

A-didactical situation in multicultural primary school

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Riassunto. Obiettivo primario della ricerca sperimentale condotta, è stato quello di studiare il comportamento cognitivo degli alunni di età compresa tra 3-10 anni nel compito di risoluzione di problemi algebrici. Più specificatamente si è cercato di studiare attraverso un'analisi parallela effettuata su differenti contesti matematici creati ad hoc, come incidono i diversi tipi di struttura logica argomentativa di un problema sulla condotta di risoluzione da parte degli allievi coinvolti (tipi di soluzioni, passaggi dell'algoritmo risolutivo ed errori) e quindi come questi possano influenzare il loro processo di apprendimento di abilità specifiche relative ai concetti trattati.

Il presente lavoro si inserisce in un progetto di ricerca più ampio, portato avanti dal GRIM di Palermo e mirato essenzialmente all'analisi degli ostacoli storico-epistemologici e didattici riferiti ad alcuni concetti fondamentali presentati nella scuola primaria (ma non soltanto) e riferiti all'algebra.

L'attività proposta è stata realizzata secondo il quadro teorico di Brousseau e prende in considerazione ambienti didattici multiculturali trattando nello specifico una possibile comparazione tra alunni italiani e cinesi.

Concetto scelto come ambito matematico specifico è stato quello di *pensiero aritmetico*, pensiero algebrico, e quindi un primo approccio alla variabile, all'incognita.

I dati raccolti sono stati analizzati parallelamente secondo un'analisi quantitativa e qualitativa. L'analisi quantitativa è stata condotta mediante l'utilizzo del software di statistica non parametrica CHIC 3.0.

Abstract. Primary objective of the experimental research was to study the cognitive behaviour of pupils aged 3-10 in the of resolution of algebraic problems. More specifically to study, through a parallel analysis carried out in different mathematical contexts created ad hoc, how the different types of logical argumentative structure of the problems engrave on the resolution behaviour of the students (types of solutions, passages of the decisive algorithm and errors) and therefore how these can influence their process of learning of specific ability related to the treated concepts.

The present work is inserted in a broader research project, conducted within the GRIM of Palermo and essentially dedicated to the analysis of the historical-epistemological and didactic obstacles related to some fundamental algebraic concepts introduced in primary school (but not only).

The activity we proposed have been conducted according to Brousseau's theoretical framework taking into account a multicultural didactic environments, with a parallel analysis and a comparison between Italian and Chinese pupils.

The selected concept that constituted the mathematical specific milieu was bushels, *arithmetical thought*, algebraic thought, and so a first approach to the concept of variable and unknown.

The collected data have been analyzed in parallel according to a quantitative and qualitative analysis. Quantitative analysis has been conducted using the software CHIC 3.0. for non parametric statistics.

Key words: algebraic thought, arithmetical thought, Variable, unknown, Chinese writing language, multiculturalism, Jiuzhang Suanshu (Nine Chapters).

1. Introduction and research objective

Timothy Glowers words, "that that all children with an early mathematical education given by a prepared and enthusiast personal teacher would grow loving it" (Glower, 2004) once again highlight, if still necessary, the need for criticism, reflection and awareness on the part of the teacher from the first years of school; in order to enhance in students and therefore in pupils themselves, an aware reasoning in the solution of mathematical problems, devolving in them a

productive thinking that aims at fully understanding. We therefore wish to pass from a mere “learning” to “realizing”.

The invitation to try “alternative” teaching strategies to the usual routine in all education levels, starting from kindergarten is stressed by several researches in mathematics education (D’Amore, 2000, 2001); researches that underline the poor cognitive effectiveness of standard teaching methods and propose in several conceptual ambits different methods and applications that aim at a greater involvement of pupils and therefore at an aware acquirement of knowledge in turn in the game.

In the last years we have witnessed a strong revision of contents, objectives and teaching methodologies of mathematics and physics, pushed on by the complexity, multiplicity and versatility of their meanings and applications and by deluding results obtained by 15 years old Italian pupils in international tests (PISA 2003, 2006) and by the “vocational” crisis highlighted by the low numbers of freshmen in scientific faculties.

In order to acquire this goal, several Italian and international researches have discovered and tested concrete learning environments that enhance meta-reflection on contents proposed in the classroom and foster motivated, meta-cognitive and cooperative attitudes that are often absent in traditional teaching.

Scientific teaching (from school textbooks to didactics methodologies) and in particular the teaching of mathematics, often has not evolved with the general cultural improvement, ignoring in many cases both motivation, learning and understanding problems (more and more evident in pupils of all school levels) and the results of cognitive and didactic research; results that give more and more interesting elements on both levels of analysis.

