

Paper for ICME11 TSG 27
Developing of teachers' professional knowledge of mathematics.
Historical, present and future perspectives.

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Abstract

Mathematical content knowledge and use of it in teaching could perhaps be called teachers' professional knowledge of mathematics. The importance of developing such knowledge continuously has become better realised due to changing conceptions of mathematics and increased research on mathematics learning. Feasibility of different models for this has not been discussed sufficiently and belief in RDD and dissemination is still well alive. The article analysis this situation and proposes different approaches based on research and carefully designed proposals for environments in schools, as well as trust in teachers as being professional if their environments allow it. The report shows studies and identifications from Iceland and a continuation of those as well as model development in Norway, which are still going on.

Keywords

Mathematics teaching, school development, holistic perspective, professionalism, models for analysis and development, teachers' contributions.

Introduction

Teachers' knowledge of mathematics, their use of mathematics in teaching, and what they have to know, do all pose fundamental questions for general education in mathematics. Not least are they essential when many countries face a serious lack of teachers with mathematics expert-knowledge and many well educated teachers, at least in Europe, are approaching retirement. However, agreeing on the importance and the seriousness does not say anything about directions or ways of approaching this field of problems nor choosing research strategies.

The following contribution does stress the importance of teachers' strengths no less than weaknesses. It is based on a conviction that the heavy responsibility, which societies give teachers, has to be paralleled with trust and respect. Teachers must be both trusted and challenged in being professional in caring for their students' learning, and being reflective and thoughtful in such processes. Also trusted to be able to develop lifelong in their profession,

being able to collaborate and coconstruct important knowledge within every school. They do however need considerable support for creating or strengthening an encouraging environment for this. To follow these statements I will introduce very briefly three connected research projects and following designs and models of development processes. The first started in Iceland before 1990 as an inquiry into how mathematics teachers could be challenged and supported to grow professionally within their natural working environment, that is their school. By growing professionally as mathematics teachers referred to growing in their understanding of mathematics and mathematics' interaction with other subjects, as well as growing in their capacity to create learning opportunity in mathematics for their students. Through several studies this stretched over more than a decade (Kristjánsdóttir, 2003b). The question was then reopened from a different perspective in one of the larger cities in Norway in 2002 (Kristjánsdóttir, 2005) continuing in the period 2006-2008 in collaboration between the University of Agder, Nord-Trøndelag University College and Vestfold University College in the network *Holistic Perspective on Mathematics Learning* (Kristjánsdóttir, in progress 2008). These connected research projects will be introduced as space permits in this paper.

To clarify background and inquiries it is necessary to offer some historical insight and analysis of what can be considered important to care for and how to avoid holding contradictory expectations to teachers or having controversial ideas about their roles and responsibilities.

Teachers' frequent meeting with proposals for changes

Practically all mathematics teachers have experienced, or read about, a row of attempts from authorities for initiating changemaking in their work. Starting with the 'new math' in 1960s where they were informed by mathematicians and mathematics educators that there was time for new content and new teaching strategies to improve learning of mathematics (Kilpatrick, 1997). This first massive initiative set the scene or modelled methods for attempting to make teachers change their mathematics teaching. Even though the 'new math' faded away, the model of educating teachers in times of change, was established to stay. People questioned the new math, however not questioning the model for influencing teaching on a large scale.

This model was the RDD or research-development-dissemination, which has frequently shaped curricular reforms in most countries. It counts on 'experts' to prepare new curriculum, not always being knowledgeable about the school situation within the country. The curriculum

is then implemented in particularly chosen schools with engaged teachers who react on and develop the material further. And later the curriculum and material are extended, through a so called dissemination, to all schools. Howson, Keitel & Kilpatrick (1981) discussed early the origin of RDD's underlying assumptions and consequences, as well as pointing out alternative perspectives. However RDD has survived to modern times and is often tacitly assumed to be the only model possible for large systems such as general education.

