

## STUDENTS' NOTEBOOKS AS A SOURCE OF RESEARCH ON THE HISTORY OF MATHEMATICS EDUCATION

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The objective of this paper is to reflect on the first results of research that has been carried out by GHEMAT – Grupo de Pesquisa de História da Educação Matemática [Mathematic Education History Research Group] in Brazil, which considers students' notebooks as a source of research for the history of mathematics education. Material rarely used in historical investigations on mathematics education, students' notebooks prove to be very rich documents to analyze teachers' pedagogical practices.

For the theoretical-methodological discussion on notebook utilization, the work takes as examples material developed by students between 1930 and 1980 in Brazilian schools. This is the time during which education suffered the influence of movements intended to make school mathematics international. The first, from 1908, with the creation of CIEM-IMUK, had the mathematician Felix Klein as President; the second one, known as the Modern Math Movement, began in the late 50s.

How do we analyze students' notebooks in order to have signs of mathematics teacher's pedagogical practices? How do we analyze the historical changes that occurred in pedagogical practices through the students' notebooks? How were the international proposals felt in the classrooms? Such questions constitute a guiding element for the present communication and we seek to answer to all of them within the sphere of cultural history. Thus, the concept of culture is of essential importance.

A classic text by anthropologist Clifford Geertz ("A interpretação das culturas") presents a discussion on the multiplicity of meanings through which the concept of *culture* is approached. Thus, for his studies, Geertz delimited what constitutes the term, seeking support in the semiotic interpretation: "I understand culture as those meaning webs woven

by man as well as his analysis of it; therefore, not as an experimental science in search of laws, but as an interpretative science, in search of meaning” (1989, p.4).

On the other hand, with the purpose of using anthropological studies to better understand educational issues, the concept of *school culture* is built up. Also in relation to this concept, similar to what occurred with the inventory carried out by Geertz on culture, there are many meanings assigned to school culture. In this text, we opted to borrow the concept given by the historian Dominique Julia (“A cultura escolar como objeto histórico”): “A set of *rules* that defines knowledge to teach and procedures to implement, and a set of *practices* that enables the transmission of such knowledge and the incorporation of those behaviors” (2001, p. 10).

Thus, considering the expression *school culture* in a wide sense, far from intending to homogenize various cultural environments, it becomes clear, as the elements that move school routine are looked into, in its meaning web, which give a sense to the actions developed in it, in a historically situated manner. There seems to be no theoretical-methodological advantage to first build a network of elements that could be present in the different school institutions, dissolving the concept of school culture, in a multiplicity of terms, derived from the idea of the existence of *school cultures*. The concept of school culture, which becomes more substantial at every investigation, may guide the research’s view towards a necessary distance from instances that at first glance appear so much familiar to us and, at the same time, show to be so enigmatic for the comprehension of pedagogical practices developed in other school times.

The theoretical-methodological apparatus that uses the concept of school culture aims at a better comprehension of what happens inside the school institutions’ “black box”. It is about a trend that has increased in education history studies: to comprehend more closely the dynamics inherent to school day by day and the shared meanings of the actions developed in it in a certain historical time. It is worth saying: the intention is to interpret and historicize cultural practices. But, according to the historian Dominique Julia’s words, “the history of cultural practices is indeed the most difficult to rebuild because it does not leave any traces: Is what is evident in a given moment necessary to be said or written?” Such difficulty leads to historical source issues. Which elements can be erected as traces of

cultural practices developed within the school environment? Or by another: which sources should be given priority in the study of school cultures from other times?

The answers to such questions point to difficult to be found materials. Textbooks, student notebooks, teacher notebooks, tests and exams and much school documentation related to the class, but that quickly vanished when no longer used in the school routine.

On the other hand, having found traces of school practices, how do we proceed to write a cultural history on such practices?

The study of practices has already been theorized by historians such as Michel de Certeau and Roger Chartier. Such theoreticians provide a valuable theoretical instrument for the historical investigation of cultural practices. In particular, analysis categories created by these historians, like: *appropriation, representation, tactics and strategy* constitute the fundamental analytical arsenal for this work.

The study of school culture, specifically mathematics teachers' pedagogical practices shall have, in this study, the students' notebooks as a source of research. The most varied circumstances allowed GHEMAT, over several years, to compile a set of mathematics school notebooks covering the period between 1930 and 1980. They will be taken as traces that can reveal the dynamics occurred in school routines from other times. Times of influence of international movements for changes in school mathematics. How can school notebooks reveal changes in school culture?

The comprehension of the changes, alterations occurred in school culture is the subject matter of historical studies. Moreover, when this study is carried out, it broaches the understanding of school culture itself. As emphasized by Julia, the school culture "can not be studied without an accurate analysis of the conflicting or pacific relations it maintains, at each period of its history, with the set of cultures contemporary to it: religious culture, political culture or popular culture" (2001, p. 10).

