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The notion of promediation in the Calculus learning

Carlos Rondero Guerrero

Universidad Autónoma del Estado de Hidalgo (México)

rondero@uaeh.reduaeh.mx

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Carlos Rondero Guerrero
Universidad Autónoma del Estado de Hidalgo-México
rondero@uaeh.reduaeh.mx

Abstract

From the epistemological perspective the notion of promediation, considered like a germinal idea is analyzed where the different concepts from average are come off, like the arithmetic mean, harmonic and geometric among others. On the other hand it looks for to realize his epistemological rescue for the learning of the Calculation, for which it is necessary to consider some of his connections, which makes possible showing evidences of different historical moments where the notion evolves.

Of beginning the epistemological rescue was realized from the conceptual perspective of expressed Archimedes in its work "the Method", where the study of the balance, this sustained indeed in the notion of the physical equilibrium, and whose mathematization this given by means the promediation notion. Constructus epistemological fundamental where it are expressed the promediation is "the excess and the defect", which allows Archimedes to relate the mechanic to the mathematician (Torrija, 2002). It is pronounced in its study on the balance of the planes, the calculation of the center of gravity and the quadrature of the parabola, among others aspects (Dijksterhuis, 1987). Another sample of the procedures of Archimedes is given through the relation with the sums of powers trying to establish quadratures, results equivalent to the defined integrals (Edwards, 1979),

$$1+2+3+\cdots+n=\frac{n}{2}(n+1) \qquad \int_0^a x\,dx = \frac{a}{2}$$

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n}{6}(n+1)(2n+1) \qquad \int_0^a x^2\,dx = \frac{a^3}{2}$$

The same results can be obtained starting off of the extreme calls of Bernoulli:

$$\int n = \frac{n}{2}(n+1) = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$\int n^2 = \frac{n}{6}(n+1)(2n+1) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

When an equal partition becomes of the integration interval [0, a], and the area under the curve like an average of areas of rectangles calculates, taking as height the value from the function in the right end of each subinterval and the base the size of the same.

If in particular the function is had $y = f(x) = x^2$, defined in the interval $[0,1]$, when doing an equal partition of this interval of the form, 0/n, 1/n, 2/n,…, n/n, is had stops the $x_k = \frac{k}{n}$, corresponding value of the function is, $f(x_k) = \left(\frac{k}{n}\right)^2 = \frac{k^2}{n^2}$ and the size of each subinterval is $\Delta x_k = \frac{1}{n}$. The total area will be given approximately by,

$$A = \sum_{k=0}^{n} f(x_k)\Delta x_k = \sum_{k=0}^{n} \frac{k^2}{n^2} \cdot \frac{1}{n} = \frac{1}{n^3} \sum_{k=0}^{n} k^2$$

Of where considering the sum of Bernoulli $\int n^2$, it is had, $A = \frac{1}{n^3}\left[\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right]$

That is,

$$A = \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2},$$

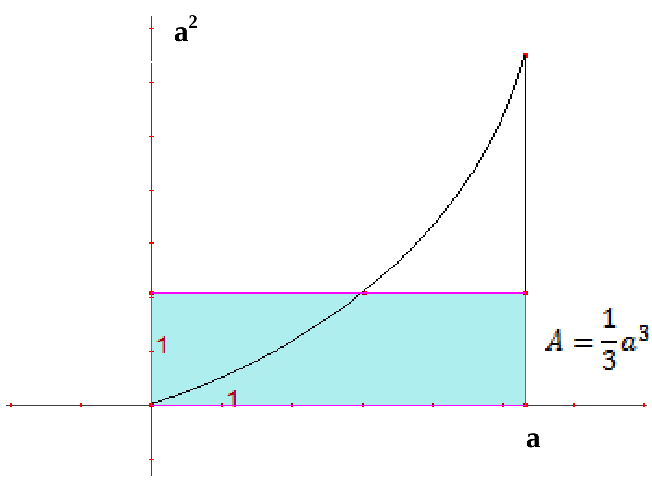
So that if the limit is taken when $n \to \infty$, it is obtained that the area $A = \frac{1}{3}$, that is to say,

$$\int_0^1 x^2 dx = \frac{1}{3}$$

On the other hand, in the case of the defined integral of the function $y = f(x) = x^2$, in the interval $[0, a]$, given by

$$\int_0^a x^2 dx = \frac{1}{3} a^3$$

Graphically it is possible to be represented like,



It is possible to identify the area under the curve $y = f(x) = x^2$, in the interval $[0,a]$, like the rectangle of equivalent area $A = \frac{1}{3}a^3 = \frac{1}{3}a.a^2$, that is to say, this rectangle has base *a* and height $\frac{1}{3}a^2$, that is indeed the height average.

These essential elements by all means have their later theoretical implications in the calls theorems of the average ones for derived and integrals from a continuous function *f(x).* Their statements are omitted.

$$\frac{f(b) - f(a)}{b - a} = f^{'}(c)$$

$$\int_a^b f(x)dx = (b - a)\overline{f}(c)$$

We can stand out that in the first case, the theorem affirms the existence of a value $c \in (a,b)$, of way like the instantaneous reason of change in this point is equal to the change reason average enters *a* and *b*.

Whereas in the second case, the theorem affirms that a value $c \in (a,b)$, in which the function has a value average $\overline{f}(c)$, given exists by, that is not other that the height average of function *f(x),* defined in the interval $[a,b]$.

Another epistemological fact important to stand out, is the referred one to a central result of the average age given by the school of Merton, is the one that talks about to the call theorem of the middle speed, (Edwards, 1979), where the distance that uniformly crosses an accelerated body is given by,

$$s = \frac{1}{2}(v_0 + v_f)t \text{ ,}$$

Where the speed uniforms is given by the average or the arithmetic mean of the initial speed v_0 and the end speed v_f, in the time interval *t*. Oresme, proved the Rule of Merton, with an associated geometric verification to the area of a trapeze. Looking that the central argument of this theorem is given by the promedial concept.

Conclusions

- Some of these excellent elements of epistemological character are to rescue themselves for their incorporation to the Didactics of the Calculus, in order to show how it is that the average concept is fundamental in the structuring of an argumentative speech that gives sustenance him to the Calculus learning.

- The promediation notion turns out to be fundamental in the construction of knowledge mathematical related to the Calculus.

- The Calculus learning requires the explicit incorporation of the excellent paper that plays the notion of promediation for its own Didactics.

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