Strategies for Solving Word Problems on Speed: A Comparative Study between Chinese and Singapore Students

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Abstract: The study was conducted to investigate strategies that Chinese and Singapore students used for solving word problems on speed. A test comprising 14 word problems on speed was administered to 1002 Singapore and 1070 Chinese students from Primary 6 to Secondary 2. A two-way country×grade ANOVA revealed that there was a significant interaction. The strategy analysis indicated that the Chinese students performed better than the Singapore students because they used algebraic strategies more frequently than the Singapore students. The Singapore students performed better than the Chinese students on one problem because the Primary-6 Singapore students successfully used model drawing and unitary methods. The Singapore students were found to use model drawing, unitary, guess-and-check, etc. more frequently than the Chinese students. However, the success rates of the use of these strategies were lower than those of the algebraic strategies that were used more frequently by the Chinese students. The study has some implications for the teaching and learning of speed, algebra, and problem solving in schools.
Keywords: Word problems, Speed, Cross-national comparison, Problem-solving strategies

Introduction

Problem solving has been included in Singapore syllabi since 1992 (Ministry of Education (MOE) (Singapore), 1990a, 1990b). It is explicitly postulated that "the primary aim of the mathematics programme is to enable pupils to develop their ability in mathematical problem solving" (MOE (Singapore), 2001a, 2001b, p.5). About twelve heuristics such as "act it out", "draw a model/diagram" etc. are suggested in mathematics syllabi (MOE (Singapore), 2001a, 2001b). However, in China, the teaching of mathematics put more emphases on the learning of basic knowledge and the training of basic skills, also known as "Two Basics" (Zhang, Li, & Tang, 2004). Problem solving is taught after the teaching of basic mathematical concepts and techniques to illustrate their applications to the real world. The new curriculum standard (BNUP (China), 2001) has included problem solving as one of the four aspects of mathematics teaching and learning. A comparative study between the two countries is meaningful for getting insight into what are the advantages and disadvantages of the different ways of teaching problem solving in schools.

Word problems on speed are selected because they are application problems of various mathematical concepts from primary to university levels. The mathematics of change and variation, and in particular, the study of motion, is a fundamental concept

that underlies elementary algebra and calculus instruction (Bowers & Nickerson, 2000). Speed has been studied as conceptual problems (Acredolo & Schmid, 1981; Piaget, 1970; Zhou, Peverly, Boehm, & Lin, 2000; Zhou, Peverly, & Lin, 2004) with children at the age of 5 to 12 years. However, very little work has been done with word problems on speed as those included in textbooks. Though several studies include rate problems as a specific model of multiplication and division (Bell, Fischbein, & Greer, 1984; Fischbein, Deri, Nello, & Marino, 1985; Greer, 1992), the word problems on speed included in these studies are only the simplest one among the 13 categories of motion (speed) problems Mayer (1981) identified. Mayer analyzed word problems on speed in secondary school mathematics textbooks, but Mayer did not investigate how students actually solve the problems and what difficulties they may have. This study seeks in part to fill these gaps. Another reason is that word problems on speed are both included in mathematics textbooks in China (Jiang, 1998a, 1998b; People's Education Press (PEP), 1992, 1993a, 1993b, 1994) and in Singapore (MOE (Singapore), 2000a, 2000b; Teh & Looi, 2002a, 2002b).

The comparative study was conducted to answer the research question "Being taught various problem solving heuristics, do Singapore students perform better than Chinese students in solving word problems on speed?"

A problem-solving strategy model (Table 1) including nine strategies was developed from a review of textbooks and syllabi in the two countries and the Concepts in Secondary Mathematics and Science (CSMS) project (Hart, 1981). Here, strategies refer to the methods or problem solving procedures that direct the search for a solution (Krulik & Rudnick, 1988). The problem-solving strategy model was used to identify strategies used by the students.

Table 1
The Problem-Solving Strategy Model

No.	Strategy category	Definition
1	Arithmetic method	It is used where the subject writes down a mathematical statement involving one or more operations on the numbers given in the problem (Fong & Hsui, 1999).
2	Algebraic method	It is used when one or more unknowns are chosen as variables and equation(s) is set up.
3	Model drawing method	It is used when the solution is suggested by or follows a model or a diagram (Kho, 1987).
4	Guess-and-check	It involves the following two steps: (a) Make a guess of a certain answer or the unknown in the problem based on an estimation; (b) Check if the constraints given in the question or implied from some of the question statements are satisfied. If all the constraints are satisfied, the guess is correct; the

