The School as a "Laboratory"

Giovanni Vailati and the Project for the Reform of the Teaching of Mathematics in Italy

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> *So that the students will be open to the beneficent action that mathematics must produce on their intellects, it is necessary for them to see at once the admirable function of this discipline as a developer, not only of the mind, but of awareness and of human dignity* [Vailati 1910, p. 36].

At the beginning of the twentieth century, forty years after the Unification, the high level of Italian research in mathematics was internationally recognised. This is demonstrated, among other things, by the fact that in 1908 Rome hosted the IV International Congress of mathematicians. On that occasion, Henri Poincaré wrote:

> For thirty years now the Italian mathematics movement has been very intense, in Rome as well as in universities in other provinces; I would have to cite a large number of names that will hold a very honourable place in the history of science.[1]

However, the high level achieved of scientific research contrasted in a peculiar way to the ongoing devaluation of mathematics in secondary schools. This is shown by official reports and final examinations as well as by the various legislative measures relative to teaching programs, the last of which was the Orlando Decree of 1904, which allowed students in the second year of *liceo* the option of choosing between Greek and mathematics, "releasing congenitally incapable students from a useless burden"[2]. This decision, which was severely criticised by the various teachers' organisations, was abolished only in 1911.[3]

Besides the evident deficiencies in secondary school teaching, several factors pointed to a pressing need for reform: the changed social and historical context; the influence of reform movements in other European countries – especially Felix Klein's movement in Germany and Gaston Darboux's in France –; the increasingly active participation of teachers in the political issues of education; and, finally, the remarkable increase in the number of pupils enrolled in secondary schools (from 18,231 to 94,572 between 1861 and 1901). To address this situation, in 1905 Minister Leonardo Bianchi appointed a Royal Commission for the reform of the secondary school system. This body was made up of professors, teachers and Ministry inspectors, and its objective was to conduct a comprehensive inquiry into upper secondary schools, and present proposals regarding the most urgent and appropriate changes to be made. Despite the difficulties and conflicts within the board itself, in February 1908

[1] Cf. "Lettre de Henri Poincaré au Journal "Le Temps" sur le 4e Congrès International des Mathématiciens." *Rendiconti del Circolo Matematico di Palermo*, XXVI, 1908, p. 20.

[2] "Programmi di matematica per i ginnasi ed i licei." *Bollettino Ufficiale del Ministero dell'Istruzione Pubblica* XXXI, II, n. 52, Roma 29 dicembre 1904, p. 2851.

[3] Cf. in this regard the wide debate about this matter during the Congress of the Associazione Mathesis held in Milan in 1905 on 21st-22nd April, in *Bollettino della Mathesis*, 1905, pp. 37-91.

the Commission presented a draft law proposing, on the one hand, a professional technical school with three-year courses enabling entry to the technical institutes, and, on the other hand, a three-year course for the *scuola media unica* (lower secondary school, or middle school, common to all schools), excluding Latin as a subject, which would grant students access to the three different kinds of upper secondary school: *liceo classico* (with Latin and Greek), *liceo scientifico* (with two modern languages and a broader science syllabus), and *liceo moderno* (with Latin and two modern languages).

The syllabi for mathematics, and the instructions on teaching methods – original and panoramic in their approach – were written by Giovanni Vailati (1863-1909), a mathematician who belonged to the school of Giuseppe Peano and cultivated a great variety of interests that range from philosophy to the history of science, to psychology and sociology, to the problems regarding education.

Here, I will concentrate on the following points: Vailati's criticisms of the secondary schools of the time, the pedagogical and methodological assumptions that he starts from in formulating his project for reform, his concrete proposals for a renovation of the teaching of mathematics, the criticisms of his proposals, and the results effectively achieved.