Widening the different environments a little more with which school culture maintains relations, it will be possible to think of those established with academic culture. *Academic culture* is a term coined by Aparecida Duarte (2007), in her doctor's degree dissertation, in which she studied the dynamics of relations between mathematics and mathematics education, at the time of MMM. She points out the specificities of the norms and practices adopted in Mathematics higher education. And it seems to be relevant for the

study of the actions that gained ground during international movements towards the transformation of the mathematics curriculum. The academic culture will be the start point for the effort to promote a substantive change in the school mathematics. In times of what could be considered the first movement towards the school mathematics modernization, the Commission led by Klein sought the introduction of Differential and Integral Calculus in the secondary education. The integration of the mathematic branches of Arithmetic, Algebra and Geometry was one of our proposal's purposes. From the Modern Math Movement, new themes should constitute a subject of elementary education, among which: Logic, Set Theory, Algebra, Integral Calculus, Probabilities, Statistics and Approximate Numeric Calculus. How have such relations imposed on school culture been incorporated by it? What alterations did they bring to school culture? What do students' books say about it?

Students' books have been the subject of studies carried out by historians like Jean Hébrard. In his text "Por uma Bibliografia Material das Escritas Ordinárias – o espaço gráfico do caderno escolar – França, séculos XIX e XX", published in the Revista Brasileira de História da Educação, no.1, of 2001, Hébrard reveals that the school notebook is the school student's common instrument since the XVI century. And moreover, in the Jesuit schools' fundamental document, in the *Ratio studiorum*, the notebook is designated by the expression "white book". However, this author emphasizes that this presence of the notebook in schools is not followed up. In schools, a place where reading, writing, counting are studied, the notebook will be virtually absent by the XIX century. Then, it will be in France, in the first third of this century that the school notebook generalization will occur in the elementary school.

In his historical study about the school notebook's course, Hébrard also emphasizes that from 1960-70, the school notebook gave ground, to a great extent, to the index notebook. Such change, according to this historian's studies, brings significant alterations to the students' daily work in the classroom and should be studied in depth.

Another relevant study about school notebooks is the doctorate dissertation by Silvina Gvirtz (1996), which analyzes the school discourse through elementary school notebooks, in Argentina, between 1930 and 1970. The researcher considers that the notebook is structured as a result of three core axes: time, activities and discipline content

and, in this manner, she develops her analysis, seeking regularities. In a later moment, she observes the discontinuance in the use of notebooks along the period in order to elucidate the work carried out in the first moment.

In her conclusions, Gvirtz highlights the need to develop studies on school notebook contents with the purpose to better comprehend the so-called “taught curriculum”, particularly if the investigation is performed in a certain curricular discipline.

The first results to be presented in this paper include considerations on how the curricular proposals of two major international movements that sought to modernize school mathematics were taken to classrooms. Different from analyses that consider the innovative proposals as pedagogical failures, the study that takes notebooks as sources of research reveals the dynamics in the classroom, the tactics developed in the school day by day to start the strategies built to alter school mathematics in the early and middle 20<sup>th</sup> century. The notebooks substantively show that pedagogical practices are cultural practices and, as such, they represent the result of creative consumption of representations imposed on the school environment.

### **School notebooks and the first international movement for teaching Mathematics**

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At the end of the 1920s, at Colégio Pedro II<sup>1</sup> in Rio de Janeiro, the resolutions emanated from 1908 for the creation of the International Commission on the Teaching of Mathematics began to gain a voice (CIEM/IMUK). The key figure who conducted the readings of the international discussions was Euclides Roxo, professor of mathematics and director of the establishment. Roxo made use of the international ideas brought by Félix Klein to CIEM/IMUK elaborating a proposal to unify the branches of mathematics – Arithmetic, Algebra and Geometry – into a single school subject called “Mathematics”.

Like Klein’s writings and also using the manner the Americans read the proposals to change the teaching of mathematics, in the publication of textbooks – merging branches, from a concept of *function* to be taught intuitively starting the first year of high school, of

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<sup>1</sup> Colégio Pedro II, in Rio de Janeiro, was founded in 1837, modeled after French schools, to serve as a model for Brazilian secondary education. This two century old institution still exists today as a place of learning, under federal inspection. Pedro II referenced reforms, textbooks and education regulations for all the institutions that intended official recognition.

the articulated teaching of intuitive Geometry with Arithmetic and Algebra, flowing into Differential and Integral Calculus at the end of the five grades of high school – Euclides Roxo, besides writing a didactic work to guide professors in the teaching of the new proposal to modernize school mathematics, became the main political figure in the government of Getúlio Vargas, with regard to the national guidelines for teaching mathematics. As a direct adviser to the Minister of Education and Public Health, Francisco Campos, Roxo made it obligatory throughout Brazil to teach a single school subject – Mathematics – under the guidance of the modernizing ideas inherited from CIEM/IMUK. Besides the merger of the three existing mathematics, using the function concept as a unifying element, they also included the heuristic methodological treatment of teaching the mathematical concepts<sup>2</sup>.

