Getting Others' Perspectives through the Hermeneutic Effort; A Theory of Understanding for Planning the Problem Solving Teaching Approach

Masami Isoda
University of Tsukuba

This paper proposes the importance of hermeneutic effort for humanizing mathematics education from the viewpoint of the subjective-to-objective for proposing the teaching approach for implementing humanization as a kind of problem solving teaching approach in Japan. Chapter 1 (Part 1) describes the hermeneutic effort. In chapter 2 (Part 1), the hermeneutic effort is illustrated by the case of dialectic in classroom.

These chapters are introductory chapters for explaining the theoretical base of a theory of understanding for planning the problem solving teaching approach in Part 2 which was omitted here. Part 1 is used for developing the local teaching theory for planning the problem solving approach in Part 2 which was developed through the adaptation of procedural and conceptual knowledge to the teaching practice, and integrate it from the viewpoint of hermeneutic effort.

Those works were originally written and published in Japanese with rich examples and case studies from 1991. Five books have been published in Japanese on this local teaching theory. Those works are being prepared for the publication in English as for the local teaching theory on the lesson study movements in the world.

Bibliography in Japanese

For Part 1

Isoda, M. (2002). Hermeneutics for Humanizing Mathematics Education, Tsukuba Journal of Educational Study in Mathematics. University of Tsukuba, 23. 1-10

Isoda, M. (1994). Logic of Argumentation, Hokkaido University of Education edited, Education and Laguage, Tokyo Syoseki.

For Part 2 in relation to Concept and Procedure, and Local Theory for Understanding

Isoda, M. edited (1996). Lesson Study for Problem Solving Approach at Elementary School Mathematics, Tokyo:Meijitosho.

Isoda, M. et al edited (1999). Lesson Study for Problem Solving Approach at Middle School Mathematics. Tokyo:Meijitosho.

Isoda, M. et al edited (2005). Lesson Study Aimed for Understanding at Elementary School Mathematics. Tokyo:Meijitosho.

Isoda, M. et al edited (2008). Lesson Study Aimed for Argumentation at Middle School Mathematics, Tokyo:Meijitosho.

Isoda, M. et al edited (2008). Lesson Study Aimed for Argumentation at Middle School Mathematics, Tokyo:Meijitosho.

Part I. Hermeneutic Effort in Mathematics Classroom; Getting Others' Perspectives

Chap. 1 Humanizing Mathematics Education with Hermeneutic Effort

Mathematical activity which was a discussed topic with regard to educational content in the early 20th century, became a taught subject in Japan during WWII, and throughout the world during modernization (see, Freudenthal: 1968, Kline; 1973 and Wheeler 1975). After modernization, it has been more emphasized in other words in curriculum documents, for instance mathematical reasoning and mathematical communication for characterizing the meaning of mathematic and humanization, and characterizing students activities. In mathematics education research, constructivism is an epistemology for knowing, understanding, and modeling what human activity is.

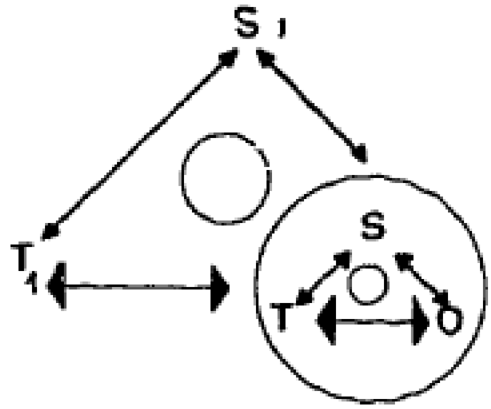
Hermeneutics is a general theory of interpretation by human view for all sciences from art and literature to natural sciences, and one of the ways to characterize the humanity with subjective understanding to others. Because it is a fundamental theory of the methodology of qualitative studies, many of mathematics education researches are deeply related to it or implicitly applying it. On the other hand, there are not so many researches that explicitly applied it for mathematics education. In this paper the author uses the word of hermeneutic effort with the special meaning. In this chapter, hermeneutic effort is defined and applied for analyzing mathematical activity and enabling to explain it more subjective perspective for humanizing mathematics education.