1. *Limits and deficiencies of the secondary schools*

Vailati's proposals derived from his lucid analysis of the defects of the Italian secondary school system. Above all else, he found that teaching was based on passive learning, which turned the schools into *factories for rote learning*, where the student was overly engaged in accumulating information *(accipere*), and not engaged enough in comprehending (*concipere*). In Vailati's words:

> The final result of this system of intensive breeding, all too similar to a system of nutrition that is used barbarously in the countryside of lower Lombardy to obtain the prised goose livers, is too often reduced to … producing in all students, and often in the most intelligent ones, a repugnance of anything having to do with school… [Vailati 1906, S III, p. 293].

There was little interaction between the humanistic and scientific cultures, with a disproportionate number of lesson hours devoted to the teaching of Italian and Italian literature. Vailati wrote:

> The final effect is that of increasingly favouring and accenting the division … of cultured people into two classes, one class who write and speak well about what they are ignorant of, and the other who can neither write nor speak properly of what they do know; that is, the creators of words that are harmonious and empty, on the one hand, and on the other hand, scientists of barbarous language and uncultured souls [Vailati 1900, S III, p. 263][4].

Classes were overcrowded.

[4] Vailati's writings published in 1911, edited by M. Calderoni, U. Ricci and G. Vacca (*Scritti di G. Vailati, 1863-1909*, Leipzig: J.A. Barth, and Florence: Successori B. Seeber,) were republished in 1987, with the addition of 25 unpublished texts in *Giovanni Vailati, Scritti*, edited by M. Quaranta, 3 vols., Bologna: Forni. It is to this edition that reference is made with the letter S.

Further, there was a lack of facilities supporting teaching activities, such as libraries or laboratories. Finally, schools were not supplied with good books (dictionaries, encyclopaedias, editions of the classics, etc.).

2. *The pedagogical and epistemological assumptions at the base of the reform project*

The programs that Vailati set forth and the methodological instructions that accompanied them attempted to remedy these defects and reflected an original epistemological vision of mathematics in which various motives and needs converge. His association with Peano and his school gave rise to his solid mastery of mathematical logic, the idea of deductive and systematic rigour, his reflections on language conjoined with a deep interest in education and the history of mathematics, and a sincere desire to democratise knowledge; these characteristics form the bases for all of his work. The pragmatism of Charles Sanders Peirce had a strong influence on Vailati, who saw it as an instrument in the struggle against senseless problems and against metaphysics; in particular, it was the operative and functional criteria for giving meaning to the propositions that he makes his own (as Pierce said, "the rational meaning of every proposition lies in the future"[5]). Intertwined with pragmatism are positivistic requirements: the realisation of a scientific *humanitas*, an appreciation of the applicative aspects of knowledge, the foundation of teaching on a positive knowledge of man (biology, psychology), in the constant awareness that the cognitive process proceeds from facts to abstraction. At the base of all is the Herbartian assumption that the aim of teaching is the formation of character.

2.1. *The operative experimental method and the school as laboratory*

What Vailati proposed was a "school as laboratory", not in the limited sense of a laboratory for scientific experiments, but rather "a place where the pupil is given the opportunity, with the guidance and prompting of the teacher, to train himself to experiment and solve problems, to measure and above all, to measure himself, and to test himself against obstacles and difficulties intended to increase his intelligence and cultivate his initiative" (Vailati 1906, S III, p. 292).

In particular, mathematics teaching should adopt an approach that is experimental and active, and, since the process of learning moves from the concrete to the abstract, pupils should never be forced to "learn theories before knowing the facts to which they refer". On the contrary, they should show that they know *how to do things*, not merely *how to repeat things*. The kind of lesson most likely to achieve this objective is the Socratic, which allows teachers to guide their students towards discovery of mathematical truths, while at the same time stimulating enquiry and reflection. Further, in a "school as laboratory" it is necessary as well to exploit moments of play during the process of learning, which far from "diminishing the dignity of the science of mathematics" (Vailati 1899, S III, p. 261), instead increases its attraction. Manual activity, appropriately directed, can serve to "practise the various skills of observation, discernment, attention, and judgment", and constitutes an excellent antidote to the common misconception that one knows something simply because one has learnt certain words (Vailati 1901, S III, p. 265).