The first analyses with school notebooks with the purpose of reading the transformations in the school culture, which occurred under the influence of the first movement to renew the teaching of mathematics in Brazil, considered the production elaborated during the first cycle of high school in Brazil. The notes contained in notebooks produced between 1935 and 1938 for students<sup>3</sup> in the first official lower secondary school in the state of São Paulo<sup>4</sup> were used as a case study. Altogether, seven notebooks were analyzed, divided into Arithmetic, Algebra, Geometry, Trigonometry and exercises for the teaching of the 2<sup>nd</sup> to 5<sup>th</sup> year of the Fundamental Course. Unfortunately, it was not possible to count on the notes from the first year.

At first glance, we see that the notes in the notebooks are divided between *lessons* and *exercises*. Thus, there are notebooks for each of these modalities of pedagogical practices for teaching mathematics. In the first case, the lessons notebook expresses the teacher's class, his explanations, his narrative for presentation and development of themes

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<sup>2</sup> In-depth studies of the action by teacher Euclides Roxo can be read in works like VALENTE (2004).

<sup>3</sup> Former professor at the Polytechnical School of the University of São Paulo, Clovis Bradaschia was a student at the old school in the capital from 1934 to 1938. Very organized, he kept his writings, which are in excellent condition, becoming a valuable source for the study of mathematics education during the Francisco Campos Reform. The material served as a source of research for the dissertation by ALVAREZ (2004).

<sup>4</sup> This old school in the capital, currently the State School of São Paulo, was inaugurated on September 16, 1894. With the Colégio Pedro II and the Normal School of São Paulo, it became one of the main learning institutions in Brazil.

with a mathematical content. The notebook, collated with interviews by its manufacturer<sup>5</sup>, indicates the day-to-day lessons:

It was all written by hand, I mean, the professor gave the lessons and we had to take notes. We had to write the notebooks and show it to the teacher. And they graded the notebook. The lesson and title were generally in red and the rest was in black or blue (BRADASCHIA *apud* ALVAREZ, 2004, p. 117).

Indeed, from spaces to spaces in the student's notes, we see the date, the teacher's initials and grade, alongside the pages of the notebooks. The presentation of the notebook texts also reveals the practice of "making clean copies". Certainly, for each class, after the dictation of the lesson, the student should copy his class notes at home, re-elaborating them in the notebooks that would be graded by the teacher.

However, contrary to what may be imagined from this activity of copying the lessons dictated by the mathematics teacher, to writing in the school notebooks to be graded by the teacher, as a repetitive practice, or even one without any meaning for school learning, it is necessary to consider the context and process in which the production of the notebooks was involved. Through the interviews conducted with the owners of the notebooks and with other former students, it will be possible to better understand the dynamic that involved the elaboration of this type of school material as an important part of the teaching and learning process for mathematics.

Upon mentioning the memories the owner of the notebooks has about his mathematics professor, he recalls:

He taught the subject like this: he supposed, admitted that everyone was up to date with the subject. So, when he was going to give a new theorem, he supposed the rest was already known. So, he would open the roll call list and say: "Whoever, theorem." And he would give theorem, and say: "Demonstrate." We would enter the classroom with the theory already known. I was called several times: sometimes, I did poorly, others, I did well. He would give the grades in the students' notebooks. We already knew that's how it was. You had to go, not only accompany everything, study from the previous material, but you had to go to the next class knowing, because he would call, statistically. He'd call one, two, three, four. But you could be one of them (BRADASCHIA *apud* ALVAREZ, 2004, p. 140).

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<sup>5</sup> Clovis Bradaschia conceded an interview to researcher Tana Alvarez on February 19, 2004.

Interviewing another former student<sup>6</sup> from the same school and with the same mathematics teacher as the owner of the school notebooks, from 1934-38, being analyzed now, Tana Alvarez gathered the following information:

He dictated. The students took notes and had to make clean copies in their notebooks too. And he dictated in a manner that was for you to take notes. And in the next class, he called the students. He had this big book and he would keep turning the pages. Then he'd turn around and the class would remain in suspense. He did it on purpose. Then he'd call. "Joe Smith, come here." You'd go up, show your notebook and he'd grade it and sign it. And then, he'd make an argument about the material in that notebook. That was systematically it and he would give a test every month (CRETELLA JÚNIOR *apud* ALVAREZ, 2004, p. 140).

The interviews reveal a pact, a dynamic agreed upon between students and teachers in everyday classes of mathematics. And this dynamic was mediated by the notebooks. The students' notes, written in the notebooks, will trigger the pedagogical relationship between the teacher and students. The notebooks are the means through which the subject demonstrates it was studied outside the classroom. More than that, they represent the vehicle for personal contact and conservation between the student and professor.