1.1 Hermeneutics in Mathematics Education Research

In mathematics education researches, major resources which proposed hermeneutics were Jahnke, H. N. (1994) and Brown, T. (1997). Brown clearly described its historical development and focused on current theoretical issues in relation to language for developing research methodology. Jahnke introduced hermeneutic effort as an activity of the recursive process of interpretation for explaining the role of history for education as follows.

> My principal thesis is that the concept of "hermeneutics" is suitable to describe the pedagogical interaction between synchronous and diachronous culture. ……My thesis implies the claim that the historian's perspective represents an important element of an appropriate teaching culture. Seen under the aspect of method, history of mathematics, like any history, is essentially a hermeneutic effort. Theories and their creators are interpreted, and the interpreter is always aware of the hypothetical, even intuitive character of his interpretation,. Interpretation itself

takes place within a circular process of forming hypotheses and checking them against the text given, in the case of history of science, the objects of his process of interpretation, the scientific subjects (individuals and groups), are again hermeneuticians who interpret fields of objects. Of course, this view of scientific work will be adequate with varying precision in different times and different fields. But if we do not understand it too narrowly, this description can very well be advanced. Scientific interpretation, too, is now subject to the circular process of forming hypotheses, testing and revisiting them. He who is concerned with history, thus, has to do with a complex network of interpreters, problem fields, and interpretations (theories) which I have represented in a little diagram and which I should like to name the "twofold circle". This diagram consists of: Primary circle in the right bottom representing the circular relation between a scientist(s), a theory (T) and a field of objects (O) and a large secondary circle representing the historian (S1), a historical interpretation (T1) and the primary circle as his field of objects. My proposition that the teacher should know and understand something about the historian's perspective if he takes history of mathematics into the classroom, refers precisely to the problem that he/she must be aware of this twofold circle and able to move within it. Only this will enable him and his students to acquire a certain freedom against the subject matter to form hypotheses and to be ready <u>to think oneself into other persons who have lived in another time and another culture</u>. For me, this thinking oneself into another person and into a different world seems to be the core of an educational philosophy, providing a basis for historical contents in mathematics teaching.



(Jahnke, 1994: 154-155)

For clarifying human activities from the view point of hermeneutics, here we introduced Jahnke's perspectives which focused on hermeneutic effort which characterize trying to get others' perspectives. At the same time, for clarifying the meaning of hermeneutics within the diversity of the meaning of hermeneutics, here, we introduce the word of the hermeneutic effort.

1.2 The Hermeneutic Effort for Clearifing Humanization

Based on various theories of hermeneutics and examples of interpretation (e.g., Isoda et al. 2000, Isoda/Tsuchida 2001), Isoda characterized hermeneutics effort as an activity according to four principles: "Understanding," "Getting others' perspectives (the assumption of the positions of others / imaging others' minds)," "Instruction from experience (self-understanding)," and "The hermeneutic circle." "Understanding" is one's interpretation regarding a text or other object. "Getting others' perspectives (the assumption of the positions of others)" means that the appropriate interpretations of a text is only possible through a subjective approach whereby we assume the writer's (or speaker's) position, feelings and sympathetically attempt to put ourselves into the position of another (writer or speaker). "Instruction from experience (self-understanding)" means that when one interprets assuming the position of another, one's own subjective opinion (at times, one's preconceived opinion) is reflected – in other words, one obtains an understanding (instruction) about one's self with comparison of others' perspectives. "The hermeneutic circle" refers to the cycle of hypothetical interpretation and confirmation from further readings such as textual interpretation whereby understanding the particulars contributes to the whole, and overall understanding contributes to understanding the particulars, but broadly refers to the fact that a recursive or multi-layered advance in interpretation leads to a more objective interpretation: if we have some understanding, we apply it to new situations and if it is applicable, it will become more objectively correct.

In particular, "getting others' perspectives (the assumption of the positions of others)" and "instruction from experience (self-understanding)" are acts subjectively carried out through the empathy of the interpreter toward the object of understanding; accordingly the objectivity of interpretation can be stipulated in the subjectively shared act of empathy. Through this, one can see man attempting to recognize mankind as an existence able to think from another's perspective – an existence equipped with a nature that empathizes with others – and comprehend human acts by such human subjectivity.