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[5] Ch. S. Peirce, "What Pragmatism is", *Monist*, 15, 1905, in *Collected Papers of Charles Sanders Peirce*, Cambridge: The Belknap Press of the Harvard University Press, 1960, p. 284; cf. the review by Vailati in S I, p. 362.

The effectiveness of a teaching system which moves from the concrete to the abstract is particularly evident in the teaching of geometry. *Intuitive method* was the name usually given to the method to be followed in the first stage of teaching, but Vailati preferred to use the term *experimental or active geometry*, since this is more indicative of the way in which it differs from the rational geometry to be studied in the upper high school courses: drawing, the use of simple mathematical instruments, and small experiments allow students to discover some of the properties of geometrical figures and stimulate their desire to understand why this is so, making learning more interesting.

2.2 *The unity of mathematics and scientific humanitas*

In Italy following the Coppino Decree of 1867, the widespread opinion that "recurring to algebraic notation for expressing geometrical facts or relationships constitutes a kind of contamination or an attack on the purity of Euclid's treatment" resulted in the application of algebra to geometry being presented in school at a point when all of geometry and the most part of algebra had already been dealt with. Instead, according to Vailati, "not only applications of algebra to geometry but vice versa as well, applications of geometry to algebra must be discussed as early as possible", [6] so that the student perceives at once the profound unity of mathematics and becomes used to confronting the same problem with various methods and choosing that which is most suitable in any given situation.

The dialogue between scientific culture and humanistic culture that was so close to Vailati's heart can be achieved by means of the history of mathematics, which, besides allowing the student to perceive the unity of knowledge, can contribute to taking the pedantry out of the exposition, and to "rendering the teaching more fruitful … more efficacious, and altogether more attractive" (Vailati 1987, S II, p. 10). At the same time, by showing the origin and the development of the scientific concepts and theories, the historical method becomes a kind of antidote for all forms of dogmatism.

3.3 *The dialectic between rigour and intuition*

As far as rigour in the teaching of mathematics is concerned, Vailati observed that the application of new research on the foundations of elementary geometry to education had made it evident that "the logical correctness of a proof is not something that depends on the number or the quality of the assumptions or hypotheses which are used in it, but depends rather on the way in which these are applied". What is important is that "each hypothesis, or assumption which … is used be clearly recognised, and formulated explicitly" (Vailati 1907, S III, pp. 305-306), and the only indispensable requirement for the rigour of the proof is that the postulates are compatible among themselves. Far from discouraging geometric intuition, the competent teacher will "discipline and refine it" in order to avoid the errors that can arise from a "rash and instinctive trust in it" (Vailati 1904, S III, p. 268).

Furthermore, Vailati maintained that deduction should "definitely not [be used] to demonstrate propositions that students already find quite obvious … but rather to use these propositions to arrive at others which they do not yet know". In this way, deductive reasoning would be seen by them as a means of "economising" on experiences and

[6] G. Vailati, "L'insegnamento della Matematica nel nuovo ginnasio riformato e nei tre tipi di licei", *Il Bollettino di Matematica*, Anno IX, 1910, p. 57.

experiments and, without these, of "foreseeing" the results, and thus as a means of discovery.[7]

2.4. *Differences between methods and the contents of the programmes for the three types of* liceo

Vailati also takes on the problem of up to what point the method and contents of the three upper secondary schools – *liceo scientifico*, *liceo classico*, and *liceo moderno* – should differ, since the first must furnish the instruments necessary for pursuing scientific studies, while the other two must aim to develop the capacity to reason with precision and rigour. Convinced that there is no conflict between these two ends, he proposes for the *classico* and *moderno* secondary schools a broader programme of mathematics compared with that provided for in the actual programme of the time for secondary schools. For those who choose one of these two kinds of school,

> …the notions of mathematics learned in the middle school represent, in a certain sense, the columns of Hercules of scientific culture; while, in contrast, for the students of the other branch of secondary school [i.e., *scientifico*], more particularly destined to prepare young people for faculties of science or the polytechnic institutes, they are the only way to acquire higher knowledge.[8]