It is therefore useful to show as a possible path for an aware growth of the concepts encountered by students in the classroom, the “transformation” of the classroom into a laboratory that allows pupils not only to “carry out” tasks but also to project, construct and handle different tools. Discussion, argumentation, hypotheses formulation, control of the validity of such hypothesis, continuously defining an equilibrium between an operative phase and a gradual theoretical arrangement in order to foster pupils passage from visual evidence to a more and more rigorous argumentation (Di Paola et al, 2007).

For this objective it is important, especially in primary school, that pupils get in contact with a knowledge made up of rich and motivating situations so that they can reinforce there insight to reach in further studies axiomatic arrangements.

The scientific laboratory can be a useful instruments to know and analyse from a pedagogical point of view the environment in which the child lives; in this sense it can be used by the teacher as a means to promote paths to approach and understand the surrounding environment and therefore of every day life phenomena.

As regards the intellectual development of the pupil, the laboratory activities can foster through exercise and repeated productions of objects some important psychological functions as movement control, memory abilities, perceptive activities and creative thinking.

One of the open problems of the “new” school is to interpret the behaviour of students introduced into multicultural classes.

The issue of “multi-cultural” classes is in Italy a phenomenon that, even though rather new, is in wide expansion: the integration of foreign students in the Italian classes had, in the recent years, an increasing rate and has become no more an exception but, on the contrary, an inevitable reality. *«The present situation requires therefore to consider and to reorganize an idea of education in balance with the new needs and resources, in order to strengthen the trend of differences integration, the change and mutual adaptation, an open trial correlated with identities recognition and acceptation and with incorporated knowledge»* (Canevaro 1983).

The differences that could be detected in the class activities from this point of view, will turn into sources that enrich the whole class. In these relationships the teacher has to play the essential role of a “knowledge mediator”.

In the specific case of Mathematics, a greater attention was paid in recent years to the problems of the didactics in a multicultural school milieu and these themes were included into several school programs and described in many papers.

It becomes evident in this context, that the starting point of any activity facing problems which have arose out of the presence of cultural differences in the class, is to specify and highlight all moments that characterise cultural models of integration: pupil's previous knowledge, his motivations, his expectations and abilities, his personal and intellectual characteristics; all that constitutes the necessary prerequisites of every correct pedagogic intervention. (Garcia Hoz-Guerriero-Di Nuovo-Zanniello 1997, 238-239).

2. Theoretical framework and research questions

The study of the History of Algebra and the its relationships with Geometry and Arithmetic is in continuous evolution. Many are in fact the researchers that work to analyze the development of this discipline, trying to delineate a general framework that can allow to individualize the fundamental steps for the development of the algebraic thought in relationship to the arithmetical and geometrical ones.

In this context many national and international research's work underline the complex problematic regarding the passage from the arithmetical thought to the algebraic thought and so the birth and the evolution of the sense of the variable' concept for the students.

In the phase of transition between arithmetical thought and algebraic thought, they verify then as some epistemological obstacles strictly connected to the passage from a meaningful semantic field, precedent, (the arithmetic) and the syntactic construction of a new language (the algebraic one) can delay the development of the algebraic language and so the algebraic thought. (Spagnolo 1998)

These experimental analysis allowed us to underline a different behaviour of the students in relationship to: the logical structure of the proposed problematic situation, the type of study course attended and the origin country (different culture, different system education, different teaching's strategies...).

One of the open problems is so to interpret the obtained results, in presence of multicultural classes.

In this sense, the history of the western Mathematics is not the only approach that we have to consider for the understanding of the teaching/learning processes connected. The theoretical framework analysed in the precedence research comes to be interpret in ampler way, more general.

In this context, the choice to study the Chinese mathematical thought is due to the fact that the Chinese culture, as regards Natural Language, Philosophy, Logic etc., is the most distant from the western culture; to analyze the reasoning schemes used by the Chinese student in the resolution of a mathematical problem allows us not only to reflect on the differences of argumentation adopted in the two countries in the resolution of a same assignment but, above all, it allows us to reflect on our cultural reference system, the Western one.

The problem regarding the sense of variable could be connected with some particular aspect of the Chinese culture, for example the structure of the ideograms in the written Chinese language and the logical-argumentative schemes adopted by the Chinese students in the class.

- Do the Italian and Chinese students, in the resolution of particular problems, put in evidence different resolution strategies reported to the effect of their origin culture (Natural Language, logical-argumentative schemes, algorithms, etc...)?
- Is it possible to underline these differences in the passage from the arithmetic thought to the first algebraic ones?
- Can the study of such differences help the understanding of the phenomenon of teaching/learning in multicultural situation?

To be able to interpret the comparative study between the Chinese thought and the Italian one in situations of teaching/learning in multicultural perspective, I refer to the studies of J. G. Gheverghese (1987) and U. D'Ambrosio (2002). For the observations regarding the languages and particularly the differences between the Chinese language and the Indo-European languages I refer to the following works: Chemla (2001), Needman (1981) and Granet (1988).

