ICME11-July 6-13, 2008- Topic Study Group 15

Teaching and learning of Discrete Mathematics - Indian scenario

Ambat Vijayakumar, Department of Mathematics
Cochin University of Science and Technology, Cochin- 682 022 INDIA
Email : vijay@cusat.ac.in ; ambatvijay@rediffmail.com

In this presentation, we shall focus on the following aspects,

- Discrete Mathematics - Old and New perspectives.
- How Discrete Mathematics is different as a mathematical discipline.
- The state of the art of Discrete Mathematics in India in its various stages of learning - ancient to modern.

Introduction

In this section we shall briefly mention the earlier thoughts which lead to the origin of discrete mathematics.

Let us, first, very briefly look into the interaction between mathematics and other fields of knowledge. In most of the cases, this interaction has been two way. The obvious one is the application of mathematics to other areas and the less obvious one is that these domains stimulate the development of new mathematics. The traditional example of this two way interaction is between mathematics and physics. It is well known that problems in physics were the main motivation for the growth of calculus. The notions of 'continuity' and 'differentiability' were the mathematical foundations.

However, to study the interaction among a different type of applications [6], as in social sciences , ecology, planning, chemistry, computer science etc. it was found that continuous mathematics was less capable of effectively explaining and solving the problems. The understanding and the solution of such problems required a totally different approach . This motivated the origin of a new kind of mathematics - **discrete mathematics**, which broadly consists of the topics such as graph theory, combinatorics, lattice theory, formal languages, automata , designs etc.
In this presentation we shall confine this term mainly to graph theory and combinatorics. These subjects were not found in any stage of mathematical education as a full fledged discipline until early seventies.

Discrete Mathematics. Old and New perspectives

The remarks by Roberts, F.S. [6] , " I have chosen to emphasize the "discrete" types of mathematics , because I feel that they are the relevant mathematics for many social, biological and environmental problems. " emphasizes the significant role of discrete mathematics and the need for its inclusion in a discussion on mathematical education.

A discrete mathematical model , in particular a graph model, can be fitted to describe any phenomenon which involves 'relations'. The origin of such thoughts dates back to the eighteenth century when the renowned Swiss mathematician Leonhard Euler presented the solution of the now popular ,'Konigsberg bridge problem' during 1735. Since then the subject has grown in its rich theory and varied applications [1, 6].

A very significant turn of events is observed recently, in graph modeling problems due to the varied nature of its applications. Apart from the classical and traditional applications mentioned above , it has entered the arena of internets , world wide webs, neural networks, metabolic networks, networks of citations between research papers etc. A surprising commonness is found in many of these types of networks. A new kind of study is emerging in the field of networks(graphs) due to the enormity of its order (number of vertices). Consequently the focus has shifted from the analysis of small graphs and the properties of individual vertices in such graphs to large – scale statistical properties of graphs. Also, actual drawing of networks with a million or a billion vertices is a problem of great difficulty even with modern tools. Newman , M.E.J. [4] has discussed in detail these issues and the modern trend of research with the help of some new terms such as small world phenomenon , power law of distributions , scale free networks etc. Thus, discrete mathematics is now widening its scope and applicability.

Information and Communication Technology (ICT) based methods of teaching are slowly entering many of the schools in India and else where, as a supplementary to the traditional 'chalk and board methods'. Consequently, mathematics has become more 'visual ', which has helped the students as well as teachers to understand mathematical concepts more clearly. This is already in use for some topics in traditional areas such as geometry, arithmetic etc. Attempts may be made to have such packages for discrete mathematics.

How discrete mathematics is different as a mathematical discipline

Background of the students

Numbers and geometrical shapes could possibly be the two concepts that the children are first exposed to when they enter schools. Though they may not have any concept of quantity, they will be able to, recite the numbers from one to ten or twenty initially , do some basic arithmetic operations, recite multiplication tables etc. later and draw some geometrical shapes such as triangles.

In the classes from 5-10 , the focus is more on Euclidean geometry, vector methods in geometry , trigonometry, arithmetic, finding the least common multiple and the greatest common divisor of integers, linear equations etc. and calculus (both differential and integral), matrices and various techniques of solving linear equations , differential equations etc. in classes 11 and 12. Students then join colleges for degree courses in arts or science, engineering or medicine .

At the end of the school studies, children who are at the prime period of their youth (hence difficult to effect any change in their mind set) and most of their teachers have the following firm conviction that ,

- There is NO mathematics without the notions of continuity and differentiability .

- Mathematics has no applications (even Alfred Nobel firmly believed so!)
- There are no good career options in mathematics.

It is at this point that the significance of discrete mathematics becomes prominent. All such apprehensions could be easily disposed off by citing examples from discrete mathematics.

Some methods of proof

Let us now look into the various methods of proof used at the school level.