Further, recalling Klein, Vailati observes that the distinction between elementary and higher mathematics is often due to reasons of a historical nature, and does not correspond to any criteria of didactic suitability. In particular, he introduces in all three upper secondary schools the concepts of function and derivative because of the important role they play in applications to other sciences, and in the *liceo scientifico*, that of the integral as well. Probability theory and its applications are inserted into the *liceo moderno*, aimed at preparing young people for economic and social activities and studies; on the other hand, in the *liceo classico* the emphasis is on Euclidean geometry, accompanied by readings from the original works of the great geometers of antiquity, with the aim of offering a more complete picture of classical civilisation, one not limited to literature and art.

3. *Criticisms of the reform project and the results obtained*

The reform proposed by the Royal Commission, and especially the unification of the middle school, was considered too radical, not only by conservative thinkers, but also by the majority of the members of the National Federation of Secondary School Teachers, associations of a political nature created in 1901 with the aim of safeguarding the economic and legal rights of teachers and promoting the improvement of secondary schools. The criticisms aroused by the reform were essentially two. First, a school organised in this way would have two goals that would be difficult to reconcile because it would accept both students planning to leave school to take up a trade as well as those intending to continue on to high schools, and then to university. Second, a common middle school would postpone

[7] G. Vailati, "Sugli attuali programmi per l'insegnamento della matematica nelle scuole secondarie italiane", in *Atti del IV Congresso Internazionale dei Matematici, 6-11 aprile 1908*, Rome: Tip. Accademia dei Lincei, 1909, 3 vols,, III, p. 485; see also http://www.subalpinamathesis.unito.it/storiains/uk/excerpts.php.

[8] Vailati 1910, op. cit., p. 53.

the teaching of Latin, further weakening the *liceo classico* whose important function was to educate those who would take up influential roles in government and in society.[9]

The mathematics curricula prepared by Vailati attracted criticism as well. The absence of a treatment of the theory of proportions, or of a rational treatment of arithmetic, the excessive fragmentation of some parts of the programme, and the abolition of descriptive geometry were particularly lamented within the Associazione Mathesis, an association of mathematics teachers founded in 1895-1996 for "the improvement of schools and the betterment of the teachers from scientific and didactic points of view". Moreover Vailati's suggestion of an experimental and hands-on teaching of geometry gave rise to an interesting debate about methodology with two distinguished geometers, Giuseppe Veronese and Beppo Levi, who maintained that at the lower level of secondary school the teaching of geometry should be essentially intuitive.

Experimental teachings, according to Levi, are exclusively informative teachings, and can present "a grave danger with respect to the education of the mind". As an example he cites the measurement of the hypotenuse of a right triangle with sides of 3 and 4 cm., and that of the diagonal of a square of side 1, and observes:

> What right do you have to force the student to not attribute equal values of certainty to the two experiments, from the first inducing the Pythagorean theorem … while from the second permitting no kind of precise conclusion, arbitrarily stating that that measure is only approximate? [10]

Veronese's criticism took on rather harsh tones:

> If the mathematics teacher can be the drawing teacher (as he could be, according to the programme) you would have a teaching [appropriate] for an art or trade school, and it is sufficient to examine the textbooks for geometric drawing, as it is sufficient to have participated in the examinations of technical schools, to get an idea of what kind of geometric notions would be given!"[11]

These discussions were linked to a broader international discussion on the roles of experiment and intuition in the teaching of mathematics in secondary schools, which came together in a inquiry promoted by the International Commission on Mathematical Instruction in 1911[12] and found its expression in Italy in two opposing schools of thought with different views on mathematical research: the algebraic geometry school of Corrado Segre and the mathematical logic school of Peano.

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[9] National Federation of Secondary School Teachers discussed at length the criteria for the reform of secondary schools during its fourth national congress (Milan, 25-28 September 1905) and then again during the seventh national congress (Florence, 25-27 September 1909): cf. *Quarto congresso nazionale degli insegnanti delle scuole medie. Milano, 25-28 settembre 1905*, Pistoia: Tip. Sinibuldiana G. Flori & C, 1905, pp. 380-382 and *Settimo congresso nazionale degli insegnanti delle scuole medie. Firenze, 25-27 settembre 1909*, Assisi: Tip. Metastasio, 1910, p. 308.

