Doing Mathematics – authentically and discrete.
A perspective for teacher training

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School has to qualify students to use mathematical knowledge and competences in a flexible way in different situations. For that they have to understand fundamental concepts and techniques of mathematics. They have to be able to use standard-procedures to solve problems in well-known situations as well as to use flexible different strategies to solve non-routine problems. Learning the basics lies in the past, learning critically thinking, solving problems and making decisions using mathematical reasoning is one of the requirements of today's world. Therefore teachers are necessary, who be capable to engage their students in building up these competences. But doing this – what seems like a banality – is only possible, if they get such competences by themselves and had been fully award of such learning processes.

A short view on the current research status is more likely to show deflating datas: Only 7% of the teachers fall back on educational theories by making an educational decision, while 81,7% orientate themselves by their own workexperience (Terhart et. al 1994, 196). It became even obvious, that teachers frequently don´t know by themselves, why they´ve acted in a special way. Explications, which are given afterwards, don´t turn out to be executive for acting. This indicates, that many fields of professional abilities cannot be explicated. With this exists less reason for the assumption, that scientific knowledge takes influence into the practice of acting in lessons and might explain, that the structure elements of scientific knowledge don´t look alike with the professional acting.

If the scientific formation of theory neither helps to be executive for acting nor as an instrument for reflecting processes, how is it possible to defend such a phase in universities as a main part of the teacher training? The reference of the scientific basis for the development of professional competences can´t be enough by laying down the described empirical findings. Radtke and others (1996) combine, that both fields of knowledge first stay unconnected in the individual cognition and that they will be connected later on in the sequel of their professional practice. But is it useful to learn ahead, connected with the hope, that this kind of knowledge will be useful for future acting? Neither in a motivation-psychologically way nor in a learning-theoretically perspective this could be a wise training concept.

An integrative approach like a connection of both training-phases is first priority. The fields of knowledge of theory and practice have to be connected from the first acting in lessons and first pedagogical reflection. A study from Nölle (2002) underpins this demand. Her empirical study shows, that courses of studies with a marked theory-practice-integration improve professional competence. With interviews she questioned graduates of a course of study with conventional, theoretical orientation (conventional study with two school-relevant practical trainings) and graduates of a course of study with practice-integration (for example periodical test and reflection of didactical constructions in several lesson-situations together with teachers in practice) about their imaginations of a lesson. It turned out, that as well novices needed categories to structure their lesson-situation regarding to significant critera. The episodic elements, which are taken from the practice, take effect as a specific function. They establish anchors for the categories and enable graduates to carry out the transfer of theoretical knowledge in forms of acting in lessons. In this case the reference to practice proves to be as a factor to connect theoretical knowledge and professional ability. It can be point out, that own experiences of lessons in schools and universities could be modified and doubt through the union of episodic anchors in casuistically lesson-situations and the theoretical reflection.

Unfortunately, the fusion of theory and practice as only one phase of the teacher training couldn´t define as an immediate achievable aim. However thinking in small steps, it might help already to shorten „the long way between knowledge and ability“ (Mandl/ Gerstenmaier 2000) before enter into profession by putting a strong practice reference in the conventional teacher studies (see Bohnsack 2000, 56). As far as that goes continue to exist a need for action to reform the first part of the teacher training, so that „the reference of the study for the later job description“ (Terhart 2000, 83) will be visible. Unfortunately, suggestions for a comprehensive reform cannot be given here, but in view of the structure in method and content opportunities for innovations will be given.

In lectures of academic teacher training usually the *well-known subjects* of school-mathematics have been instructed from a higher point of view to a deeper understanding. In worst cases the students only reproduce standard definitions and standard procedures. The good case, that fundamental ideas will come through from a higher point of view, will be rarely seen. These changes of position are mostly like a stroke of luck.

The introduction in a *new theme* may offer students the chance to investigate a new part of mathematics in an unbiased and self regulating way. If this field furthermore offers interesting and stirring questions, which are easy to approach, and which treatment is able to strengthen the character of mathematics as well as bringing the described competences within the student’s experience, these future teachers will walk along a promising way to understand the mathematics and their peculiarities from within.