Those four principles for characterizing the hermeneutic effort were identified by the author from the referencing of various hermeneutic theories and reflecting on personal interpretations of mathematics history and a hundred classroom experiments which enable students to do hermeneutic efforts in mathematics on historical subject matter;

see classroom experiments on http://math-info.criced.tsukuba.ac.jp/Forall/project/history/

Here, four principles are illustrated at first and then, confirmed with some references.

1.3 An Example for illustrating four principles of hermeneutic effort

For illustrating the four principle of hermeneutic effort, here we read and interpret the classroom communication between Japan and Australian high school through the Internet (Isoda, McCrae, Stacey 2006). The theme was to determine the attributes of the sums of consecutive numbers (See figure 1).

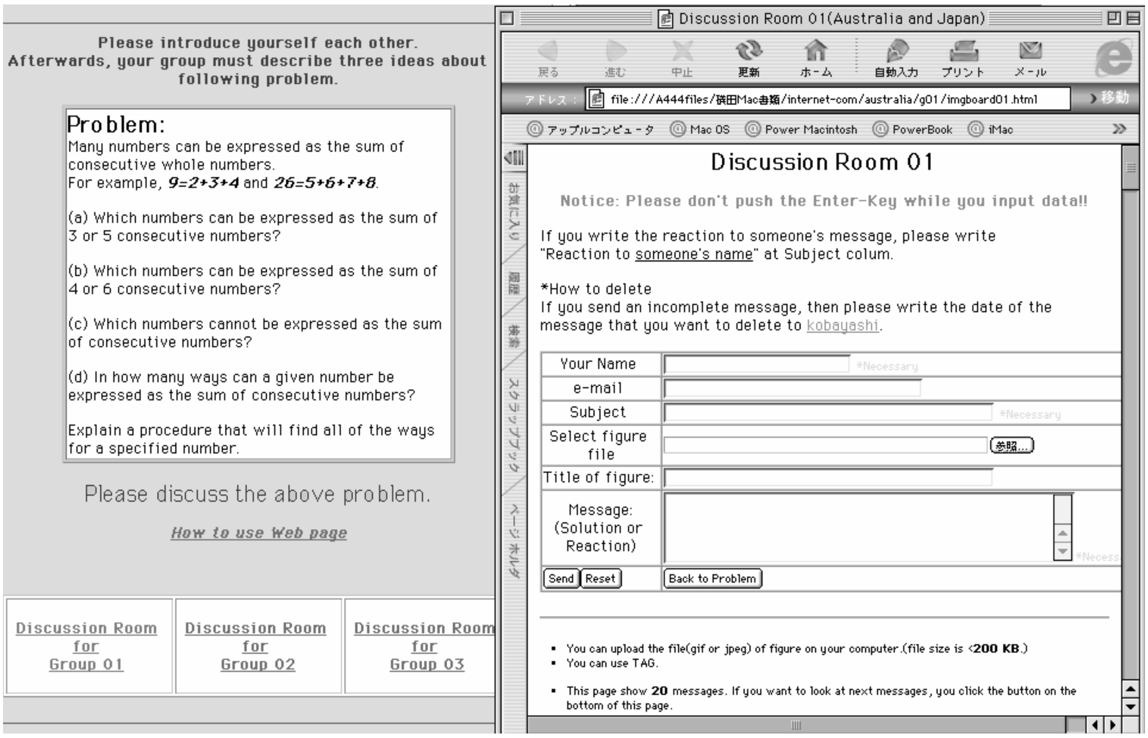


Figure 1. A Problem and Discussion Room

Internet Project name :P P [1999/10/26,08:51:49]

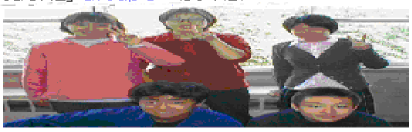
(A) We have four members in our group. They are P P , E C , R t R and D d T . We are all between 14 and 15. We go to S in Melbourne, Australia. We all play *(OMISSION)*

(B) For part (a):
Let x be the first number, y be the second number and z be the third number in the consecutive sequence. The first sequence is 1,2,3 which adds to 6. The second sequence os 2,3,4 which adds to 9. The third possible values of x,y,z are 3,4,5 which add to 12. This is because the first sequence (1,2,3) can be represented by x,y,z. The second sequence will be (x+1),(y+1),(z+1) which is the same as x+y+z+3. The third sequence is (x+2),(y+2),(z+2), adding to x+y+z+3+3. The next equals x+y+z+3+3+3, and so on, adding 3 each time. So, starting at 6, they all go up in multiples of 3 when represented by consecutive numbers. The same method applies for adding with 5 consecutive numbers – it goes up in multiples of 5.