You could say that the relationship maintained by the teacher in everyday mathematics classes, mediated by the notebooks, was the first form of pedagogical relationship which became known as "individual teaching". Thus, in the middle of the generalization of schools, through "simultaneous teaching" – a teacher for several students – which becomes more solid starting in the mid 19<sup>th</sup> Century, there was this first type of pedagogical relationship, like a sort of inheritance of the individual form of teaching – one teacher, one student. By calling the student to the front of the classroom, alongside his desk, with his school notebook, the teacher maintains a relationship of individual teaching, correcting the notes made from already given classes and asking his student about the acquisitions of knowledge from already taught content. We also add that the lessons notebook served as a reference to the students' studying. They contain the mathematical texts to be studied for tests and exams, and that should be consulted for doing mathematical exercises. That means at that time the role of the textbook was minimal. The manuals are more useful to the teachers, especially those in large schools, in order to elaborate their lesson plans, than they are to students. In this dynamic of pedagogical practice, the students

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<sup>6</sup> This is José Cretella Júnior, interviewed on March 31, 2004.

do not see the school manuals as study guides. That role was left to the class notes on the lessons given by the teacher, which were then rewritten in the notebooks. In that sense, it also seems the lesson notebooks have greater pedagogical importance because they were checked, and from time to time they were also graded. In the case of exercise notebooks, only the teacher's initials counted. This different weight attributed to the two types of notebook can be better understood by collating testimony from former students about the mathematics teacher's thoughts and pedagogical actions at that time:

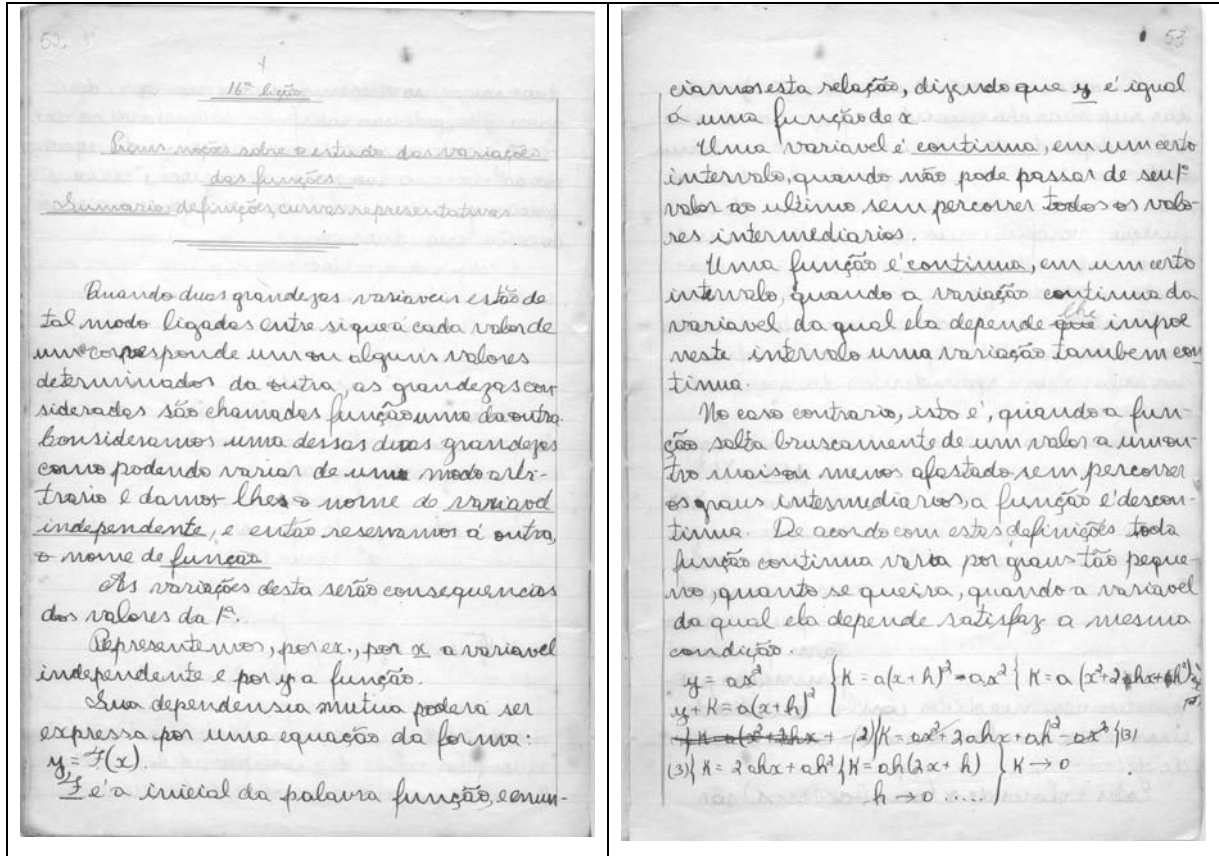
He would give exercises on the blackboard and guided a little. Then it was a little better. He'd say: "Write an exercise." It was a problem. He'd call you to the blackboard and you had to go. It was the so-called practical part. That's how mathematics has to be: deductive: Starts with a theorem you have to demonstrate, get to the thesis. And Mr. Cruz was like that, tough. He would reprimand fiercely and was demanding of the student. We studied three times a week. Mathematics is what we studied the most. (Cretella Júnior and Marco Napoli interviewed by ALVAREZ, 2002, p. 140).