Principle of Mathematical Induction : The word 'induction' means the generalization from particular cases . To prove statements involving positive integers, this is a well suited principle. In schools this method of proof is used (in class XI) to find a formula for the sum of positive integers etc., certain inequalities involving integers and divisibility problems .

The method of 'reductio ad absurdum' (method of proof by contradiction) : This is a traditional method of proof since the time of Euclid. But in school curriculum this appears only in the proof of irrationality of square root of 2.

Proof techniques in Graph theory

There is a feeling among the students and teachers that those mentioned above are the only methods of proof. The topics and techniques of discrete mathematics help us in molding a totally different mind set in them. Since, most of the proofs in graph theory are of 'constructive ' type, this is accepted with a different taste. We need to do things differently to attract good students to study mathematics seriously.

From puzzles to problems

I strongly feel that to make the teaching of any subject effective and to make the understanding of the subject stronger it has to be taught with a historical flavor. Further , various notions and themes have to be very carefully interwoven to make it attractive. In a class this can be done by drawing diagrams to convince the students how certain theorems and theories are central and how they are inter connected. Such lectures leave a long standing impact on the minds of the students. Most of the concepts in graph theory originated from simple looking puzzles. The Konigsberg bridge problem and the related diagram tracing puzzles (leading to the Eulerian graphs and unicursal graphs), puzzles on chess boards (domination theory) , seating problems, knight's tour problem and icosian games (Hamiltonian type problems), six people at a party problem (Ramsey theory) , storage problem and four color problem (chromatic theory of graphs) are a few to mention. I have discussed these types of puzzles in very many different forums of different age groups with varying level of rigor . It is observed that the audience is well motivated and inspired. Starting with such puzzles, we can slowly move on to more challenging mathematical ideas so that the transition of thought from puzzles to problems is smooth. As the level of maturity and understanding of the students reach a certain point we can convince them the need for a rigorous theory and proof. To my knowledge and experience, this transition can be clearly shown and confidently taught only in discrete mathematics.

As a case, the various stages in the development of graph theory with a historical perspective can be taught at the appropriate level with the help of some interesting and inspiring books like [1]. After discussing the bridge problem and the Euler's polyhedral formulae one can introduce the diagram tracing puzzle: Can one draw a diagram without lifting a pen and without re tracing it , that is , in one continuous stroke. If not , what is the minimum number of lifting required ?
This is a typical situation where students can be highly participative inside the class , which is now seldom found in schools (this is an important point that we stress in the meetings of teachers to make the students confident in the topics). After giving some diagrams where this can be done very easily, we can give some in which it cannot be done in one stroke. The rigorous theory behind such observations need not be explained at the school level.
Another innocent looking puzzle is that of coloring regions which was initially posed by Francis Guthrie in 1852 when he was a fifth or sixth grade student, which later on became quite popular as the ' four color problem '. This could be mentioned as a case how simple looking problems are actually very challenging. Such stories and historical exposition could break the monotony in teaching. In [8] , I have explained in detail the importance and unsuccessful attempts to solve the four color conjecture with historical anecdotes , which is written in a regional Indian language (Malayalam), which received much popular attention and appreciation .

Thus we have seen how discrete mathematics is different in its applicability, methods of proof and approach of the themes.

Research and Teaching of Discrete Mathematics - Indian scenario

India is a country with a great scientific and mathematical heritage since the Vedic times (B C. 5000) and which still continues to be in the forefront of many disciplines. With the decline of the Roman empire the center of mathematical research began to shift to India[7]. India had a rich tradition of 'combinatorial thoughts' also since the works of the ancient scholar **Pingala** (3rd century B.C). In his '**Chandas- Sutra'** he considers the method of finding the number of combinations obtainable by taking one, two etc. letters out of a given number of letters. He mentions a term, **'Meru Prastara'** which is the same as the modern **Pascal's triangle. Mahavira (650 A.D)** also had some contributions to the theory of combinations in his magnum opus **'Ganithasara Samgraha'** [5]**.**

In [2], the authors mention " It is strange that there is almost no material relevant to combinatorics in the literature of the classical Western civilization. All the evidence points to the fact that the originators of the subject came from the East. The main stimulus came from the Hindus". It is also known that the modern concepts of **permutations and combinations** – formulas for the number of permutations of an n- set and the number of k- subsets of an n – set , date back to **Bhaskara** (1114 -1185 AD) and **Brahmagupta (**6th century) respectively. The drawings that the Indian women, especially in Tamilnadu (the part of India where the legendary mathematical mystic Srinivasa Ramanujan was born) make using rice powder in the front yard of their houses as a sign of welcome and prosperity called 'Kolams' are nothing but drawings of Graphs.