(C) Please reply to our suggestion for the answer of (a) and suggest spmething for part (b).

[FIGURE] Group 3 –(58 KB)

(D)



[SUBJECT] **We are Japanese**
[NAME] R K [1999/10/26,14:06:06]

[MESSAGE] Hello. Our group has 5people. *(OMISSION)*

Internet Project name :P P [1999/10/27,08:26:45]

(E) P P is the Vice president of the *(OMISSION)* be a lawyer or a business man and R want to be rich and possibly a scientist or an engineer. Please reply to our suggestion for part (a) in our last letter, we look forward to hearing from you. :) :) :) :)

□ Reacton and our idea to solve (a) ,(b) name :Y K [1999/10/30,22:20:51]

(F) #As you know, we are not so good at writing English. So please let us know if you don't understand.

(OMISSION)

We read your message. The answer is same as ours. But we solve it in a differnt way. I think this way is easier than yours. You used three letters. But to use only X is easier. I'll show you our way.

---- part (a) -------

(G) #In this problem, we have to think 3 consecutive numbers and 5 consecutive numbers separately.

< 3 consecutive numbers >

Let the first number be X. As three numbers are consecutive, the next number must be (X+1).
In the same way, the last number must be (X+2). So the sum of these 3 numbers is...
X+(X+1)+(X+2)=3X+3=3(X+1)
X will be natural number. (It can be taken for only integer which includes negative numbers.)

(OMISSION)

(H) Gentlemen and Gentlemen! (You are only boys) I hope you will understand the meaning of this expression.
Actually, when X=2 3(X+1)=9, when *(OMISSION)* Anyway the answer is <u>multiple of 3 bigger than or equal to 6</u>

< 5 consecetive numbers >

(OMISSION)

(I) #Question from us (1)
About "A" consecetive numbers. When "A" is an odd number, you can express the sum as multiple of "A".
When "A" is an even number, you can't express the sum as multiple of "A". Can you tell us why?

---- part (b) -------

#We considered part (b) in the same way. I'll show you waiting for your pointing out our mistakes.

< 4 consecetive numbers >

(OMISSION)

(J) The expression 2(2X+3) means that when X increases 1, the answer increases 2.
The answer is <u>multiple of 2 bigger than or equal to 12</u>

< 6 consecetive numbers >

(OMISSION)

#Question from us (2)

(K) We considered this problem over an basic condition. It is that the "numbers" means natural numbers.
But as I discribed before, "numbers" can be taken for integer which includes negative numbers.
If "numbers" means integer, how does the answer change?

---- Message -----

(L) Are you happy? Be happy! *(OMISSION)*

□ Internet Project name :P P [1999/11/03,07:30:13]

(M) If negative numbers were included then the answer would be the same, but include all the answers as a negative as well as the positive.

part (b)

(N) The lowest number is 10 , this is because the numbers can be represented as (x, x+1, x+2, and x+3.) This is for the addition of 4 consectutive numbers.This works out as 4(x+1.5) As with your solution for part (a), It goes up in multiples of the amount of adding consecutive numbers, in this case, 4. This meanns the values are 10. 14. 18. 22. 26, etc...

For 6 numbers...

(OMISSION)

☐ Let's think about part (c)! name :Y K [1999/11/05,21:05:34]

(O) We read your letter. Your answer of #Question from us (2) was perfect!
If negative numbers are included, there is no minimum value. I think we discussed enough about (a) and (b).
But, have you discussed on #Question from us (1) in your group?
I will tell you the answer of it in the next letter. Please think about it again before the next letter comes.
Anyway, we want to go to part(c).
(OMISSION)

(P)
This chart means 1+2+3+4+5+6
0
0 0
0 0 0
0 0 0 0
0 0 0 0 0
0 0 0 0 0 0
It is similar to a right angled isosceles triangle.
Instead of counting all the points, calculate its area.