The discrete mathematics offers such an „untouched“ field for future teachers: “Discrete mathematics offers a new start for students. For the student who has been unsuccessful with mathematics, it offers the possibility for success. For the talented student who has lost interest in mathematics, it offers the possibility of challenge. Discrete mathematics provides an opportunity to focus on how mathematics is taught, on giving teachers new ways of looking at mathematics and new ways of making it accessible to their students. From this perspective, teaching discrete mathematics in the schools is not an end in itself, but a tool for reforming mathematics education” (Rosenstein 1997).

The special significance of discrete mathematics could be described with the following aspects:

- The tasks are easy to approach, have a stimulative nature, induce processes of active problem-solving and offers approaches on every level
- Model and reality are closed-fitting, so that processes of modeling could be conducted on different levels
- The mathematical concepts grow up from handling with real objects
- Fathoming different kinds of possible concepts and models is followed by the formal construction of the concepts

To spread out these qualities, the tasks of discrete mathematics have to be presented in shapes of learning environments.

A learning environment

- has to offer self-regulating learning (in sense of a social-constructivistic learning theory (Boekaerts 1999, Cobb & Yackel 1996, Ernest 1997, Gergen 1995)
- includes easy approachable tasks (Hußmann 2002, 2003a)
- offers genetically concept formation (Hußmann 2002, 2004)
- are structured with fundamental ideas, central conceptions, different representations
- stimulates mathematical processes like reasoning, problem-solving and modeling

For example in the USA and Hungry discrete mathematics is part of the school curricula. Reports of experience about the role in the academic and school education like the following aren't rarity:

"Participants reported changes in their classrooms, in their students, and in themselves. Their successes taught us that discrete mathematics was not just another piece of the curriculum. Many participants reported success with a variety of students at a variety of levels, demonstrated a new enthusiasm for teaching in new ways." (Rosenstein, 1997)

This final speech for discrete mathematics explains, that subject-specific didactic and subject-specific science have to be closely in teacher training. Of course there could be found good reasons for considering other disciplines – analog to discrete mathematics. A legitimation for such curricula decisions should be a reason imaginations about aims for teacher training.

Discrete mathematics from visibility of standards for teacher training

In Germany there are two communities for mathematics - DMV (german community for mathematician) and the GDM (community for didactics of mathematics) -, which are formulating standards for teacher training. In a joint statement (DMV 2001) they formulate the following demand: teachers should get knowledge and competences, which will go beyond the level and extent of the content in the school curricula. Merely than they are capable to react on questions and ideas of the students and answer broader questions in an adequate and competent way. But this demand should be seen in the light of a holistic image of mathematics. This includes competences like modeling, problem-solving and argueing, which are focused in the new standards. Furthermore the processes of concept formation have to give in student's hands. Hence part of the subject-specific systematic is the familiarity with the epistemic processes of mathematics.

Up from the beginning of the academical training - in the subject-specific training as well as in the didactical training - there should be provide opportunities to the students to do mathematics in an explorative way and to discover mathematical structure on their own paths. They have to experience and reflect their own concept formation processes. In this manner they do research on their own learning processes, theoretically and practically. The impact of academical teacher training could be seen in school practice, if self practiced learning is model for teaching in school. In this way the episodic potential of the learning situation could be visible in later teaching situation. Learning teaching is reflecting learning.

From the subject-specific perspective this kind of teaching could be described as elementary mathematics from a higher point of view. This implies the following principles:

- The difference between doing mathematics in life and doing mathematics in school is only gradually, not principally
- Students are not only consumers of ready-to-use mathematics, they create mathematics in every singular learning situation by their own
- Teachers have to experience doing mathematics and learning in the described manner and reflect it theoretically

Learning discrete mathematics – a concept for teacher trainings

Regarding to these aspects there is one thesis – not empirical proofed yet - which includes and allowed for them all: the themes of discrete mathematics should be seen as an essential and imperative link between subject-specific didactic and subject-specific science.

The following presented concept try to fulfill this claim by understanding mathematics in general not as a finished product but as an arising and a created knowledge. In this sense, the students open up their minds for the development in an individual and active way. Therefore the development of the mathematical subject is linked with the potential of the development of the individual, which means that the knowledge in the described manner will be connect with the ability.

Shaping of a learning environment

The crucial influential factors of a concept like this, which integrate theory and practice, are the following:

1. essentially the tasks (intentional problems),
2. self-regulated learning processes,
3. dialogical communication structure,
4. research-journals to write down the results of the learning processes.