[10] B. Levi 1907, "Esperienza e intuizione in rapporto alla propedeutica matematica. Lettera aperta al prof. Giovanni Vailati", *Il Bollettino di Matematica*, Anno VI, pp. 177-186, at p. 182 and 184.

[11] Veronese a Vailati, 8.9.1907 in P. Cantù, "L'insegnamento della geometria nelle scuole medie inferiori. Una lettera inedita di Giuseppe Veronese a Giovanni Vailati", *Il Voltaire*, 5, 2000, pp. 109-118.

[12] D.E. Smith, "Intuition and experiment in mathematical teaching in the secondary schools", *L'Enseignement mathématique*, 14, 1912, pp. 507-534, translated in part into Italian by G. Castelnuovo in the *Bollettino della Mathesis*, 1912, pp. 134-139.

In any case, the reform set forth by the Royal Commission was never carried through. Part of Vailati's proposals was implemented with the creation of the *liceo moderno* in 1911, which diverged from the *classico* after the second year of the *liceo,* and where Greek was replaced by a modern language and greater scope was given to scientific subjects. The programmes or mathematics and the methodological instructions were drawn up by Guido Castelnuovo, a distinguished scholar of algebraic geometry. He, as president of the Mathesis Association and the Italian delegate to the International Commission on Mathematical Instruction, tried to promote a mathematics teaching that would take into consideration the European reform movements, particularly that of Klein, the main tenets of which had been accepted by Vailati. His conception of the teaching of mathematics[13] is apparent from a number of slogans which appear often in his speeches and in his articles: "Rehabilitate the senses"; "Break down the wall separating schools from the real world"; "Teaching should walk hand in hand with nature and with life". So it comes as no surprise that Castelnuovo was one of the supporters of the project to reform the secondary school proposed by Vailati,[14] and that he adopted some elements of this project in developing the syllabi for the *liceo moderno*. In particular, he introduced the concept of function and elements of differential and integral calculus into the curriculum:

> But if we truly wish that middle school children are inspired by modern mathematics and perceive something of the grandeur of its whole structure, we much speak to them of the concept of function and indicate, albeit in a brief and simplified manner, the two operations which constitute the fundamental basis of infinitesimal calculus. In this way, if the pupil has the scientific spirit, he will acquire a more correct and balanced idea of the exact sciences nowadays […] If, instead, the pupil's mind is more disposed towards other subjects, he will at least find mathematics to be, instead of a logical drudge, a set of tools and results which can be easily applied to concrete problems".[15]

But the *liceo moderno* was short-lived: deferred until the 1920s, the reorganisation of the secondary schools was introduced in completely different terms; the positivist, liberal democratic culture was defeated by new political trends and by the triumph of Neo-idealism. Nevertheless, the school as laboratory proposed by Vailati, where the student is a "field to be sown, a plant to cultivate, a fire to feed", and where the scientific *humanitas* has a central role, can still provide a stimulating point of reference today.

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[13] Cf. L. Giacardi, *Guido Castelnuovo*, in http://www.icmihistory.unito.it/.

[14] G. Castelnuovo, "Sui lavori della Commissione Internazionale pel Congresso di Cambridge." in *Atti II Congresso della Mathesis, Padova, 20-23 Settembre 1909*, Allegato F, 1-4, 1909, at p. 3.

[15] G. Castelnuovo, "La riforma dell'insegnamento matematico secondario nei riguardi dell'Italia." *Bollettino della Mathesis* XI (1919): 1-5, at p. 5.

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L. GIACARDI (ed.) http://www.subalpinamathesis.unito.it/storiains/uk/documents.php, website which offers some key documents in the history of the teaching of mathematics in Italy from Casati's Law (1859) to Gentile's Reform (1923).