☐ Internet Project name :P P t [1999/11/08,
Discussion for part (c)
(Q) We like your idea, but we have another idea.
For this problem we will just focus on positive numbers, as you can get any numbers using negatives, eg. (−3)+(−2)+(−1)+(0)+(1)+(2)+(3)+(4)=8
(OMISSION)

On the meaning of Jahnke, H. N. (1994), there are two different dimensions of activities can be noted –the students participating communication are seen as synchronous and the observing researchers interpreting their communication dialog and data are seen as diachronous. And interpreting those kinds of activities, he enhanced the role of hermeneutics. Here, we interpret this extract for illustrating the four principles of hermeneutic effort: Understanding, Getting others perspective (the assumption of the position of others), Instruction from experience (self-understanding), and The hermeneutic circle.

The students involved in this communication each conducted hermeneutic effort. First, in (B), an answer came from the Australian side that expressed three consecutive numbers as x, y, and z, which differed from the Japanese customary ways of expressing algebraic expression. In (D), the Japanese side limited themselves to self-introduction, and in (E), the Australian side politely urged and showed concern for the Japanese side's failure to send an answer. In (F), the Japanese side gauged how the Australian side would respond to answer (G), which consisted of algebraic generality when sent. Simultaneously questions were submitted in (I) and (J). In (M), the Australian side explained using the Japanese side's ways of expression. The Japan side confirmed using the same means as the Australian side. In (N), the Japanese side expressed consecutive numbers using a diagram and explained the total, but in (O), while still being supportive, the Australian side advocated thinking of a different expression that cannot express negative numbers.

In the above communications, the students had an empathetic stance at each stage by the time the sending and receiving content from both sides were synchronized. Each side described their own <u>understanding</u>. The other side's message content indicated their understanding, the other side's mathematical ability was appraised using their mathematical expression, and an attempt was made to respond by adopting the perspective of the other side. Then, the other side was asked questions for the purpose of obtaining deep <u>self-understanding</u>. Through the <u>recursive activity of mutual</u>

interpretation, communication was synchronized. For example, Australian students answer (B) was not appropriate for Japanese and thus why they considered explanation (P) which was easier to understand. It is evidence that Japanese students were trying to get Australian students' perspectives. Each reply itself described their learned and developed content from others. Further instruction from their experience was clearly recorded in their comments sheets in each side. In fact, the students themselves applied their views of mathematics through message interpretation, and as a result, their individual views of mathematics were adjusted. For example, an impression from the Japanese side included, "I usually did mathematics alone, but discussing a problem as a group, in this way, has its own appeal and I think it's a good thing." This comment shows that the student's usual way of studying mathematics became clear through the mirror of this study activity. The instruction included 'leaning how to learn' for developing social norm among their communication. In another impression in Japanese side, "It was fun that we could talk with students in far-off Australia. We could neither see them nor hear them, but the three of them certainly exist on the other side of the ocean and were thinking about the same questions as we were. Just imagining that makes me happy," said a student. The students were delighted by this synchronized communication carried out with others on the far side of the ocean. By means of mathematics communication, the student himself reappraised his own view of mathematics through their hermeneutic effort of thinking and sharing with others. At the same time, this comment clearly explained human activity which students were imaging the existence of students in other side. Because of trying to understand other side (getting other perspectives), Australian students kindly waited reply at (E) and, on (O) and (Q), they positively evaluated previous messages at fist and then, they replied their own ideas. The whole processes developing synchronized communication with human relationship. It is a process of hermeneutic cycle; For example, Australian students used different characters for consecutive numbers on (B), thus, Japanese students imagined and have hypothesis that Australian students may not use symbolic-algebraic representation well. Japanese students asked numbers on (K) and Australian students simply answered (M). Japanese confirmed the hypothesis, and for easy understanding, Japanese used Pythagorean representation on (P).

What is clear is that the students and all of us who are interpreting students' activity engage in the activity of hermeneutics efforts. We feel empathy with the interpretation that assumes the position of others, as seen in the students' communication acts, with the instruction through self-understanding as evident in their impressions, and the ability to discuss this.

Additionally, this example for illustrating hermeneutic efforts illustrates the synchronized letter style communication which included sympathetic and competitive attitudes (Isoda 2003). For example, "We like your idea but we have another idea" on (P) shows sympathetic and competitive attitude. It is not limited in this example but historically well known on historical mathematical text such as the Method of Aristotle and the letter on probability from Pascal to Fermat. Next chapter, we will illustrate that those kinds of communication are observed in classroom communication, and analyze it from the view point of four principles, too.