These aspects will be discussed first in general, before illustrating them with an example of a discrete mathematic lecture.

Starting point and base of this kind of teaching and learning environment are complex and meaningful problems, so called *intentional problems.* These problems should open the approach to a mathematical field, for example the theory of shortest paths, and should give the opportunity to walk alone a long way through the mathematical field. In the first approach the concepts and conceptions to solve the problem will be developed by the learners on their own. After this he or she reflects about the process of problem-solving and create an adequate mathematical theory, with definitions, theorems and proofs. The problems are structured in a special way, so that the learner has to create the appropriate concepts necessarily. In this manner a mathematical theory occurs, in which the context of problem situation is connected with the models and the mathematical objects, which are created by the learner to interpret the problem situation.

The usual lesson sequence, with the phases introduction, formulation, retention, and practice in the common teaching style is irrelevant - the organization of these phases is the learners business. They work the problem out together in different groups, secure their results through documentation in so called *research journals*, and create examples and the necessary practice material either alone or with the help of a teacher.

The creation of examples and exercises are oriented in the first step through the context of the intentional problems and the individual's prior knowledge. But this kind of productive practice also aims towards structural aspects of mathematics. The structure of the examples and exercises build the size and the content of the net of concepts that must be built in order to solve the problem successfully. Therefore in the research journals the students set out in writing first their results and processes of problem-solving and second the appropriate mathematical theory with a language related to the specific field, but at the same time individual. Similarly to the classical lecture-script this journal contains definitions, theorems, proofs and examples, but all in the dress of an individual perspective.

Concerning the underlying learning theory, this conceptual approach is based on a social-constructive paradigm; hence, the assumption that *learning is a self-regulated activity* which cannot be controlled from the outside but which can be encouraged at best. As a result, the pupils have to receive diverse opportunities to activate own experiences, to study with the help of their interests, to use their own speech, in order to decide the topic and much more. Of course, each institutional general condition is an obstacle, but it is still possible to give responsibility to the learners. Goals that are prescribed from the outside are "only" documentation, reflection and presentation of the problem and the mathematical theory.

Since the degree of self-adjustment is different from person to person, individual support for the learners is required, which is organized through the build-up of a *dialogical communication structure* between teachers and pupils, and which is, at the same time, characteristic for an open lesson organization. Central aspects of this dialogical principle are the confidence in the learner's efficiency, and the concentration on the ideas and products made by the students. With this in place the right balance between the learner's construction and the teacher's instruction is found, and becomes the key for successful learning and teaching.

For this purpose it´s necessary that cognitive, social and affective factors of influence will be considered:

- A learner don´t built his knowledge passivly but activly: „the function of the cognition is adaptivly and serve the organisation of the world of expierence" (von Glasersfeld 1998, 49) (comparative to Piaget 1950, Maturana & Varela 1991).
- Subjective theories of the learners have to be viable within the social realitiy, which mean that they have to be prove in processes of communication within varios social systems as functioning, suitable and recognized (comparative Gergen 1995, Cobb & Yackel 1996).

To enable students to develop themselves in these four dimensions, a learning environment is required, in which learners are participate in essential decisions, if, what, how, when and why they learn (Weinert 1982).

Experience Combinatorical Optimation - Learning Environment to discrete mathematics

To examplify the described concept, parts of a lecture named ‚discrete mathematics - experience combinatorical optimation‘ will be presented now. The central aspect is focused on the individual concept formation. The questions of research are:

- Which factors of the learning environment foster respectively hinder the learning processes of the students?
- Are the students capable to create mathematical concepts and are these concepts comprehensible with the consolidated concepts?

This concept is empirically tested and modificated in several steps. It had been tested in a period of time between 2003 and 2007 with different learning groups in a quantity of 20 to 160 students. The presented intentional problems are published in Hußmann/Lutz-Westphal (2007). The methods of this empirical research follow the interpretative paradigm. The results are based on interpreting the research journals followed by an interview study.