Well then, in the middle of all this dynamic about teaching and learning, new winds emerge from the creation of the CIEM/IMUK, swirling about Brazil from the so-called Francisco Campos Reform. What do the analyzed notebooks tell us about the transformation of the current school culture in terms of methodology and content, regarding the teaching of mathematics?

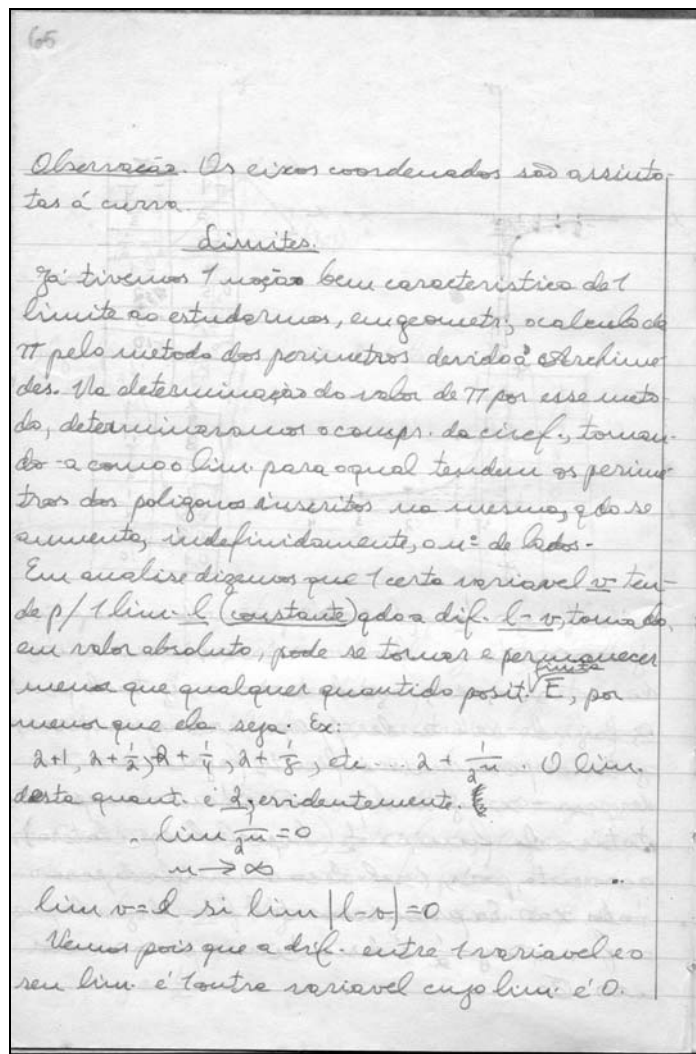
A page by page analysis of the school notebooks reveals an invariable sequence of formal and deductive education of the lessons written by the student. This corroborates the memories of former students of their mathematics teacher. Mathematics is a deductive process that aims at proving theorems and reaching a thesis. The theorems, the deductions are present in the Arithmetic, Algebra and Geometry notebooks. The exercises have less room and are a practical part that can be seen during different moments of the lesson, by calling the students to the blackboard. Greater emphasis should be given to the processes through which it is possible to elementarily and deductively reach conclusions. Thus, heuristically constructed intuitive processes have no room in notebooks. This represents the absence of this treatment of mathematical content defended by the modernization of teaching.

The dearest idea of the reform – the concept of function –, as an integrating element of the different mathematical branches, is present in the notebooks, but not like it was proclaimed in the official determinations that wanted it taught from the first year of the

secondary course in an intuitive manner. The concept was only introduced in the third grade, in the 16<sup>th</sup> lesson, as you can see in the notes below:



The initial notions of Calculus are dealt with in the 5<sup>th</sup> year of the secondary course. They are not present as a result of a formalization developed from the intuitive treatment of mathematics content representing a finish line, a summary of the course. They are presented as new content for teaching. However, even if in an exponential manner, we see the interconnection of geometry with algebra in the introduction of the concept of limits, as seen in the note below:



The first renovating movement for teaching mathematics arrived in Brazil with great force from the point of view of national education policies. Its leader was a mathematics teacher, director of the Colégio Pedro II – which at the time gave him the status of a Minister of State - author of the first mathematics textbook that explained Klein’s proposals in detail as well as the experiments they were doing in the USA concerning the integration of Arithmetic with Algebra and Geometry. All these qualifications made the writings of Euclides Roxo become national laws for teaching mathematics.