For many reasons, such as the lack of success with the 'back to basics' movement, which followed 'the new math', and Erlwanger's article (1973) on Benny's conceptions, the attention in innovative and serious studies focused in the following years more on students than teachers (Carpenter & Moser 1984). This started to change after the mid 1980s with teachers and teaching becoming a clearer focus of attention. At this time researchers knew that they had important insights into children's mathematics learning which deserved to be shared with teachers (Hiebert & Carpenter, 1992). Instead of approaching them in a semi-alien way (content and teaching strategies both new to them), the approach was now different, literally sharing with teachers insights into children's solving of mathematical problems, innovatively and through reasoning explicitly. What grasped the participating teachers' attention was that children could solve much more complicated problems in mathematics (number operations) than the teachers who had long experience in teaching this, had ever offered their students. And how capable they were in explaining their own thinking about the mathematical problems and own process in solving them. The teachers having grown into habits of teaching how to calculate, and possibly never questioned their way of dealing with such problems, actually saw kids performing and reasoning in a way they had never imagined. This was powerful for the teachers. In an interview Fennema describes how the researchers realised that making such knowledge reach teachers at large would not succeed through dissemination, different strategies had to be used (Foster, 2000).

Research in classrooms

Since the late 1980s classroom research in mathematics has increased with researchers and teachers working "side by side" poetically speaking, although that is not really a good phrase. Researchers and teachers have very different "gains" in mind, very different entrances to a carefully listening audience, and very different affordances for a sustaining developing as an inseparable part of their job. And in spite of a wealth of theoretically based publications as well as numerous attempts to reach teachers through their own journals, webs, and in-service

courses, the majority of mathematics lessons in most countries do not show signs of reflect such a knowledge being held by the teachers. (Begg, Davis & Bramald 2003; Lester & Wiliam, 2002). However many exceptions can be found and Nelson (1997) described changing views on mathematics learning, and that teachers could already then:

- Start looking at own students as intellectually generative, and being capable of creating questions, developing solutions to problems and constructing theories and knowledge, not as vessels to be filled.
- Start realizing that teaching can be based on development of the students' own thinking instead of being transmitted through teaching material.
- Redefine the intellectual authority in the classroom and move it from the teacher and the book to a qualitative reasoning process in the class, where both teacher and the students participate.
- Start seeing that teacher and students can use mathematical way of reasoning to generate and validate mathematical knowledge.

Such changes are not only indicative of changes in perceptions of learning and teaching, but also of changes in perception of mathematics as a discipline and school subject. The time has passed when this was a question of existence, of what was possible. However still, ten years later, it is a question how teachers meet such statements, how they make them theirs and how they confirm it in their own classroom, with their own students, as well as in their own schools with their own colleagues and principals. How do they inform parents? How do they come to say confidently that this is true and that they can prove it with their own students?

Researchers doing classroom research as "watchers" or "videomakers" tend to minimize the problem of, as they say, the 'disseminating' of results from their studies to a large number of teachers. Or they, sometimes tacitly, believe that the teachers, who generously opened their classrooms to them, will easily inform their own colleagues in the schools in a powerful way. The RDD model is still strong, also in a researcher's mind.

Powerful phenomena interfering with mathematics teachers' development

Two very powerful phenomena have, during the last two decades, complicated the situation for of teachers' growth in mathematical thinking for teaching and their developing of appropriate roles for a more demanding and challenging learning environment. These are both largely initiated, or at least greatly influenced, by powers outside the educational arena.

Both are characteristics of modern societies in a global world. This is the rapid development of ICT and the marketdirected social situation.

In a sense it is possible to say that both have prolonged the existence of the RDD model as can be seen in authorities' approaches to schools. Furthermore the traditional way of focusing on individual teachers, simply counting on conscious and engaged teachers to come to courses, and then be tacitly expected to 'move the mountains' in their own schools, frequently without their principals even knowing about it, not to speak of supporting the teachers before, during and after the courses. In such cases a well known pattern repeats itself all over again, as explained by a teacher in an interview. "We go to the course, and listen, and then we choose what fits us in our own class."

The RDD model has in fact nothing to do with the essence of ICT, although it has been used there. It is, however, considered efficient for acting and acting fast. And ICT is a field where authorities want to move fast. Questioning further effects of the model and the underlying values, is then not the highest priority, or priority at all.

In some places, like the Nordic countries, the market driven evolution has only recently begun to influence the school system. The influence can be seen in planning, choices in administrating financial resources and a row of evaluating activities or control. Mathematics tends to be clearly mentioned in such demands, although mostly not going into an essential discussion of directions or goals for mathematics learning. If this situation is not treated carefully such efforts can be controversial to teachers' developing of of more supportive and challenging learning environments for mathematics. Particularly naming bureaucratization of evaluation, instead of increasing evaluation for learning (Utdanningsforbundet, 2007).