1.4 The four principles of hermeneutics effort in its historical development

The above four principles were identified by the author from the referencing of various hermeneutic theories and reflecting on personal interpretations of mathematics history and classroom activity. As Brown, T. (1997) described, the current meaning of hermeneutics was described by Gadamer. On the other hands, here, we are not considering current meanings of hermeneutic problems but hermeneutic efforts like historian engagement which was introduced by Jahnke, H. N. (1994) into mathematics education. Especially, historians, such as Shubring, G. (2005,1-7), described hermeneutics as for their methodology and their hermeneutics is more traditional. Here I will identify some ideas from the historical development of hermeneutics for explaining the four principles.

Gadamer (1993, Japanese translation 1995), who led the development of hermeneutics from the 1960s and in recent years, said that the development of hermeneutics began with "seeking the will of God in the Bible," advanced with Schleiermacher, D. F. and Dilthey, W., and then, the opinions of Heidegger and Gadamer.

Schleiermacher (1805, 1809, referred Japanese translation 1984) generalized hermeneutics from Protestant biblical hermeneutics to methodological theory on literature, history and other textual interpretations: "Two contrasting maxims of understanding. (1) I understand until I encounter a contradiction or nonsense. (2) I do not understand anything that I cannot perceive and comprehend as being necessary." This is an allusion to the confirmation of understanding by necessity and noncontradiction as seen by the subjective, and to the <u>hermeneutic circle</u> whereby hypothetical understanding is preserved until the acknowledgement of contradiction, with interpretation continually occuring. "The main point of interpretation is that the person must be able to make the transition from his own mind to the mind of the author." This is an allusion to <u>getting other's perspective (the assumption of the other's (author's) position)</u>. The act of seeking to interpret empathetically by assuming the mind of the author and aligning one's own mind with that is described. "Grammatical interpretation is objective interpretation, and technical interpretation (hermeneutics) is subjective interpretation." The act of aligning the mind for getting other's perspective is totally subjective. The position of Schleiermacher based on this kind of subjective interpretation hints at the inclusion of self-understanding whereby interpretation becomes a mirror that reflects the subjective understanding itself; however, I could not find that Schleiermacher himself mentioned about this.

Dilthey (1900, Japanese translation 1973) considered hermeneutics to be a methodological theory of mental science that gives objectivity to interpretation based on the subjective: "Comprehension always remains merely relative, and can never be complete." In other words, the hermeneutic circle applies. "The central point of the techniques (hermeneutics) that apply to comprehension is in the interpretation of human existence implicit in the text," "The artful comprehension of the pemanently emended in expression of the existence (life) is called 'interpretation'. In terms of the interpretation, the only expression capable of such objective apprehension is linguistic expression." The salient feature of Dilthey's understanding is the point of seeing living human testimony (activity) in the text. The perspective that recognizes objectivity in the interpreting subject's empathetic reading of this human nature is a characteristic of hermeneutics. "By comparing myself with others, I am able for the first time to experience individuality in myself." Dilthey described getting others (the assumption of the position of others) and instruction from experience (self-understanding) through empathy and the superposition of the mind by "transferring one's self into the macrocosm of the given expression of existence."

Gadamer disagrees with the hermeneutics descended from Schleiermacher and Dilthey. Gadamer (1960, Japanese translation 1986) said, "The essential character of the historical spirit is not in the restoration of the past, but rather in the mediation of present existence through thinking." For Gadamer, the emphasis is on self-understanding as a manifestation of the interpreter's present existence conducted in the medium of the work of restoring the living testimony of the past: "With all comprehension (in addition to Heidegger's understanding and interpretation), the third performance opportunity arises in order to 'comprehend one's self'." This is an expression of understanding (comprehension) from Gadamer's perspective.

Gadamer enlarged the object of interpretation from text to other area. For example, with regard to conversation he states, "Placing one's self in another's position is on all occasions an element of true conversation." (1960, Warnke 1987: Japanese translation 2000) As we already mentioned and illustrated, the subject of hermeneutic effort, whereby one aligns one's mind with another's position and understands one's own subjective thinking, is not limited to the diachronic text. After Gadamer, there also appeared movements to see hermeneutics as a philosophy that applies to natural science and all sciences. This included viewing the relationship between theory and observation as a hermeneutic circle in the natural sciences.

