In order to prove their conjecture, they gave a stronger one that is *"an optimal solution contains all the cards"*. Here they wanted to **count** the total number of distinct cards when n = 2, 3 and 4. One group proved that this number corresponds to the cardinalities of card games obtained by (S1P2). For the other groups, we had to assist them to solve their conjecture using a counting argument.

Still with (S1P2), they found a solution with 4 cards on 2 lines and 3 colors. They conjectured that it was optimal. Unfortunately, even if (P4) is asked since they claimed their results as conjecture instead of 'theorems' they did not see which kind of arguments permit (P4) to be solved.

With 3 lines, the best score is an obstruction-free card game of cardinality 8 and they conjectured its optimality.

So the distinction between (P2) (Sufficient Condition) and (P4) (Necessary Condition) is approached.

Moreover, they made hypotheses or conjectures that they checked with experiments like "*this card game is maximal", "by using this strategy, we build an obstruction free card game"* or *"with only 2 colors on each card, there are no obstructions"*, which allowed them to find counter-examples.

Differences between Experiments E1 and E2

E1 : The discussion on duplicating cards appeared only at the end of the first hour. After discussion, they conjectured and proved Lemma 1.

During the presentation of the problem, we gave an example on 3 lines. Consequently, pupils almost always experimented with 3 lines. It is why for E2 we decided to provide cards with 2, 3 and 4 lines as basis, without colors.

One pupil claimed that if you start with a 3-line obstruction-free card game, then add an arbitrary number of lines, it will still give an obstruction-free card game of same cardinality. Here we are close to (SP3). Unfortunately, since this pupil thought that (P1) with n = 3 was the main problem, he did not go far with this idea.

A group used another strategy based on geometrical schemes, but this strategy built card games with obstructions.

E2 : After the presentation of the problem, one pupil asked if we required that all cards be distinct. The following discussion provided Lemma 1.

According to the material given, the groups, here, accepted to work with 1, 2, 3 and 4 lines even if the example given at the presentation was again on 3 lines. They proved Theorem 2 when $n = 1$ and they conjectured for $n = 2$ and $c \leq 4$.

At the end of this sequence, we exhibited an obstruction-free card game of cardinality 9 as a counter example to their conjecture "Max(3, 3) = 8".

One group generalized the result with 2 colors for a card game with n lines.

Another group used strategy (S2P2). They did not get a new result. Nevertheless, this strategy forced them to represent all possible cards. So they decided to use symbols instead of colors.

Concluding remarks on Experiments

First it appears that we need to take some time to explain (P1) especially what an obstruction is. This preliminary allows logical quantifier and connector to be understood.

Even if (P3) was identified, it seems to be difficult to obtain useful strategy like (SP3). The 'naïve' strategy becomes rapidly unfeasible (by hand) when n and c grow. Consequently, there were some group which did not obtain results on 3 lines.

Solving (P1) with 2 colors allows for discussion on counting argument to prove Corollary 4 and to distinguish between (P2) and (P4). It is not surprising that no strategy was proposed to solve (P4).

One session of two hours is probably not sufficient to allow pupils of this level to find (SP3) or (SP4) strategies. We tested this situation on a longer time (18 sequences during one year). In this context, strategies (SP3) and (SP4) were developed and their corresponding results were obtained.

It appears that the use of material during experiments E1 and E2 led pupils to carry out their **own experiments** in mathematics.

Moreover, according to didactical analysis, (P1) allows **logical quantifier and connector** to be understood. Strategy (S1P2) and (S2P2) led pupils to define **algorithms**. The **counter example** concerning the **conjecture** "Max(3, 3) = 8" allowed **local and global maximum** to be discussed. The (partial) proof of Corollary 4 introduces a basic **counting argument**. The distinction between (P2) and (P4) introduced distinction between **Sufficient and Necessary conditions**.

Nevertheless, the concept of **variable** useful in a **enumerating strategy** like (SP4) was not discussed. Similarly no good **eliminating ordering** was proposed by the pupils ; they remained in a 'naïve' strategy.

Conclusion :

We saw that exercises proposed by books do not put students in an experimenter role, they are only observers. We think the obstruction situation game put students in an experimenter role and let them be active. Even if, in a 2-hours session, it did not allow them to work on all the knowledge we hoped, all the same they had worked on the experimental approach by asking themselves questions and carrying out experiments, conjectures and proofs.

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