A first approach to the research problem was brought ahead, in a first approximation, with a historical-epistemological analysis through a critical-comparative analysis of different

representative texts of the two cultures; considering the book “The Nine Chapters of Mathematical art” as the principal reference for the Mathematics in Chinese education. Particular attention in this sense is turned to the comparison of the logical-argumentative references as regard the natural language and Mathematics.

The principal theoretical reference for the methodology of the study is the Brousseau`s theory of the situations (Brousseau 1998); in a multicultural milieu it could result central in the specific phase of socialization of the cognitive styles. To put in evidence the socialization of the cognitive styles (*phase of validation* of the a-didactic situation) became then the carrying element for the comprehension of the phenomenon.

Accepted the principle by which the education comes to be "realized" around the student, considering then the social and physical milieu in which he lives, the Didactics of the mathematics has to build on the different experiences offered by the contemporary presence in class of different cultures, each one with the own mathematical inheritance, with the own mathematical knowledge, to a cultural exchange and a mutual enrichment.

The topic we deal with allows also a series of transversal theoretical reflections that need to a consider as broad as possible frame of reference that considers not only motivation/emotional side of the didactical activities but also the role of such a didactical methodology, centred on playing and recreative activities. (Piaget, 1976, Brousseau, 1998), that “disrupts” that pre-arranged context expected and feared by the pupil in which he carries out mathematics.

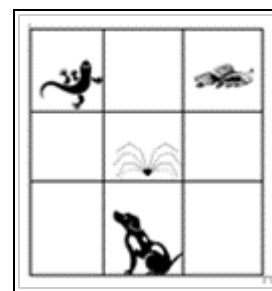
2.1 Methodology and first results

In this paper I refer to a particular game experimented with Chinese children of infancy and elementary school, *Sudoku/Magic box* opportunely simplified.

The game is referred to the *box/matrix* showed in the figure beside.

We proposed it in the classroom with other five different images of animals on the cards and a series of rules for the composition/solution of it:

1. each animal cannot be in the same line or column of its enemy (we presented the enemy animals);
2. each animal has to appear in the square only once;
3. each student has to insert in the box, all of the nine possible different animals showed in the image cards;
4. the solution has to be only one.



This is one of the possible games for a first approximation research, conducted in a multicultural milieu with Chinese students and also pupils from other countries, into the relationships between the “serial thought” and the “global thought” in the reading and understanding of the problem. In a first approximation, we could consider, this kind of reasoning connected to the arithmetical and algebraic thinking and their relationship.

We involved in the experiment about 95 children (13 Chinese students) aged 3-10 ; the age range was chosen to investigate in the broadest possible way the different behaviours and different verbalizations of the pupils in this situation.

The experimentation was divided in two phases:

1. situation/game with children of the infancy School “Ferrara” of Palermo and of the of the Primary School “Costa G.” of Palermo, first two years, to observe through quantitative and qualitative analysis (classroom experiences videotaped), the behaviours enacted by the students and the different playing strategies and the recurrent reasoning of Italian and Chinese students;

2. semi-structured interview (videotaped) to two foreign (Chinese) students, inserted in the Sicilian scholastic context at the Elementary school, regarding the same situation/game.

As I previously said the game was chosen and adapted according to a series of critical reflections and researches previously carried out within the GRIM on the same topic. To structure the game, we considered particular linguistic aspects that characterise the structures of Chinese written language: tabular representation of the phonetic characters (the matrix as mathematical reference (Needham 1981, 46)); the **possible interpretation of an ideogram as union of single elements (local vision, *Western vision*) or unitary character (global vision, *Chinese vision*)**,

Ex. **Algebra** = 代数 = 代 (to represent) + 数 (number)
[人(men) + 弋 (an arrow that points out: it represents the phonetic part)] + [(夂 (*clapping, tapping rhythmically to facilitate in counting*) + 娄 (“that is obscure”)]

the **possibility to find a first approach to the sense of variable inside the written Chinese language**

Ex. Gǔ (古) = as unitary ideograms ‘old’ composed by “ten” and “mouth” (in reference to the Chinese philosophy That which has passed through 十 ten □ mouths, i.e. a tradition dating back ten generations):

固 = to harden (annoyed and hardened), with the radical 31: □

枯 = to fade (annoyed and done harden) “From 木 (mù) 'tree' and 古 gǔ ('old') phonetic. 古 'old' also it is suggested the meaning, 'withered’

故 = reason, cause (aged, dried him and fixed him) with the radical 66

姑 = mother-in-law (elderly woman "dried him") with the radical 66 to the left.

箇 = solid thing and hardened,

(個 = old men).