		answer has been obtained or can be worked out. All processes will end at this point. If the constraints are not satisfied, the guess will be refined or adjusted, and another guess will be made, then another round of guess-and-check will begin.
5	Looking for a pattern	It involves the following three steps: (a) Several specific instances/special cases/particular examples of a problem are explored and listed; (b) A pattern (or a conjecture, a generalization, a hypothesis, or common property among those special cases) is determined by investigating the special cases explored in Step (a); and (c) A solution to the entire problem is found by applying the generalized result obtained in Step (b).
6	Unitary method	Unitary method involves finding the value equivalent to one unit of a quantity from an equating statement and obtaining the value equivalent to more units of the quantities using the value for one unit just found (Fong, 1999; Fong & Hsui, 1999; Yuen, 1995).
7	Proportion method	A proportion method is used when proportional properties (direct and inverse proportions) are used.
8	Logical reasoning	Logical reasoning strategy is used when some forms of "if-then" reasoning are used (van De Walle, 1993).
9	No strategy	It refers to the absence of a written response and where only pieces of information taken from the question are written down but without any continuing working (Fong & Hsui, 1999).

Method

1070 Chinese students (361 Primary 6, 354 Secondary 1, & 355 Secondary 2) and 1002 Singapore students (345 Primary 6, 315 Secondary 1, & 342 Secondary 2) participated in the study. The Chinese sample was from Wuhan City, China. A test was developed from an analysis of various types of word problems on speed (Jiang, 2005) and administered in intact classes. No calculators were allowed. Prior to the test, all the students have learned and completed the topic on speed.

Both quantitative and qualitative analyses were conducted. The responses were scored using a 0-1-2 scale. Two points were given to each correct answer or an incorrect answer where all the necessary steps are included but with only minor computational errors. One point was given to each answer that solved part of the

problem. A zero '0' point was given to answers that were completely wrong and to cases with no solution offered. The problem-solving strategy model was used to identify students' responses to 11 problems where workings were required.

Results

This section has three parts. The first part is about the performance comparison results. The second part is trying to explain the performance differences from analyzing the strategies the students used. The third part presents detailed results about the use of strategies for solving three problems.

Performance Comparison Results

Table 2 shows the mean scores and standard deviations of the Chinese and the Singapore students in the test. A two-way country×grade ANOVA revealed that there was a significant interaction ($F(2, 2066) = 22.88$, $p < .001$). Post hoc pairwise comparisons indicate that there were no statistically significant differences between the Chinese students at any two of the three grade levels. However, there were statistically significant differences between the Primary-6 and Secondary-1 Singapore students (Mean difference (MD) = 2.76, $p < 0.001$) and between the Primary-6 and Secondary-2 Singapore students (MD = 3.38, $p < 0.001$). There were no significant differences between the Secondary-1 and Secondary-2 Singapore students (MD = 0.62, $p = 0.37$). The post hoc pairwise comparisons also showed that there were statistically significant differences between the Chinese and the Singapore students at each of the three grade levels ($p < 0.001$). Therefore, the main difference existed between the Primary-6 and secondary Singapore students.

Table 2

Means and Standard Deviations of the Chinese and Singapore Students on the Test

Grade	China		Singapore		Mean Difference
	M	SD	M	SD	
Primary 6	21.95	5.41	19.52	4.60	2.43
Secondary 1	21.74	5.10	16.76	5.09	4.99
Secondary 2	22.20	5.05	16.13	5.72	6.07
Overall	21.96	5.19	17.49	5.36	4.47

Differences in the Use of Strategies

Comparisons in the use of strategies between different groups of students are made in terms of Strategy Percents (SP), which is the percent of a specific group of students using a specific category of strategy. Table 3 shows the SPs of the students for solving 11 problems.

Table 3

Strategy Percents (SP) of the Chinese and the Singapore Students in Solving Problems in the study

Strategy category	China	Singapore

	P-6 (n=361)	S-1 (n=354)	S-2 (n=355)	Total (n=1070)	P-6 (n=345)	S-1 (n=315)	S-2 (n=342)	Total (n=1002)
Arithmetic method	68.24	38.50	42.51	49.86	47.01	44.36	46.23	45.91
Algebraic method	15.66	50.13	48.14	37.84	0.18	0.26	4.89	1.81
Model drawing method	0.13	0	0.05	0.06	20.26	20.20	13.85	18.05
Unitary method	0	0	0	0	12.02	9.90	10.55	10.85
Guess-and-check	2.37	0.15	0.18	0.91	13.18	8.60	5.21	9.02
Logical reasoning	2.04	0.10	0.20	0.79	0.47	0.63	0.35	0.48
Proportion method	1.44	0.46	0.18	0.70	0.53	0.38	0.19	0.36
Looking for a pattern	0.05	0	0	0.02	0.34	0.23	0.13	0.24
No strategy	10.07	10.66	8.73	9.82	6.01	15.44	18.61	13.27

Note. P-6 = Primary 6; S-1 = Secondary 1; S-2 = Secondary 2.

