Exploiting next generation handheld technology: TI-Nspire as "Microworld Maker"

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Abstract: After two decades of incremental advances in the capabilities of graphing calculators, handheld technologies have recently made a leap into a new genre of educational tool – the "microworld maker." One such example is the TI-Nspire handheld device that allows for the creation of dynamic documents endowed with "hot links." The goal of a hot link is to achieve the optimum in visual proximity, immediacy, and transparency by providing two or more external representations linked together in such a way that the actions performed in one representation have virtually simultaneous discernible consequences in the others. Such hot links can provide uniquely powerful settings for exploration of connections, pattern searching, and inductive reasoning. That is, students are presented with environments where they can directly manipulate or take actions on mathematical objects and immediately see the mathematically meaningful visual consequences of those actions. We offer a variety of examples of such environments drawn from a range of mathematical areas and raise two issues of import for both teachers and developers: *mathematical fidelity* (faithfulness to the mathematical representations) and *cognitive fidelity* (faithfulness to the cognitive perceptions of the user).

Students' mathematical activity is the lens through which we can better understand the influence of technology on mathematics learning and teaching. Technological tools do not emit some special radiation that magically forges new understandings within the minds of those lucky enough to be nearby. Questions posed around technology's influence on learning and teaching mathematics must attend to an examination of *what changes technology can bring to students' mathematical activity*. Dick (2007) posed the following pedagogical "axiom" in a discussion aimed at the designers of technology tools for mathematics education:

> "Students learn mathematics by taking mathematical actions (e.g., transforming, representing, manipulating) on mathematical objects (e.g., symbolic expressions, graphs, geometrical figures, physical models), observing the mathematical consequences of those actions, and reflecting on their meanings. Students' reflections on mathematical consequences of mathematical actions on mathematical objects are the fuel for feeding the cycle of prediction–conjecture–testing that ultimately leads to proofs or refutations."

A measure of a technological tool's faithfulness to this pedagogical principle lies in how easily and clearly it is perceived to:

(a) facilitate the creation of mathematical objects,
(b) allow mathematical actions on those objects, and
(c) provide clear evidence of the consequences of those actions.

In this paper we make the case that the most recent generation of handheld devices have moved into a new arena – beyond what we will call *representational toolkits* toward what might best be described as *microworld makers*. The platform we will use to illustrate that claim is Texas Instruments latest offering, the TI-Nspire. We will discuss some issues of import for both teachers and developers that could help guide those who would exploit the new opportunities provided by these new devices.

Performance versus inquiry activity

Let us first make a distinction between two very broad categories of mathematical activity: *performance* and *inquiry*. By *performance activity*, we mean the recall or recording of information, the creation of mathematical objects, direct actions taken on mathematical objects, and procedures (sequences of actions). Examples of performance actions include geometric constructions, numerical computations, algebraic manipulations, transformations, measurements, translations between notation systems, graphing, diagramming, displaying, collecting, sorting, etc. By *inquiry activity*, we mean the mathematical sense-making that can only result from purposeful reflection on the part of the student: finding and describing patterns (inductive reasoning), conjecturing, generalizing, abstracting, connecting between representations, deducting, predicting, testing, relating, justifying, proving, and refuting.

Carrying out prescribed actions and procedures are mathematical performance activities that can be handed over to technology. Reflecting on the consequences of those actions and procedures, seeing patterns in results, describing connections between representations, making predictions, generalizing, making conjectures, testing hypotheses, justifying, explaining, defining, designing proofs and refutations– these are mathematical inquiry activities that must be built around cognitively demanding tasks posed to students. The role technology can play in inquiry activities depends on how it can be used to orchestrate environments where important sense-making questions can be asked of students.

Representational toolkits

A representational toolkit provides a collection of technological tools for working both within and between symbolic, graphic, and numeric representations. Examples of representational toolkits include computer algebra systems such as *Mathematica* or *Maple,* spreadsheet applications (since most include a range of graphing tools besides the numeric capabilities one associates with spreadsheets), and handheld graphing calculators. The widespread access to representational toolkits afforded by the availability of relatively inexpensive handheld graphing calculators provides a critical impetus to "rule of three" approaches to curriculum reform. The "rule of three" refers to an emphasis on cross-representational sense-making of the notions of function as analytic formulae, graphs of coordinate pairs, and tables of input-output values. Notable examples of these approaches were evident in the calculus reform movement in the United States in the late 1980's and 1990's (Dick & Edwards, 2007).

Mathematical activity with graphing calculators has tended to be dominated by performance tasks in the service of problem solving. In this context, the key decisions for the user are often managerial or metacognitive in nature: choosing one or more representational tools from the collection to address a given mathematical task. Inquiry activity with representational toolkits is certainly possible, but it requires thoughtful direction by a teacher (or activity author). A common theme to many inquiry activities with a representational toolkit is the examination of a series of results for patterns, often across representations. For example, many "parameter play" activities engage students in systematically manipulating the defining symbolic parameters of a family of functions such as $y = A\sin(Bx + C) + D$, observing the corresponding effects on the graphical representations. Recognizing, describing, and explaining precisely the graphical characteristics driven by each parameter are the goals of the inquiry in such an activity.