Turning the attention to research in Iceland and Norway

Although very slowly the profession of mathematics teaching has been changing. On the other hand important characteristics of quality of modern time have followed the profession for a very long time. These are essential in preserving a professionalism and to fight against modern tendency of educational authorities to give instrumentalistic prescriptions to teachers.

I choose to describe this by what I call *The teacher's spirals of developmen* (Figure 1).

Indicates that a teacher is developing simultaneously at three levels, where the foci are: the individual student, the particular class, the teacher her-/himself. The care for the individual

student's learning of mathematics includes intentions for that particular student, furthermore the students process of learning, and the evaluation. Black et al. (2003) give fine examples of teachers being able to create activities to use in evaluation **for** learning. The evaluation then calls on a revision or improvement of the teacher's intention for each student etc. In the case of some students this is explicit by the teacher, in other cases more tacit.

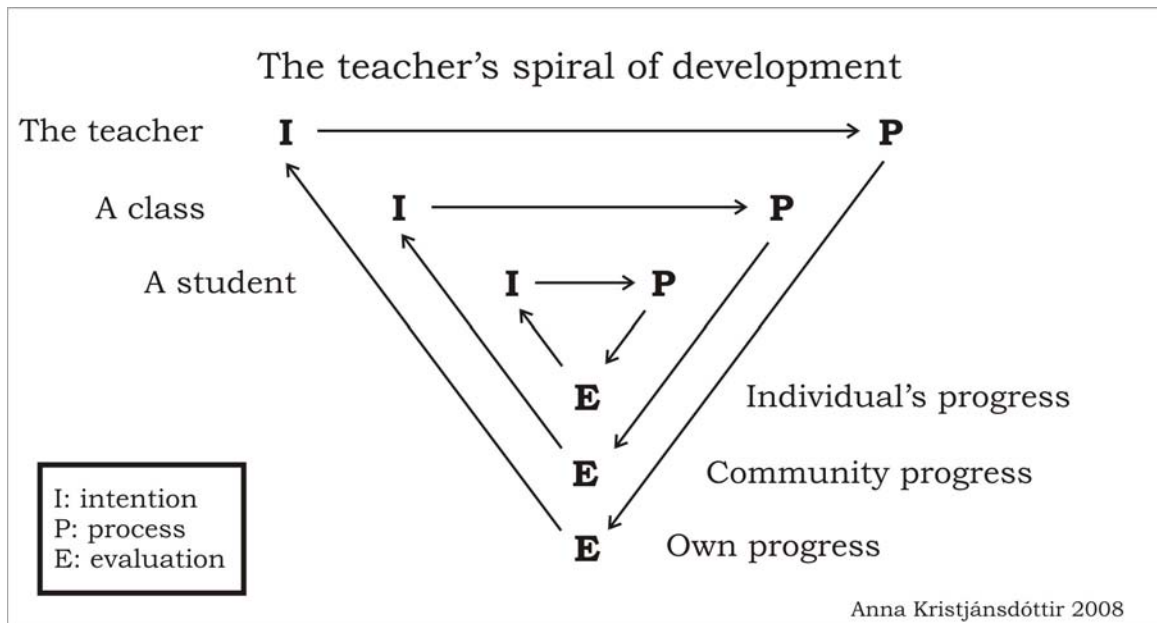


Figure 1

The teacher also holds intentions for for the student's class, as a learning community. The intentions may be clear or vague, the process is visible and sometimes in traditional routine, in other cases more exploratory. The evaluation of the class as a learning community of meaning making and mathematizing is not often made explicit and reported on. Rather average scores and behavioural characteristics of the students are made explicit. However clear visible intention, process description and evaluation of the learning community and it's development as such are important part of the teacher's spirals of development as well as being most important for the students. Wenger (1998) claims that the student's identity develops through engaging in community with others, in efforts to make meaning of the mathematics they are studying. And Solomon (2007) informs how students, even strong in mathematics, can develop lack of interest in the subject because the learning community they experience makes them feel excluded by norms in discourse and attitude. Here researchers have a job to do with teachers in identifying directions and steps in their community towards a more explicit sharing of the developing of mathematical communities of students.