[a]The numbers are the percentages (% is omitted) of the cases (No. of problems (11) × No. of students) where the group of students used the specific kind of strategies.

Data in Table 3 show that:

(1) The students from both countries used the arithmetic strategies in most of the cases.

(2) For the Chinese students, the second most frequently used strategy is algebraic strategies. For the Singapore students, the second most frequently used strategy is model drawing methods followed by unitary and guess-and-check methods.

(3) The use of other three kinds of strategies (i.e., logical reasoning, proportion method, and looking for a pattern) was very rare.

(4) The Singapore students had no strategy in more of the cases than their Chinese peers.

(5) For the samples from the same country, the trends were different. For the Chinese sample, the secondary students used algebraic strategies more frequently than the arithmetic strategies. Though the Secondary-2 Singapore students did use the algebraic strategies more frequently than the Primary-6 and Secondary-1 Singapore students, they used the algebraic strategies even much less frequently than the Primary-6 Chinese students. The secondary Singapore students still used the arithmetic, model drawing, unitary, and guess-and-check methods as the Primary-6 Singapore students.

Therefore, the strategy analyses generally show that (a) the Singapore students used a greater variety of strategies for solving word problems on speed; and (b) the Chinese students used algebraic strategies more frequently than the Singapore students, especially the secondary Singapore students.

Strategies for Solving Three Problems

As examples, this part will show the detailed results about the use of strategies for solving two problems (Appendix A). Problem 1 is a typical algebraic word problem like Cows and Chickens Problem in Kaur (1998). Problem 2 describes a round trip. Knowledge of inverse proportion could be used for solving it. Problem 3 involves

fractions to represent the relationships among the distances. This kind of problems were found in a popular workbook written by Fong (1998).

For solving individual problems, we do not only use SP for comparison, but also use Success Rate (SR). SR is the proportion of the specific group who could use the strategy to get the correct answers to a problem. It is used to measure how successfully the strategy is used by different group of students. If the Success Rate Difference (SRD) between different groups of students is more than 10%, it is taken as high. If the success rate of a strategy is lower than 30%, the strategy is taken as inappropriate for solving a specific problem because a majority (70%) of the students using the strategy could not reach the correct answer. Table 4 shows the SPs and SRs in the use of strategies for solving the three problems.

Table 4

Strategy Percents (SP) and Success Rates (SR) of Chinese and Singapore Students in Solving the Three Problems

Strategies		China				Singapore			
		P-6 (n=361)	S-1 (n=354)	S-2 (n=355)	Total (n=1070)	P-6 (n=345)	S-1 (n=315)	S-2 (n=342)	Total (n=1002)
Problem 1									
Arithmetic	SP	8.31	3.11	4.51	5.33	21.16	25.71	31.87	26.25
Method	SR	6.67	0	0	3.51	1.37	0	1.83	1.14
Algebraic	SP	46.54	82.20	80.85	69.72	—	0.32	13.16	4.59
Method	SR	92.26	91.75	89.20	90.88	—	100	80.00	80.43
Guess-and	SP	18.84	1.69	1.69	7.48	66.38	41.90	27.49	45.41
-check	SR	97.06	100	100	97.50	96.51	93.18	97.87	95.82
Logical	SP	12.19	0.56	1.12	4.67	3.48	5.08	3.22	3.89
Reasoning	SR	75.00	100	50	74.00	83.33	31.25	54.54	53.85
Model	SP	0.28	—	—	0.09	3.48	8.25	3.51	4.99
drawing	SR	100.00	—	—	100.00	25.00	30.77	16.67	26.00
Looking for	SP	—	—	—	—	0.29	—	—	0.10
a pattern	SR	—	—	—	—	100.00	—	—	100.00
No strategy		13.85	12.43	11.83	12.71	5.21	18.73	20.76	14.77
Problem 2									
Arithmetic	SP	30.75	22.88	21.97	25.33	54.20	46.35	43.27	48.00
method	SR	0	3.66	0	1.11	0	0.68	0	0.21
Algebraic	SP	31.86	53.39	62.82	49.25	—	0.32	11.70	4.09
method	SR	87.83	67.20	63.23	70.02	—	100.00	65.00	65.85
Guess-and	SP	2.77	—	—	0.93	21.16	11.11	6.43	12.97
-check	SR	80.00	—	—	80.00	90.41	74.29	72.73	83.08
Proportion	SP	15.79	5.08	1.97	7.66	5.22	3.81	1.46	3.49
method	SR	82.46	94.44	100.00	86.59	83.33	75.00	40.00	71.43
Model	SP	—	—	—	—	7.83	3.81	2.05	4.59