The first exposition of graph theoretic terminologies (strongly regular graphs) in India was in the late fifties due to the works (at the Indian Statistical Institute (ISI) , Kolkatta) of R.C.Bose (1901-1987) and his colleagues , as a consequence of the distinct techniques of applying finite fields to solve problems in agricultural experiments. This could be termed as the first systematic approach of the usage of algebraic techniques in combinatorics in India in modern times, during late 1930s. He had a novel approach to solve problems in Latin squares, association schemes and combinatorial design theory.

The disproof of the famous conjecture (1862) of Leonhard Euler on the 'mutually orthogonal Latin squares ' by Indian mathematicians R.C.Bose, S.S.Shrikhande and T.Parker in 1959 was a major breakthrough [3]. This ushered the study of ' combinatorial designs' in India. The following extract from [3] regarding this achievement is in support of the need for further consolidation of teaching and research in discrete mathematics.
" In recognition of the significance of the then recent disproof of a famous conjecture concerning latin squares made by Euler in 1782, an editorial in the New York Times of April 27, 1959 stated that , it would be a serious mistake to suppose that modern mathematics is far from real life. Actually there has never been a time in history when mathematics was so widely applied in so many different fields for so many vital purposes as is true now". What was true in 1959 holds stronger now.

As a consequence of all these, many aspects of discrete mathematics are taught at the under graduate and the post graduate levels since late 1960s in colleges and universities in India . Since 2005, mathematical modeling is in the syllabi of higher secondary courses (standards 11 and 12) as prescribed by the National Council for Educational Research and Training (NCERT) - an apex body of Government of India on school education. The various stages of mathematical modeling and its cyclical nature are mentioned with examples. At present the teaching of mathematics in schools in India is based on the National Curriculum Framework (2005)[9] which envisages a close interaction among various levels of mathematics teaching. The NCF recommends that the students' life at school must be linked to their life out side the school.

The concept of 'mathematics laboratory' has recently entered the school curriculum.
A document on 'Mathematics laboratory in Schools- towards joyful learning' was brought out by the Central Board of Secondary Education (CBSE) and some guidelines have also been provided [10]. This document is primarily aimed at creating an awareness among schools as to how the introduction of mathematics laboratory will help in enhancing the teaching- learning process in the subject from the very beginning of school education.

Some of the ways this can contribute to the teaching- learning process are

- It helps the students to build interest and confidence in learning the subject by providing greater scope for individual participation.
- It provides scope for greater involvement of both mind and hand which facilitates cognition.
- It enables the teacher to demonstrate, explain and reinforce abstract mathematical ideas by using concrete objects, models, charts, posters etc.

Many concepts in graph theory can be effectively taught with the help of mathematical models and by encouraging them to ' see and believe'. In fact the same model can be used in explaining very easy as well as most difficult examples. The level of easiness and difficulty of some problems- which can converted in terms of existence of a necessary and sufficient condition or non existence of such conditions can be effectively demonstrated. Also my approach of introducing a theme in three stages - the motivation, theory and then applications (MTA) is also found to be well received.

In my university , we have a ' Centre for Science in Society' which attracts many school students and teachers. This centre has many scientific exhibits, posters on science, models etc. To some extent we are thus fulfilling our social commitment as a teacher. We also have now an **Academy of Discrete Mathematics and Applications (ADMA)** which is a professional body of discrete mathematicians which organizes 'Graph theory day ' for the past four years.

The Department of Science and Technology (DST) , Government of India has also come up in a big way to support activities related to discrete mathematics. Recently , **n- CARDMATH** (**n**ational **C**entre for **A**dvanced **R**esearch in **D**iscrete **Math**ematics), was established by DST which funds and coordinates the research programmes in Discrete Mathematics from different parts of India.

To conclude, this presentation will show case the status of teaching, learning and research in discrete mathematics in India .

References

1. N.L.Biggs, E.K.Lloyd , R.J.Wilson , Graph theory 1736-1936, Oxford University Press (1976).
2. N.L.Biggs, E.K.Lloyd , R.J.Wilson , The History of Combinatorics, In Hand book of Combinatorics, Eds.R.Graham.et.al, Elsevier, 2163-2195, (1995).
3 Laywine, C.F, Mullen, G.L. Discrete Mathematics using Latin squares, Wiley (1998).
4. M.E.J. Newman, The Structure and Function of Complex Networks, SIAM Review, 45, 167-256, (2001).
5. S.Parameswaran , The Golden Age of Indian Mathematics, Swadeshi Science Movement (1998), Kerala, India.
6. F.S.Roberts, Discrete mathematical models with applications to social, biological and environmental problems, Prentice Hall (1976).
7. D.J.Struik, A Concise History of Mathematics, Dover (1967).
8. A. Vijayakumar, Ganitha sasthrathile chathurvarna prasnam (the four colour problem in mathematics) , Mathrubhumi Weekly, January, 2006.
9. National Curriculum Framework 2005- Position paper , National Focus Group on Teaching of Mathematics, NCERT (2006).
10. Guidelines for Mathematics Laboratory in Schools , CBSE (2007).