The third level concerns the developing spiral of the teacher him-/herself. There have always been teachers who set goals for themselves in their teaching, independent of singular classes which they work with. They decide that something they have read about, learned from others about or discussed with colleagues, is so important to them that they want to integrate it into own teaching. Such a decision may be known to colleagues or not. They sometimes succeed, on the other hand sometimes not thoughtful enough about what they are trying to encompass. In such cases they may not be successful. The reason can at times be traced to simplified ways of introducing new strategies or content to teachers, more like a selling of ideas than exploring them with teachers. The teacher may then not have adapted ideas thoughtfully, in particular the mathematics which is involved, nor rethought former conceptions. If that is the case she/he is unguarded against meeting problems that may arise in the students' functioning in an unknown environment or tackling unknown problems. And the teacher then unable to react on such problems within a path that the innovation might offer.

The teacher's spirals of development refer to individual teachers. However all levels gain from this development being a matter of a concerned collegial discourse. All levels gain from being explicit and well recorded as a part of joint constructive effort within each school.

To the research carried out in Iceland and later continued in Norway.

Although changes in mathematics teaching have been more thoroughly analysed and reported on through research during the the last 15 years, many indications of the changes to come could be 'sensed' as approaching much earlier. Results of research on children's solving mathematical problems in the early 1980s and a variety of investigations into alternative teaching of mathematics, in particular in United Kingdom (Cockcroft, 1982).

Late in the 1980s the RDD model was beginning to show serious lacks of sustainability and qualitative professional development for teachers. At best the effect was minimal, at worst teachers were frustrated and sensed controversy. (Sigurgeirsson, 1987). Individual teachers did not have sufficient strength to undertake changes, although they had evaluated them as interesting, nor did they have any power within their schools to engage other teachers with them on a necessary substantial scale of participation and time.

ICELAND

The author started in late 1980s her inquiry into a sustainability in mathematics teachers' learning process as part of their profession. Which were their main, possibly unnoticed, obstacles? What sort of security and challenge would suit their circumstances in the schools? Were there any unused affordances for a continuous learning process? Which role would a much closer collaboration of teachers play? It should be noticed that these inquiries were not only educational, they had much to with the mathematics, as content, strategies and culture.

As a beginning of a design process my question was how to help lowering the threshold to innovating confidently. The first attempt was creating an environment where teachers could explore their own learning of mathematics within areas or with tools which they did not know, this being done parallel to their exploring childrens' working along the same tracks. Around 20 experienced primary mathematics teachers and a large group of voluntary children aged 6-13 participated in the first course 1988. The teachers worked in groups in exploring mathematics outdoors, on computers, as collaborative solving on problems etc. and also with their group of children, choosing activities for them, making draft plans, refining them and discussing with the course's leader each 3 hours session they had had with the children. They could dwell with individual children, go on and watch other groups as well as each other's communication with the children on mathematical issues. Something, which they had never experienced in their deacades of teaching. This turned out to be powerful for the teachers' mathematical awareness and thinking, as well as acknowledged by them as increasing considerably personal strenght for continuing with their own students. The teachers strongly wished to establish a network for the year to come under guidance of the leader, to be able to share and inspire each other, something that had not been planned but was made possible at their request. On their own initiative several of the teachers presented in journals or presentations what they had experienced and learned, thus showing additional type of confidence. This model for mathematics courses continued for about a decade, once with a group of Swedish teachers together with Icelandic teachers and children.

The next steps in the design-based inquiry had two differnt intentions and directions, both important. The first one of helping to create a sustainable platform for possible networks of mathematics teachers across schools. This meant taking the initiative towards establishing a mathematics teachers association and forming it with the teachers for the first years. The other to start identifying affordances for developmental work on mathematics within schools.

Such projects started in 1994 and several models were tried out during the coming years, each lasting from 1 year to 2 ½ year most frequently with several schools participating in the same project.. The principals were from the beginning considered a natural part of such projects and were members of the core group from most of the schools. Their interpretation of their role varied and so did the teacher´s interpretation of their principals participation. But was clearly stated that the principals were not there as knowledgeable about mathematis and mathematics learing. They participated at leaders of the schools pedagogical policy and as holding the administrative power. If a school had an ICT person, she/he participated. So far the focus had been on offering teachers tacitly a support which could be identified that they needed, however had not asked for or declared that was missing. The courses with children were not to teach the teachers how to teach children mathematics, they created an environment to grow together and stretch their own boundaries. That was highly valued.