All these ideograms are formed therefore from two meaningful parts that give a new meaning, but at the same time one of the parts also have phonetic value and it communicate the sound.

and also some **typical reasoning schemes showed in the previous experimentations with Chinese students** and referred to an algorithmic approach to the solution of a mathematical problem including “one problem multiple solutions,” “multiple problems one solution,” and “one problem multiple changes.” (Cai, 1999, 2007)

The selected data, are analyzed both quantitatively, through the analysis of the protocol, and qualitatively with single case studies. For the quantitative analysis I used the software for inferential statistic Chic 3.0 (Classification Hiérarchique Implicative et Cohésitive).

Trough this quantitative analysis of the collected data, in the presentation of the article, the proposed game will be exanimate in relationship to the results underlined previously in other relevant works conducted in multicultural milieu with Chinese student of different ages; research work realized within the GRIM and coordinated by the Professor F. Spagnolo.

In this sense, the game of *Sudoku/Magic Box* seems to confirm, even though it is a first approximation, results previously discussed in other research works: compared to Italian students, Chinese pupils present a different kind of reasoning in the following items: problem reading data, data organization, “type” of language used to describe the solution...

We can therefore consider the situation/game as a first good instrument of investigation. In particular, the collected data relative to the Chinese students, seem to confirm a concrete, pragmatic behaviour, already highlighted in the works of Chemla (2001) and Spagnolo (2002); behaviours strictly bound to procedural thinking, to algorithm through which students use each single case (each animal proposed in the game) not only as simple procedural description (each case as a particular problem) but also as a representative of all the possible series, connected through the construction of an algorithm; typical reasoning of construction adopted in the written Chinese language.

This kind of strategy is evident comparing videotapes with the data analysis; it does not appear analysing strategies adopted by Italians student.

As regards this aspect of our research there are other interesting aspects regarding the way to “read” the *box/matrix* and discuss it in its “solution”.

The most evident difference between Chinese and autochthonous argumentation is that Chinese students seem to use mainly a pragmatic way of reasoning. During the game it often happens that they try in deed to show the truthfulness of a particular assertion with a sketch or a particular “operation”. The Italian students instead used to justify the adopted strategy, a kind of “local reasoning”, with “theoretical” reference to the scheme of the situation. The chosen “theoretical” references, result to be more and more formally rigorous during the game.

From the analysis of the strategies adopted by the students (Italians and Chinese) in the resolution of the assignment, it is also possible to underline the ability of the Chinese children to read and therefore to interpret the *box/matrix* proposed in a holistic way, with a global vision of it.

They show therefore attention to the particularity of each single case, each single card image presented in the game, reading the table, in a unitary vision.

They underlined immediately, as first step of the game, what was important for the solution of the game, the essential elements of the situation that were the data meaningful for the problem.

Examples of question to Chinese students were:

... *We have one “non influence” animals that we can consider only at the end;*

... *We have animals that can be posed only in one part of the box.*

If the Italian children prefer a strategy based on attempts and errors, looking for first step, the single relationships among the various image cards (animals) in the game and working on the box for lines or columns and only after in parallel, through lines and columns; the Chinese student, maybe only because, as we just said, of the relationships that is possible to find between this kind of situation/game and some of the linguistic aspect of the Chinese written language (as I introduced previously we can refer to the type of writing and reading of an ideogram), underlines a more uninhibited attitude, working immediately in parallel on lines and columns and reading so the box in an unitary way.

Other interesting considerations can be driven from the videotaped classroom experience, in particular the interview to the two Chinese students. From this further qualitative analysis, evidently comes out how, in the two culture (Italian and Chinese), the meaning of the term “To think for cases” is interpreted. **Is it a behaviour connected to the arithmetical thought, to attempts and errors?** In this sense the proposed activities could allow critical and more careful reflections on the possible correlation between Chinese language, entirely “abstract” and with an “algebraic nature” (in the mathematical meaning) with a complex syntax, Chinese thought and mathematical reasoning (logical-argumentative problematic) adopted by students in class to solve a mathematical problem. In according infact with idea, the hypothesis, that exists a strong correlation among the Chinese language, at least written language, and the mathematical thought (Spagnolo 2002); this correlation involves also the behaviour of students in class when they solve a mathematical problem and to support Hok Wing and Bin (2002, 223-224) who sustain that the Chinese students furnish the tallest performances of the world in assignments that ask the application of mathematical abilities I could consider, as I said previously, this analyzed situation/problem as starting point for future more specific and deepest researches in this context. Particular attention in this sense will be turned, in the future developments of the research to the depended analysis of the *algebraic nature* of the written Chinese language and the correlation that it could have in the main study of the difficulties showed by the student (western and Chinese) of different grades in the passage from the Arithmetical thought to the Algebraic one, from “To think for cases” in the arithmetical acceptation to the final formalization; problems already well documented and discussed in literature when looking at Western research.

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