As shown above, the four principles, "understanding," "getting others' perspectives (the assumption of the positions of others)," "instruction from experience (self-understanding)," and "the hermeneutic circle" are known on the historical development of hermeneutics. Here we use those four principles for describing human activity on mathematics education and for developing good teaching practices.

Part I. Hermeneutic Effort in Mathematics Classroom; Getting Others' Perspectives

Chapter 2. For Getting Other's Perspectives

-Hermeneutic Effort in Dialectic Discussion on problem solving approach

Here, for illustrating the significance of hermeneutic effort, the dialectic communication in classroom are shown for explaining the importance of getting others' perspectives and knowing ways of argumentation in classroom which is developed through the teachers for constructing mutual understanding.

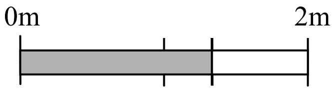
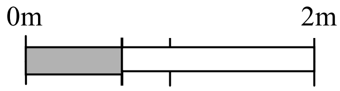
Japanese problem-solving approach in mathematics classes is comprised of both individual solving of an unknown problem using students' previous knowledge (known or learned), as well as a whole classroom (or a group) work that utilizes individual thinking in problem-solving. In particular, the classroom work, which is aimed at using others' thoughts (individual problem-solving) and reshaping them into something that can be shared publicly based on values of mathematics such as simple, understandable, reasonable, general, easier and so on. If one focuses on the communication of information within the group as part of the process of group work, one notices that this is a process of interaction between individuals. If one views this as a process of each individual cognition, it becomes evident that this is a process of reviewing one's own thinking as perceived based on information from others. This chapter focuses on the difficulties of students for getting others' perspectives and analyzing the methods of argumentation used by the teacher among children who are arguing the correctness of their own thoughts during group work, in a manner that makes it easy to elicit both sides.

First, the dialectic communication of fraction in classroom will be described and ways of argumentations will be illustrated.

2.1 Case Study: Divisional (Partitive) Fractions vs. Quantitative Fractions

The 45 minutes class (first lesson) and the 15 minutes class (second lesson) at fifth grade were taught by Hideaki Suzuki (the Sapporo Elementary School Attached to Hokkaido University of Education, in 1992) regarding the problem of "making (creating) a 2/3 m piece of tape from a 2 m piece of tape". This exercise is known for its very low percentage of children and even higher grade students of giving a correct answer (Noda: 1981). It involves discriminating the different understandings between divisional fractions (fraction in partition; n parts from among m equally divided parts of the whole) which studied at grade 3 on the 1989 curriculum and quantitative fractions (n parts from among m equally divided parts of a unit quantity (such as ‘1 m’), where m<n is also possible such as ‘3/2 m’) which studied at grade 4 on the 1989 curriculum. Both of them were learned in the previous grades. But in the case of the classroom children, the actual results of a previously implemented test showed only one out of 38 children gave a correct answer. In those children answering incorrectly, some simply misread the question, or others simply focused on the “from 2 m” part and automatically applied the procedure of divisional fractions.

The result of pretest at the day before the class: Make a 2/3 m piece of tape from a 2 m piece of tape

	
37 students	1 student

The class proceeds in two groups: 1) Matsuura’s Group, those who follow the divisional fraction method of thinking, whereby they come up with an answer that is “two parts of the three equal parts of 2 m (it means 4/3 m as quantity)”, and 2) Minamiyama’s Group, those who follow the quantitative fraction method of thinking, whereby they come up with an answer that is “2/3 m”. Groups were developed by each student’s individual solution. Groups exchange ideas while attempting to convince the other side. Through the argumentation, students change their idea and move their position. What follows is an overview of how a student Suzuki repeatedly says the same thing in an attempt to persuade from the perspective of the listener, whereas the student Minamiyama fails to persuade from the perspective of the listener. This overview focuses on the intervention by the teacher: Teacher chose speakers among students who raised their hands and intervened for synthesizing parallel discussion. Otherwise the communication goes parallel and will be in discrepancies.