NORWAY

The opportunity to widen the scope of inquiries into challenging and supportive environments for teachers' understanding of mathematics and mathematics learning, opened for the author in Norway in 2002. One of the largest cities there had decided on a competency raising project for mathematics teaching and learning (about 40 schools, primary and lower secondary). We did as researchers at the university college not want to enter the programme the community had decided as that had already been planned, but offered to analyse through questionnaires on net some of the conceptions held by teachers, students and parents on mathematics and mathematics learning (Kristjánsdóttir, 2003a). This was to facilitate understanding the schools' situations and both affordances and obstacles there could be for the community project. Similar questions had been created by the author several times in Iceland. However all suggestions of questions were presented for a careful scrutiny in this new environment and culture. The questions were both multiple choice and open.

Analysing the teachers' replies gave many indications which were important for a project trying to raise the competencies, expecting the schools and teachers to take it seriously.

- Some teachers had participated in in-service courses in mathematics, but within each school only few, down to one, in each type. The courses were mostly very short.
- Some teachers had carried out project work in mathematics, but rarely more than two or three mentioned the same kind of project withing each of the schools.

- In naming resources they used for trying new ways in their teaching of mathematics, the Norwegian mathematics teachers journal and the Norwegian mathematics teachers' association were hardly mentioned, although both are of high quality.
- Being asked about attitude towards collaboration of mathematics teachers within their school, they were mostly positive or very positive.
- Asked about their ideas of quality in mathematics teaching and the project for raising competency to work, many replied optimistically and named important issues.

Instead of judging this result as indicating lack of knowledge, I focused on the knowledge, experience, ideas and hopes the teachers' replies indicated. However it was clear that professional environments for developing teaching expertise in mathematics was almost non-existent in the schools and the knowledge, which was considerable, was fragmented and private. It was clear that courses for individual teachers would not help in this respect.

By studying the teachers replies it seemed unlikely that a written report would be influential, neither would a oral presentations do. My suggestion was to present the results through action, inviting schools to include most of the mathematics teachers in a developmental process, trying out innovative activites in class and reading and discussing. This meant that each school would be united in collaboration with all/most teachers of mathematics and other teachers welcome. It furthermore meant that the courses, which we designed particularly to present a model of school development in mathematics, be led by the principal as a pedagogical leader and administrator in collaboration with a professional advisor on school development in mathematics. Figure 2 shows the model that was designed for this.

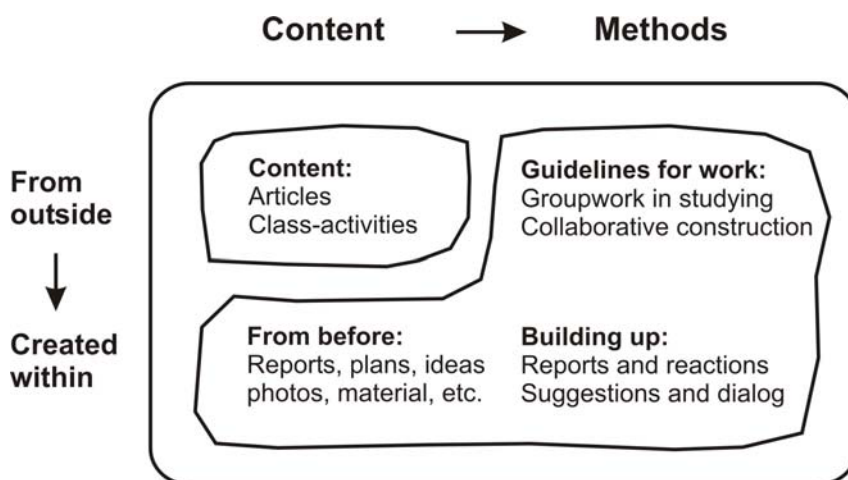


Figure 2 Model for School development in mathematics (Kristjánsdóttir, 2004)