drawing method	SR	—	—	—	—	40.74	41.67	14.29	36.96
Unitary	SP	—	—	—	—	2.90	0.95	1.17	1.70
method	SR	—	—	—	—	40.00	100.00	100.00	64.70
Logical	SP	4.43	0.56	1.13	2.06	1.45	1.90	0.58	1.30
reasoning	SR	93.75	100	100	95.45	100	50.00	100	76.92
No strategy		14.40	17.80	12.11	14.77	7.25	31.75	33.33	23.85
Problem 3									
Arithmetic	SP	81.44	40.96	42.54	55.14	3.77	7.30	19.30	10.18
Method	SR	61.90	40.69	53.64	54.58	23.08	8.70	12.12	24.51
Algebraic	SP	8.86	49.44	48.73	35.51	—	—	0.88	0.30
Method	SR	34.38	49.14	54.91	50.53	—	—	66.67	66.67
Model	SP	0.28	—	0.28	0.19	52.75	51.75	38.01	47.41
drawing	SR	0.00	—	100.00	50.00	65.93	44.79	47.69	53.68
Unitary	SP	—	—	—	—	42.61	39.05	36.55	39.42
Method	SR	—	—	—	—	68.03	56.91	52.00	59.49
No strategy		9.42	9.60	8.45	9.16	0.87	1.90	5.26	2.69

Note. P-6 = Primary 6; S-1 = Secondary 1; S-2 = Secondary 2.

For Problm 1, we can have the following abservations:

(1) A higher percentage of the Singapore students used the inappropriate arithmetic strategies (SR < 5%) than that of the Chinese students (SPD = 21%). It is surprising to find that the higher the grade, the higher percentage of the Singapore students used this inappropriate strategies.

(2) A majority (70%) of the Chinese students used the effective algebraic strategies (SR > 80%). However, only about 5% of the Singapore students did so. The Chinese students used the algebraic strategies better than their Singapore peers (SRD > 10%).

(3) Up to 45% of the Singapore students used the guess-and-check strategies. However, only 7% of the Chinese students did so. The students from the two countries used the guess-and-check strategies equally successfully (SRD = 2%).

(4) About 5% of the Chinese students and 4% of the Singapore students used logical reasoning method. The Chinese students used the logical reasoning method more successfully than their Singapore peers (SRD = 20%).

(5) A higher percentage of the Singapore students used the inappropriate model drawing methods (SR < 30%) than the Chinese student (SPD = 5%).

(6) One Singapore student was found to use the “looking for a pattern” method to solve this problem.

(7) Similar percentages of the Singapore students and the Chinese students had no strategies (SPD = 2%). However, for the students at the three grade levels, the trends are different. For Primary-6 samples, a higher percentage of the Chinese students had no strategies than their Singapore peers (SPD = 9%). However, for secondary samples, higher percentages of the Singapore students had no strategies

than those of their Chinese peers (SPD = 6-9%).

For Problem 2, we can have the following observations:

(1) A higher percentage of the Singapore students used the inappropriate arithmetic method (SR < 2%) than that of the Chinese students (SPD = 23%). Most of them used irrelevant procedures like "(120+40)÷2 = 80", "120+40 = 160, 160×2 = 320", etc.

(2) A higher percentage of the Chinese students used the appropriate algebraic strategies (SR > 65%) than that of the Singapore students (SPD = 45%). The students from both countries could use the algebraic strategies equally well (SRD = 4%).

(3) A higher percentage of the Singapore students used the guess-and-check methods than that of the Chinese students (SPD = 12%). The Primary-6 Singapore students used this method more successfully than the secondary Singapore students.

(4) A higher percentage of the Chinese students used the proportion methods than that of the Singapore students (SPD = 5%). The Chinese students used the proportion methods more successfully than the Singapore students (SRD = 15%). For the Chinese students, the higher the grade, the more successfully they used proportion method. However, for the Singapore students, it was reversed.

(5) About 6% of the Singapore students used the appropriate model drawing (SR = 37%) and unitary methods (SR = 65%). However, no Chinese students were found to use these two kinds of methods.

(6) Very few students from the two countries used logical reasoning strategies (CN 2%; SG 1.3%) though their success rates are high.

(7) A higher percentage of the Singapore students had no strategies than that of their Chinese peers (SPD = 9%).