The lecture is separated in eight chapters:

1. Shortest path problems
2. Minimal spanning trees
3. Chinese postman problem
4. Travelling-Salesman Problem
5. 4-Colour-Problem
6. Combinatorial games
7. Matchings on bipartite graphs
8. Networking problems

In the beginning of every chapter the students get between one and four intentional problems. To give an example, the first of three intentional problems in the chapter ‘Shortest path problems’ is the following one:

Problem 1. Take a traffic-map of your favorite town (i.e. figure 1) and choose two stations. Now imagine you are at station 1 (i.e. Richard-Wagner-Platz) and you want to go to station 2 (i.e.). Search the shortest path between the two stations.

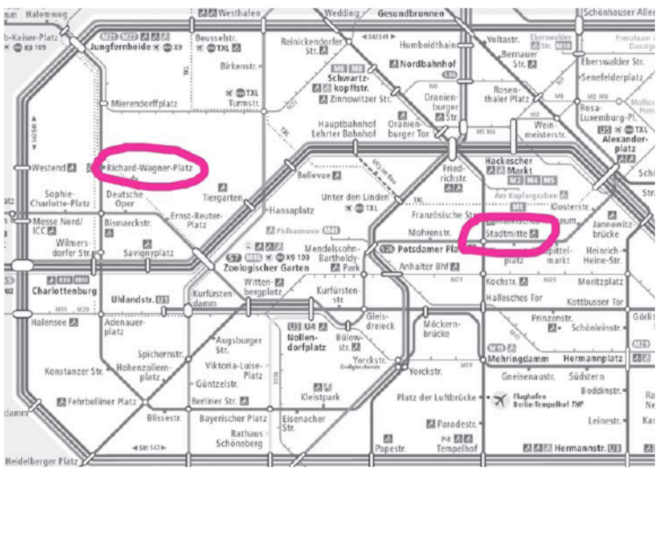

Figure 1.

- How do you work, describe your procedure?
- What are the central steps to find the shortest path between two stations?
- Which aspects do you have to consider finding a shortest path?
- Develop an algorithm for finding a route between any two stations.
- Test your algorithm with different examples, easy ones and 'crazy' ones.
- Compare your algorithm with other ones and develop a possible better one.

The other two intentional problems deals with finding shortest path going by car, by bike or by feet and exploring shortest paths sending data-packages. Aims of this chapter are exploring the model of weighted graphs, developing first ideas of greedy algorithm and formulating the dijkstra algorithm and the breadth-first search, incl. showing their correctness.

In the first approach to the problems the students work in groups, explore and formulate algorithm. After this they present their solutions in one or two other groups, discuss and modify them. In the final step of the students work they formulate the mathematical concepts they used to solve the problems in their own words.

After this the research journals are collected, corrected and giving back to the students. By means of the individual constructs and the documented research-processes of the students the mathematical theory will be presented as an overall picture by the docent.

Some Results

In generally, the results of the empirical study depend on the quantity of the learning group. Orientated at the questions of research the following extracts try to give an example of the results:

1. Which factors of the learning environment foster respectively hinder the learning processes of the students?

> The research-journals demonstrate, that the students at the basis of single examples and proceedings developed all central concepts which they could transfer and use in a varity of situations. The journals pointed out, that the students were able to identify and to correct mistakes. In the processes of formalisation of their developed mathematics there have been presented difficulties in bigger, but not in smaller groups. This observation could be related with the teaching competence of tutors. A better part of the students review the problems as very stimulating and estimate the possibility to learn on their own ways as promotion of their self-confidence. It could be shown, that the students are motivated without any guiding and instructing by the teacher. They build up various cognitive competences, especially modeling and problem-solving.

2. Are the students capable to create mathematical concepts and are these concepts comprehensible with the consolidated *concepts?*

> The study of the journals and the interviews shows, that the learners are capable to explore the mathematical concepts by their own. They generalize the concrete solution to appropriate concepts and connect them to a mathematical theory. It was amazing, that all central algorithm and the central concepts of graphs, (spanning) trees and matchings are developed in an excellent way, very near to the consolidated concepts. In the field of formulating proofs the students show some

difficulties: The reasoning based often on the specific situation and the representation had been seen as the mathematical object, for example two or three examples – so they argue – are sufficient to prove a mathematical theorem.

In a follow-up examination a half year and one year later it could be shown, that some parts of the theoretical concepts couldn't make explicit, but a better part of the students could apply the concepts in various situations.

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