The authors role was designing the model and making or choosing with colleagues the material to be used by the teachers. Also running a course, 2-3 times 3 hours per term over 2 years. As there were no mathematics educators available with experience in working with schools, normal teachers from schools were replacing such professionals. That was the only choice available for “mediating the knowing of the situations in schools and offering guidance. As an aim to reach high this was not a good choice, however as a part of identifying processes of affordance as well as with obstacles, even serious misinterpretation, this was a generous possibility. As the community leaders had not originally planned a collaboration with the university college, one can say that this was in most ways a winning situation for analysing for the present situation and future. It should be noted particularly that teachers' reading together and writing together is a central part of the model. The question in teachers' professional development is not whether they read, but what and how. Similarly there is not a question if they, as professionals, write or not, only why, how and what. Naturally this has to grow through action, it cannot be explained in words only. The value has to grow with the teachers seeing the importance of growing and constructing together.

There were many aspects which the teachers in the first project in Norway were pleased with, such as the reading of articles and discussion, as well as many of the particularly chosen activities for classes. They were also pleased to have the "course" placed in their own school and stretching over 8-10 weeks with 2 hours extra work per week, although they many times complained that the time that was reserved for this was sometimes taken by the principal for unexpected meetings. The model for school development in mathematics does focus on building up within the schools a sustaining learning environment for teachers, not for a period but as an integrated way of working. Frameworks, roles, routines and use of human resources are all important factors which cannot be discussed further here because of too limited space. However Gellert (2007) does particularly mention how hard routines are to change. Some aspects turned out to be hard to understand the importance of, both for many teachers and their superiors in schools and educational leadership within the community. This was not unexpected, however had to be tried out and identified in reality. For example that school development in mathematics does not happen:

- if only few of the teachers participate
- if the principal is not one of the leaders
- if the teachers only share their experience orally, without writing

- if the teachers cannot identify former material of quality to put into their base
- if the teachers do not construct together and consciously develop further
- if the teachers do not value each others' contribution and act constructively on them

Holistic Perspective on Mathematics Learning

The concept of school development in mathematics is not only important for the teachers in schools, it is also vital for a continuing learning process of beginning teachers in schools. A second round with questionnaires and courses to introduce the model for school development in mathematics, took place in a collaboration of the University of Agder, Nord-Trøndelag University College and Vestfold University College in the period 2006-2008. This time the professional advisors were experienced college teachers of mathematics with pedagogical knowledge and having worked with teachers although not on a school development. The analysis of difficulties in understanding of the concepts in the first project were treated openly in this collaboration and we have managed to increase considerably our understanding of necessary aspects in the roles of both principals and professional advisors. We have also developed further the themes which we have used to introduce the model of school development. They are linking mathematics learning to constructivism, ICT and communication (3 different courses).

Having at this time collaborators in working with the schools of higher quality it was possible to initiate with them a larger picture than a growth within individual schools. The model showing Holistic Perspective on Mathematics Learning was created, see figure 3. It has a three levels, on each one role is in focus, another officially supportive and the third undervalued and frequently treated as an outsider in the case of collaborative efforts to improve the learning and teaching of mathematics. These are the parents on level 1, professional advisors for school development in mathematics on level 2 and last but not least the reserachers on level 3. To clarify the last statement, it does not say that researchers do not try to influence and make their results in research known, they however mostly address own colleagues and secondly teachers. The last one has shown to have very little effect (see ref. on page 3), and in fact they ought to know that individual teachers coming to their presentations and courses do neither have the facilities nor the power to make any real change in their schools. The holistic perspective is drawn up to help to identify lacs, misinterpretations and controversial opinions and actions.