The analysis, even though initial of school notebooks containing notes from mathematics classes at the most prestigious lower secondary school in the state of São Paulo, revealed important elements of the subject’s school culture in the 1930s. One of

these is the segmentation of the branches, indicating non-adherence to the unified mode of teaching mathematics. The notebooks showing the various lessons reveal the Geometry, Arithmetic and Algebra courses given separately, even though the same teacher was giving the courses. The collated notebooks with testimony from the former students also show the prevalence of a deductive concept of mathematics with regard to its teaching. From other research already carried out, everything leads to believe that the distinction between the teaching of mathematics in the early grades (elementary) and teaching it in the secondary course was precisely one of deduction. While the first was the place for intuition, the second was characterized by the deductive processes which students should be taught in any of the branches of mathematics.

It is an error to say the lower secondary school teachers of the city of São Paulo were unaware of the Francisco Campos Reform in the 1930s. The modernizing proposals, little or almost never practiced in the classrooms, from the reading and analysis of student school notebooks, must have some other interpretation. It emerges when you use the school notebooks as a precious source for the dynamic analysis of the teaching and learning of mathematics during the times of high school mathematics teachers in São Paulo, as we attempted to show.

### **School Notebooks and the Modern Math Movement**

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Forty years have passed and we are in the 1960s. The second international movement for the teaching of mathematics is created. Since the 1950s, different American and European initiatives put in discussion modernizing proposals for teaching mathematics in school. These initiatives are subsidized by organizations like the OCDE – *Organisation Européenne de Coopération Économique* and the NSF – *National Science Foundation* which, in a great part, enabled significant actions to trigger the change process proposed by the Movement. It became known in the history of education as the MMM – Modern Math Movement.

Once again, the modernizing proposal aims at unifying the different branches of mathematics that are still fragmented in the teaching of mathematics. However, the unifying concept, which in the first movement was *function*, is no longer present. At this

time, the proposal for unification is through the mathematical structures, especially the algebraic structures. Algebra stands out and a unifying element role, in a way occupying the place that the function concept occupied in the first movement. Another characteristic of MMM, resulting from the dominance of Algebra, is the use of the set theory and the valorization of mathematical language and its symbology. One of the conclusions of the Royaumont<sup>7</sup> Seminar explains the role the symbols together with the set theory play in the new proposal:

These symbols and their use will give a new look to school mathematics. But that is not the intended purpose. The symbols are necessary because they represent concepts that provide more clarity and more precision to thought and because they link and unify the mathematical concepts for the student who sees them reappear in each of the branches studied. Furthermore, they are fundamental later on in the study of university mathematics. (OECE, 1961, apud GUIMARÃES, 2007, p.37).

With regard to the methods, the guidelines proposed are geared towards the learning process and the need for active participation by the students. An attempt is made to break from the mechanization and memorization so common in the teaching of mathematics. Valorization of understanding, learning by discovery, intuition and rigor are the methodological recommendations present in MMM. Other highlights are experimental work and the use of concrete and flexible materials as a first necessary phase in the learning process that permits axiomatic development in a second moment.