Technology active performance tasks

We describe a mathematical performance task as "technology active" if its successful completion requires the use of technology. Dick et al (2003) discuss two fundamentally different ways that a performance task might be technology active in the context of representational toolkits such as graphing calculators.

1) A task is technology active on the "back end" if the toolkit essentially extends the computational reach of the user. That is, the sheer number of computations or the complexity of computations required to carry out the performance of the task in the manner chosen by the user may require technological assistance. For example, if the solution to a calculus task leads to the evaluation of a definite integral that is impractical or perhaps impossible to calculate by paper-and-pencil antiderivative techniques, then the use of the numeric computational capabilities of a graphing calculator or CAS can provide the needed assistance. Note that the use of the technology here surfaces at the final computational stage of the solution, hence the descriptor "back end."

2) A task is technology active on the "front end" if the toolkit is used to make a switch in representations as an initial strategy. That is, the original representation presented in the task may not afford the same opportunities for solution as a different representaion. For example, given a function defined as a symbolic expression $f(x)$ and questions regarding the behavior of one of its antiderivatives, one might be unable to analytically determine a closed form expression (either by pencil-and-paper or even with the assistance of a CAS). However, by graphing $y = f(x)$, the user is afforded the opportunity to make visual interpretations of graphical behavior with direct implications for the behavior of the antiderivative, such as the changes in sign of $f(x)$ indicating relative extrema of its antiderivative. Here the use of the technology arises early in the solution, hence the descriptor "front end."

From representational toolkits to microworlds

A software *microworld* sets up a constrained environment where the mathematical rules of engagement are enforced by the software coding. Within such an environment the tools available to the user are primarily for use with the objects created by the system, and these objects are the virtual versions of the abstract objects in a mathematical system. For example, dynamic geometry systems such as *Cabri* or *Geometer's Sketchpad* provide tools for measurements and constructions that are specialized within a rather comprehensive microworld for a mathematical system (in this case, Euclidean geometry). Other examples of microworlds may focus on a much smaller set of objects with only highly constrained actions possible. For example, Java applets might provide virtual versions of concrete manipulatives such as base 10 blocks.

Just as with representational toolkits, both performance and inquiry activities may be pursued within a microworld. However, it seems clear that the actual design of a microworld may be explicitly driven by the kinds of inquiry activities that can be supported. In the case of representational toolkits, the managerial decisions are external – indeed, the choice of whether or not it is appropriate to use the technology at all may be the most significant. In a microworld, the managerial decisions are internal – the tasks themselves are posed within the technological environment, so its use is a given.

Graphing calculators brought the power of visualization to a handheld technology scene that had previously been dominated by numerical computation. The ability to easily produce graphical representations is the most salient feature of graphing calculators. Dynamic geometry packages such as Cabri and Geometer's Sketchpad took this a significant step further by allowing dynamic manipulation of virtual geometric objects in a constrained environment. For example, the direct manipulation of a polygon on screen can result in corresponding changes in the numerical measurements of some important characteristics such as the area and perimeter. If we accept the pedagogical axiom proposed in the introduction to this paper, then a strong case can be made for microworlds that provide opportunities for students to take such mathematically meaningful actions on visual models and to see the mathematically meaningful consequences of those actions.

The potential value of these action-consequence linkages in providing compelling learning experiences for students is greatly dependent on how directly accessible and visually obvious the results are. The *visual proximity* and temporal *immediacy* of the results of an action itself are attributes of the technology that can aid the student in making connections. In general, the closer in space and time the consequences are to the original actions as perceived by the user, the better. Heid (1997) uses the term *transparency* to refer to another construct related to the efficacy of technology: "The degree of transparency is the extent to which the technology being used highlights the mathematics that is being studied rather than obscures it." (Heid, 1997, page 6).

A new generation of handhelds: TI-Nspire

Over the last two decades, graphing calculators have undergone an incremental evolution, primarily in the sense of particular tool refinements and enhancements or in general improvements to the user interface. Some handheld devices have gone so far as to provide the user access to both a representational toolkit as well as dynamic geometry, such as TI's Voyage 200 or the Casio Classpad 300. However, in these instances, the approach is essentially multiple-purpose rather than integrated. Much like an audio device that allows the user to play either tapes or CD's, such handhelds simply combine the software onto one machine, allowing the user to choose between the toolkit or the dynamic geometry microworld.

Texas Instruments' latest offering, the TI-Nspire, might appear to simply be the next incremental advance beyond the Voyage 200. One finds the usual representational toolkit (in both CAS and non-CAS versions) as well as dynamic geometry on board. At first glance, the two biggest distinctions appear to be in terms of user interface and organization:

1) the TI-Nspire adopts a document model (similar to Word or Excel) where the user can save multi-page work sessions, and

2) the TI-Nspire employs a "Navpad" that plays the role of a mouse or trackpad, with corresponding analogies to the top-level navigation and control operations that users find on most personal computers (such as a screen-active pointer and global recognition of "control C" for copy and "control V" for paste).