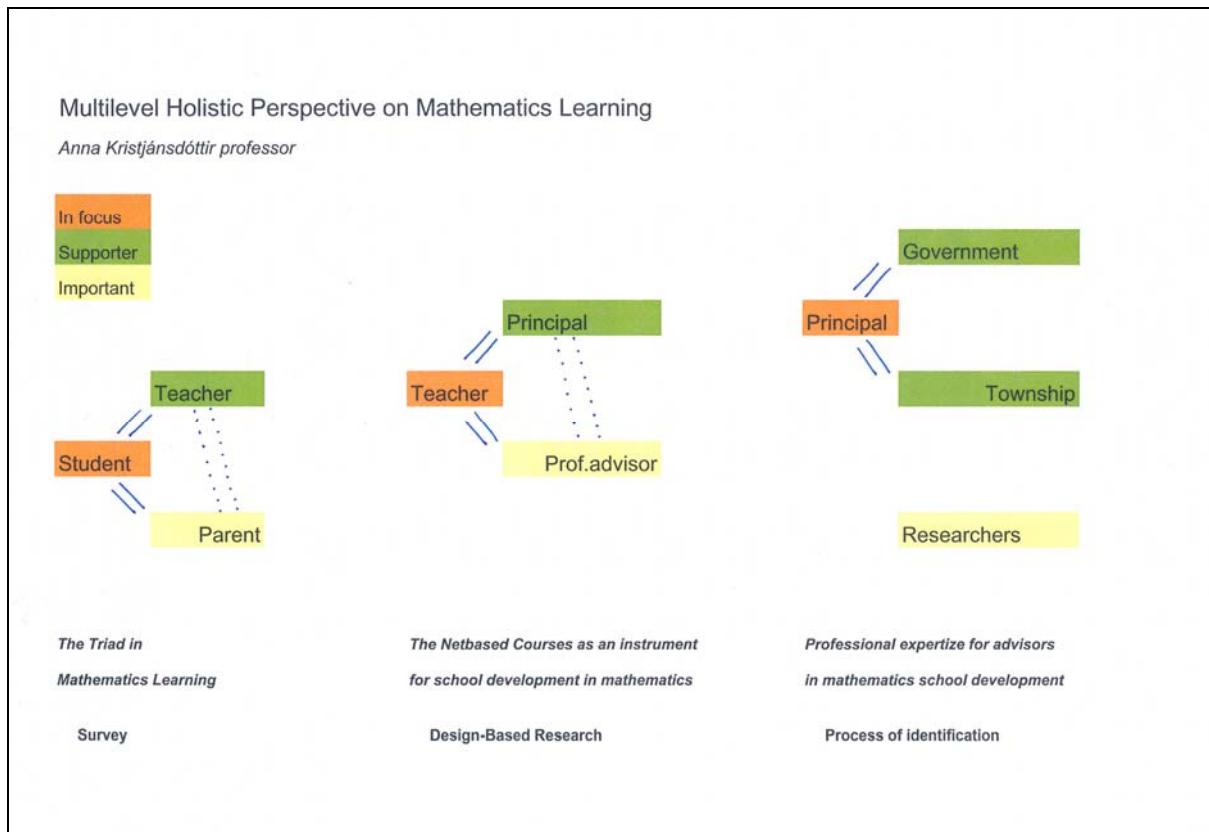


Figure 3

In the spring 2008 we have interviewed almost all the principals involved in 2006-2008 as well as some teachers from each school. Their understanding of the model for school development in mathematics, and their description of how they gradually came to understand this better and better through watching the teachers and their collaborative efforts, has been very generous to our understanding as researchers. It may have been supportive that the Norwegian educational authorities have years after the model was launched within our project, increased their document publishing on the importance of school development generally and the principals' particular role for ensuring a lasting effect. Certainly the school development concept in most of these documents is not initiated from a subject's perspective. However the principals we have interviewed valued this kind of approach and saw indeed that the model could be used for other subjects where a collaborative platform is needed.

Conclusions and finale.

"TSG 27 is concerned with teacher content knowledge and its relationship to teaching skill. Including similarity and differences between mathematical knowledge for [professional]

teachers and professional mathematicians. The nature of mathematical knowledge in different contexts is important and the function of the mathematical knowledge in each case."

There has been no comparison in this paper. This means that the author suggests that a professional mathematical knowledge of teachers should perhaps not only be compared with the professional knowledge of mathematicians, although that is highly important.

Our deeper research questions have been shaped by teachers' replies to questionnaires, and taking their replies seriously. The choice of content and ways of building up a discourse in this paper, is based on the conviction that we cannot identify mathematics content knowledge of teachers, and what might be important/necessary there without involving teachers to participate in such a study. That is why the building up environments within the individual schools, to practice professionalism, is so strongly underlined in this text. That is here considered a fundamental step and entrance.

Support for the reported research projects in Norway has generously been given by The Competency Fond of the Southern Part of Norway 2002-2007, and The Norwegian Research Board 2004-2008 [project: Mathematics, learning and teacher competency]. Our gratitude goes to them and the teachers, students and parents who have helped us to understand their perspectives and actions.

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