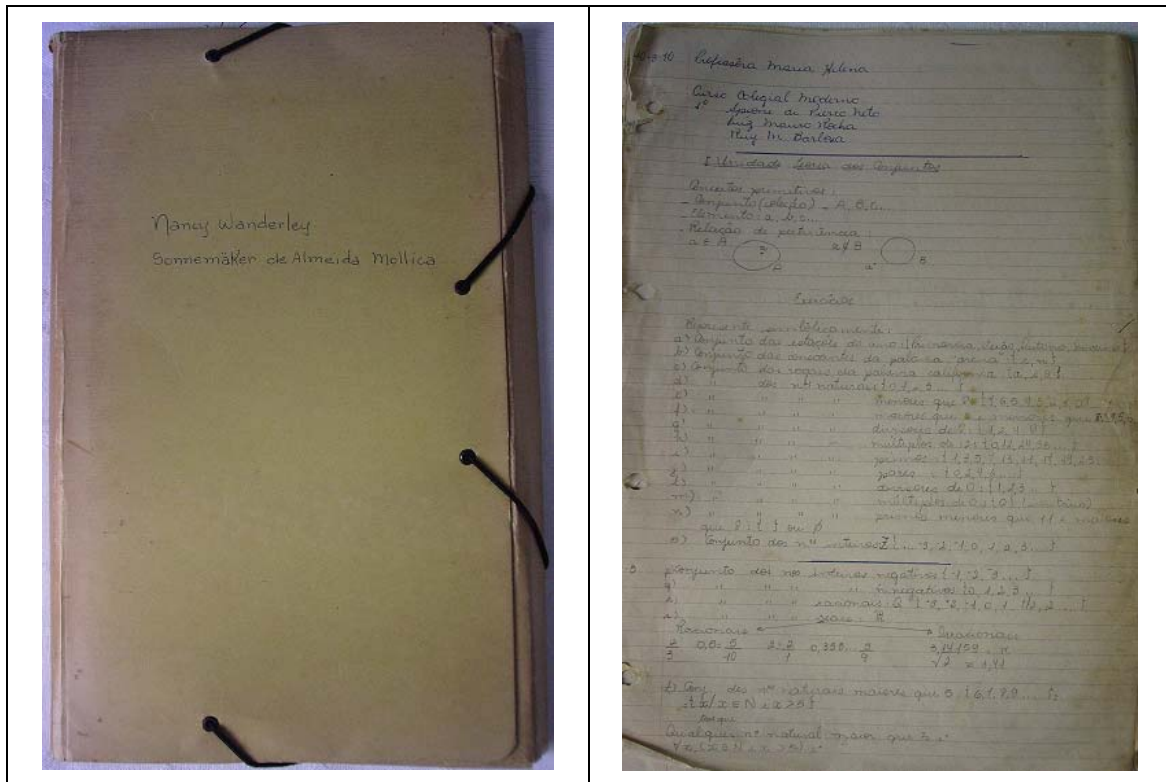
The incorporation of MMM tendencies in the teaching of mathematics in Brazil, is greatly achieved through textbooks (D'Ambrosio, 1987) and begins in the first grades of secondary education, with the publication of the "Modern Math Course" collection by Osvaldo Sangiorgi in the beginning of the 1960s. Only after a generation of student graduates with modern mathematics in the lower secondary school, at the end of the 1960s, is the 1<sup>st</sup> volume of the "Modern High School Math" course by Luiz Mauro Rocha, Ruy Madsen Barbosa and Scipione Di Pierro Neto published in 1968. Everything indicates that

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<sup>7</sup> In 1959, the *Organisation Européenne de Coopération Économique* (OECE) held the Royaumont Seminar in France, with the presence of representatives from eighteen countries, but not including Portugal. According to Guimarães (2007), this meeting reunião is certainly the most symbolic achievement of the entire reform movement, with great international influence, which received the name Modern Math. The reform proposal drawn up in Royaumont and its specification held in 1960, at Dubrovnik, with the elaboration of "A modern math program for secondary education", were strongly influenced by the dominant structuralist ideas of the time, especially with regard to Mathematics and Psychology.

this is the work that inaugurates the modernization of the teaching of mathematics in high school.

The school notebooks we analyzed as a case study for reading the transformations in school culture, which were influenced by MMM, belong to Nancy Wandereley Sonnemäker de Almeida Mollica, refer to the high school course (last three years of secondary education) and were produced from 1970 to 1972, at the Colégio Estadual Conselheiro Rodrigues Alves, in Guaratinguetá, a city in the interior of the state of São Paulo. It is a set of three mathematics notebooks in a loose-leaf file folder; each grouped according to the school year (1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> year of high school) and held together by a band. The folder's cover has the student's full name. The pages are yellowed, some folded, with tiny tears, but with clear detail regarding records and easy to understand.



On the first page of the notebook for the 1st year of high school, in 1970, there is the name of the teacher, Maria Helena and the title of the textbook “Modern High School Math”, published two years before, in 1968, and considered the first textbook to bring the modern approach to high school.

A reading of the first notes reveals the notebook was elaborated from the book identified on the first page, the textbook for the course. However, when making a comparison between the book and the notebook, we see there is book content that is not addressed in the notebook and notebook content not found in the book. This first comparison reveals the book was not followed step-by-step. The teacher has autonomy in relation to the material studied. Chapter V of the book, which introduces the exponential function and logarithms, is not addressed in the notebook. The notebook does not mention these functions. On the other hand, before beginning the study of spatial geometry, proposed in the book in chapter VIII, the notebook shows the development of a detailed study of plane geometry, which is not even mentioned or revised in the book. In short, the index of the book recorded as a reference in the notebook for the 1st year of high school does not express the subject's schedule. What then is the role of the textbook recorded on the first page of the notebook?

In a conversation with the student who produced the notebook, there is information that the notebook was made in the classroom and that she copied everything Ms. Maria Helena wrote on the blackboard. Thus, all of the exercises in the notebook were solved in class on the blackboard. It was not a clean copy notebook. It was produced in the classroom. Besides that, Nancy reveals that the teacher, Maria Helena, never took the adopted textbook to the classroom, or any other book for that matter. According to the report, the teacher entered the classroom with a small notebook, with the class prepared, and followed her notes. Nancy commented that there was great curiosity about the teacher's little notebook. With regard to the reference book written on the first page, Nancy said she used the book to study at home.