For Problem 3, we can have the following observations:

(1) A majority (55%) of the Chinese students used arithmetic strategies, especially for the Primary-6 Chinese students. However, only 10% of the Singapore students used this method. The Chinese students used this method more successfully than the Singapore students (SRD = 30%).

(2) About 36% of the Chinese students used algebraic strategies. Higher percentages of the secondary Chinese students used this method than that of the Primary-6 Chinese students (SPD > 40%). However, very few Singapore students were found to use algebraic method. The secondary Chinese students used this method more successfully than the Primary-6 Chinese students (SRD > 15%).

(3) About half of the Singapore students used model drawing method, but very few Chinese students did so. The success rate of this method of the Singapore students is not low (SR = 54%), especially for the Primary-6 students (SR = 66%). However, the secondary Singapore students used this method much less successfully than the Primary-6 Singapore students (SRD > 18%).

(4) Up to 40% of the Singapore students used unitary methods for solving this problem. The secondary Singapore students used this method much less

successfully than the Primary-6 Singapore students (SRD > 11%).

(5) A lower percentage of the Singapore students had no strategies than that of the Chinese students (SPD = 6%). This is also true for the samples at each of the three grade levels.

In conclusion, (a) the Singapore students used the inappropriate arithmetic strategies more frequently than their Chinese peers; (b) the Chinese students used algebraic strategies more frequently and more successfully than their Singapore peers; (c) the Singapore students used more varied strategies including guess-and-check, model drawing and unitary methods, etc. However, the secondary Singapore students could not use these strategies as successfully as the Primary-6 Singapore students.

Summary and Discussion

Cross-national studies provide us with an opportunity to ascertain the strength and weakness of educational systems (Robitaille & Travels, 1992), and consequently provide information about how to improve teaching and learning of mathematics (Cai, 2000, 2004; Robitaille & Travers, 1992). This comparative study has serveral implications for the teaching and learning of mathematics in schools.

First, the teaching of various heuristics in Singapore schools has equipped the Singapore students with more ways to tackle problems. This kind of practice has also made the Primary-6 Singapore students have no strategy in a lower percentage of the cases than their Chinese peers. The use of model drawing and unitary methods has converted the multiplication and division of fractions (Problem 3) to the multiplication and division of whole numbers. This kind of conversion might have removed the difficulties that the Chinese students experienced. This finding may explain the Singapore students' good performance in number and proportionality in the TIMSS studies. In the TIMSS studies, the secondary Singapore students performed the best in number and proportionality (Beaton, et al., 1996; Mullis, et al., 2000; Mullis, et al., 2004). Therefore, it is suggested to teach the Chinese students the problem-solving heuristics that are proved to be effective, such as guess-and-check, model drawing and unitary methods.

Second, the strategy analyses show that the secondary Singapore students do not use algebraic strategies as frequently as the Chinese students. Problem 1 is similar to the Cows and Chickens Problem in the study of Kaur (1998) and the Correct Answer Problem in the study of Loh (1991). The results obtained in the study are consistent with the studies of Kaur and Loh in that (a) few Secondary-2 Singapore students were found to use algebraic methods, (b) the Singapore students preferred to use guess-and-check methods and could use them efficiently, and (c) about one-third of them used irrelevant procedures. Consequently, effort needs to be made to help the secondary students recognize the strength of algebraic methods for solving word problems as well as to bridge the gap between primary and secondary mathematics (Fong, 1994).

Third, the secondary Singapore students used the arithmetic, model drawing, unitary, and guess-and-check methods less successfully than the Primary-6 Singapore

students. Therefore, effort needs to be made to help the secondary students consolidate what they have learnt in primary schools.

Appendix A

1. On Sunday, Judy went to see her grandma who lives 150 km apart. After cycling at an average speed of 15 km/h for a few hours, she got tired and took a lift from the passing truck. The truck's average travelling speed is 75 km/h. When she got to her grandma's house, she checked the time and knew that the trip took her 6 hours. Find the time she cycled?
2. Sunday morning, Rebecca and her parents went out to enjoy the natural scenery. On the way to the destination, they travelled at a slow speed of 40 km/h. On the way back, they drove at a faster speed of 120 km/h. When they came back home, they found that they had been out for 2 hours. Find the average speed for this round trip (Ignoring time around the destination).
3. Mike made a journey from City P to City Q. In the first half an hour, he covered $\frac{1}{7}$ of it. In the second half an hour he covered $\frac{1}{3}$ of the remaining journey. Finally he took another half an hour to finish the journey at a speed of 72 km/h. Calculate his average speed for the whole journey.

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