This document model and computer-like naviagation of TI-Nspire can be viewed as a nice organizational feature (for example, with presentation potential similar to Powerpoint). However, by far the most significant advance is one that is not so immediately apparent: an integrated general dynamic linking authoring feature that can be "live" across all the TI-Nspire applications used in a single document. It is this feature that allows the user to define "hot links," a critically important feature of many microworlds.

Hot links

The notion of a *hot link* (Kaput, 1992) refers to a dynamic link whose intent is to maximize the opportunity for the learner to see a connection between a mathematical action and its related consequences. The goal of a hot link is to achieve the optimum in visual proximity, immediacy, and transparency by providing two or more external representations linked together in such a way that the actions performed in one representation have virtually simultaneous discernible consequences in the others. Such hot links can provide uniquely powerful settings for exploration of connections, pattern searching, and inductive reasoning. Hot links can provide opportunities for prediction in two directions (What would happen if I made this change? What change should I make to get a specific result?)

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For example, the TI-Nspire allows the user to engage in the parameter play described above while seeing the graphical effects occur immediately on the same screen. Dependent graphical constructions (such as a tangent line to the graph at a point) are also immediately updated upon changes in parameters. Conversely, the TI-Nspire also allows direct manipulation of the graphs of certain families of functions with immediately visible changes in the symbolic parameters. But what is distinctly new here is that either the numerical values of the parameters or measurements related to the geometry of the graph can be dynamically linked as drivers of values in spreadsheets, attributes of geometric objects, or parameters in other symbolic expressions.

This means that teachers (or authors) can create, save, and distribute documents endowed with their own defined hot links to illustrate important mathematical connections across and within representations. Thus, the TI-Nspire has the potential to provide a much wider range of dynamical manipulation possibilities, but the most compelling (and distinguishing) applications will be those involving visual mathematical models that the student can directly make visually discernible changes to. A TI-Nspire document equipped with a compelling hot link shares many of the characteristics of a software *microworld* - a learning environment in which a student can engage with mathematical objects having specific action-consequence rules under well-defined constraints. Indeed, TI-Nspire could be viewed as a “microworld maker.”

Examples

Here are some examples of simple but powerful microworlds that have been created on TI-Nspire:

1) *Dynamic percent illustrator* - a single TI-Nspire page with two live line segments (that is, the lengths of either can be changed by simply grabbing and dragging an endpoint). One segment represent the base whole (with any number and units that can be chosen by the user, such as “120 books”). The other segment represents the corresponding percentage of the whole. The labels for quantities and percentages are all immediately updated with any manipulation of either line segment.

2) *Number line variable evaluator* – a single TI-Nspire page with a number line and a live pointer to one or more specific numeric values on the line. One or more algebraic expressions dependent on the number line pointer value(s) are displayed. The value of both the variable pointer and the dependent expressions are all immediately updated.

3) *What's my rule?* - a single TI-Nspire screen showing a coordinate system and two points with coordinates of both points displayed. One point can be dynamically moved and the other point responds dynamically according to some mathematical rule defined by the author. Coordinates are updated immediately.

4) *Function manipulator* - a single TI-Nspire screen showing a slider bar for a one parameter family of functions of the user's choice (example: $y = b^x$ with parameter b controlled by the slider) and the graph of the function. Manipulation of the parameter slider is immediately reflected in the graph. Furthermore, any user defined dependent function (example: the derivative) will also be immediately and dynamically changed.

Emerging issues in the new era

Two constructs that may be useful in thinking about technology use in mathematics education are *mathematical fidelity* and *cognitive fidelity* (see Zbiek et al, 2007; Dick, 2007). The former refers to the mathematical faithfulness of a computation or representation (as judged by the math community) while the latter refers to a faithfulness of a machine's representational response to the individual's intended action on the mathematical object being represented.

For example, some violations of mathematical fidelity are nearly impossible to avoid – the machine representation of the graph of a continuous function as a discrete collection of pixels gives rise to "false asymptotes" on a graphing calculator screen. When instances of mathematical infidelity are unavoidable, then it is important for users to be aware of them and their predictable consequences. Interpretations or conjectures based on machine representations can then be qualified or adjusted accordingly.

On the other hand, cognitive fidelity has much to do with user interface concerns. For example, a student might perceive a polygon as shrinking when it is the underlying coordinate scaling that is changing instead. In other words, cognitive fidelity has to do with misperception of what is happening mathematically onscreen, even if that mathematical behavior is correct. Sometimes providing additional contextual clues (such as labeled hash marks on visible axes reflecting scaling change in the example given) can make such instances less likely to happen.

These issues were formerly those of special concern to technology developers. However, with devices such as TI-Nspire, teachers potentially become microworld developers themselves and should also pay heed to these concerns.

Finally, we conclude by emphasizing that even the most compelling hot linked microworld will not deliver results in student learning unless the tasks posed and questions asked by teachers put a premium on sense-making and reasoning. This latest generation of hand-held technology can provide new fuel for mathematical inquiry by way of allowing student both new opportunities to take mathematically meaningful actions and providing more direct access to their mathematically meaningful consequences. Teachers will play a critical role in taking full advantage of those new opportunities.

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