The notes in the notebooks allow us to conjecture it was a complete course, that is, it is neither a *lesson* notebook nor exclusively an *exercise* notebook, since one of its characteristics is the presentation of subjects in the following order: title of the content, its formal definition, one or more examples and then exercises. There are also relatively frequent observations made at the end of the examples or exercises, which are highlighted by the student, generally using a rectangle in ink. These observations are summaries of the developed concept, hints on how to identify important characteristics of the content in question.



especially the selected pages, is the use of set theory language and its own symbology, identifying elements in the modern approach.

Nancy told us the school context in which the notebook was elaborated: an exclusively girls' class, where the students had been studying together since the 1st grade of junior high, with good grades and behavior. In the classroom, they all stood to wait for the teacher and only sat after receiving permission to begin the class. The mathematics classes described by Nancy were carried out in absolute silence on the part of the students, who listened to the teacher lecture and copied what she wrote on the blackboard. Questions, student participation to develop content were not part of the pedagogical practice in these classes. She said when they had doubts they resorted to books or even private teachers, but never to the classroom teacher. The exercises proposed and solved in the classroom should be done individually and later corrected by the teacher. She does not recall any group work with her colleagues. We are thus out of step. The methodology revealed in the notebooks does not coincide with the report the student gave about the teacher-student relationship. The notebook shows the gradual development of concepts, calling attention to regularities, with many examples, which translates into a methodology that considers the student in the learning process. However, the student's report shows characteristics of traditional teaching, of lecturing on the part of the teacher and passive copying on the part of the students. New sources and research must be added for a better understanding of the pedagogical practices for mathematical teaching at the time.

In relation to the unifying nature of mathematics, especially the algebraic structures, they are present in the notebook for the last year of high school, the 3<sup>rd</sup> year, before the introduction of complex numbers. When this unifying concept is given, the last year of high school, seems strange to us. With regard to the branches, there is a predominance of Algebra in relation to the other fields of mathematics, but they are presented separately: Geometry is taught at the end of the notebook in the 1<sup>st</sup> year of high school, and on three smaller loose-leaf pages, different from the others, in the middle of the notebook for the 2<sup>nd</sup> year of high school, remaining without any link to the other concepts. Arithmetic is also taught in the notebook for the 3<sup>rd</sup> year of high school, on page 58 (the pages in the 3<sup>rd</sup> year of high school are numbered), and in isolated fashion. In short, the desired unification is not

seen in the notebook records. What is most significant is the presence of set theory language and the symbology preconized by MMM.

### **Final Considerations**

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Besides the curriculum references, the official texts and even the textbooks, considered material of utmost importance in the everyday activities of the mathematics teacher, school notebooks are documents that contain precious information for the study of pedagogical practices. In them it is possible to see vestiges of day-to-day classes and conjecture on how the dynamics have changed with regard to teaching mathematics.

This communication seeks to analyze which relations two broad-based international movements to reorganize the mathematics curriculum can be read in student notebooks. Far from reading the official renewal programs, and event the texts from textbooks copied in classes and put in school notebooks, the students' notes reveal the complexity of pedagogical practices. Legislation, programs, books and school notebooks are related to each other in an absolutely non-linear and determinist manner.

Reading the first international movement for curriculum renewal from the notebooks reveals a period when this material constitutes a fundamental element in the teaching and learning process of mathematics. The notebooks will mediate the teacher's teaching and will be a reference for student's learning. The inheritance of the branches separated into Arithmetic, Algebra and Geometry is present in the different notebooks written by the students: one for each. The concept of *function* and even the introduction of Differential and Integral Calculus are present in the notebook writings. But these elements seem to be more like remnants of proposals to modify the mathematics curriculum long after the Proclamation of the Republic of Brazil (1889), with the penetration of the positivist idea, than any incorporation of novelties from the CIEM/IMUK. Regardless, this is a topic that deserves more in-depth study.

In the case of the notebooks from the 1960s, there is a clear change in the function of this type of material. Those days of teachers' lessons found in student notes that are corrected daily by the teacher are long gone. The notebook becomes the reference for notes for students so they can study at home and prepare for tests and exams since there is no

explicit textbook to guide the teacher's work in a linear and mechanical manner. It will be necessary to follow the classes, with the notes, to be successful in school evaluations. Notebooks themselves are no longer considered a means of evaluation. With regard to the appropriations of MMM proposals, notebooks reveal the presence of a new language, the set theory, together with the corresponding symbology, however without achieving the proposed objective, of being representative of unifying concepts of the branches of mathematics. The inheritance from the start of the century still remains, a single mathematics notebook, however spaces that are both limited and separated according to Arithmetic, Algebra and Geometry.

Regardless, there is much to be done concerning the analysis of school notebooks as sources of research to understand the changes in pedagogical practices of mathematics teachers. And, of course, it will be necessary to expand the repertoire of this type of material, creating a database with an increasingly more expressive number of notebooks.

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