Class Overview

Fist class (45 min.):

Scene 1: Presentation of the Results of Individual Problem-Solving

When the teacher described the problem and distributed out pieces of tape (see it as 2m each), telling students to cut the tape, complain from students such as “this is a pain” or “this is too simple.” can be heard. The majority (Matsuura’s Group) sees this exercise as a problem of divisional fractions and cuts the tape into two parts of the three equal segments of the 2 m (resulting in 4/3 m). Only two students (Minamiyama’s Group) see this exercise as a problem of quantitative fractions with the desired quantity as 2/3 m, or 2/3 m with a base quantity of 1 m (from the perspective of Matsuura’s Group, this is one part of three equal segments of 2 m).

Matsuura Group: two parts of the three equal segments of the 2 m	Minamiyama Group: 2/3m
37 students	2 students

The teacher asks students to present the cut tape on the blackboard. Matsuura presents the answer without saying anything while Minamiyama says the following:

Minamiyama (1): Basically, it's half of this (Matsuura's tape).

Matsuura Group: No, it's two thirds!

Matsuura Group to Minamiyama's Group: Wait, I get it; I'm in Minamiyama's Group, too.

Matsuura Group: The way the teacher wrote this isn't "get 2/3 of 2 m from 2m of tape"...

Suzuki (1): Teacher, this can be interpreted either way.

The teacher asks the students to indicate their positions which they agree with their Named Magnets on the blackboard. "Matsuura Group" has 22 students, "Minamiyama Group" has 4 students, "Either Way Is Fine" has 11 students (included Suzuki), and "Undecided" has 2 students. At this point, the student Suzuki is in the "Either Way Is Fine" group.

Matsuura Group: two parts of the three equal segments of the 2 m	Either Way Is Fine	Undecided	Minamiyama Group: 2/3m
22 students	11 students	2 students	4 students

Scene 2: Exchanges Regarding Each Side's Position (Each others' Opinion)

Either Way Is Fine Group: Maybe the problem is coming from the way of question. According to Matsuura Group, the answer is 2/3 of 2 m.

Minamiyama Group: But it says "from 2 m."

Either Way Is Fine Group: So the reason for Matsuura's answer of 2/3 is that since 1 or 1 m is taken from the 2 m tape.

Matsuura Group: You just divide it into three and take two of those segments.

Minamiyama Group: But it's 2/3 from the 2 m tape (Note: the "m" of "2/3 m" is missing in his explanation).

Suzuki (2): This is a 2 m piece of tape, so with 2 m, you get 2/3 m, right? Usually when you have a fraction, the base number is 1. Since it's 2/3 m here, you have to get the base to 1. It says 2/3 m, right? Since there's an "m" on it, that means 2/3 of 1 m. So it's 2/3 m from 2 m of tape, and Minamiyama first threw out this half (1 m), and I think you use two of the three segments of the remaining tape. If Matsuura's Group did this without the "m" in "2/3 m", I think it would be just like Matsuura's answer.

Minamiyama Group: Now there's the "m", so wouldn't Minamiyama be right?

Here, from the viewpoint of observers who knew which is correct, Suzuki (2) is speaking with a good understanding of both sides, and so that should cause Minamiyama's Group to win the discussion. Logically, this should have finished the discussion however the students are not satisfied.

Matsuura Group: Why is it that Matsuura's right if there is no "m" (in "2/3m")?

Some of Matsuura Group do not quite understand what Suzuki is saying. Then, Teacher asked Minamiyama to explain the meaning once more.

Minamiyama (2): I thought that 3/3 is equal to 1 m.

Teacher (1): One more time.

Minamiyama (3): 3/3 means 1 m, right?

In spite of the fact that he, Minamiyama, will not be able to persuade Matsuura Group until he clarifies the fact that this is a quantitative fraction, the "m" quantity is consistently missing in Minamiyama's explanations from Minamiyama (1) to (3) such as "3/3 is equal to 1 m" but the divisional procedure is represented as well as the explanations of Matsuyama group. Even if the "m" is clearly written in Minamiyama's note, when he explains what he has done to other students, he equates 3/3 with 1 m rather than stating "3/3 m". Minamiyama is applying divisional fractions with 1 m as the unit quantity, and is overlooking the fact that this is a quantitative fraction despite of the repeated prompting from the Teacher (1). This makes it impossible for Minamiyama to deny the "three equal parts of 2 m" idea of Matsuura's Group.

Because Minamiyama himself did not explain well, the teacher diverts the discussion away from what Minamiyama is saying in the following way and starts stirring things up.

Scene 3: Stirring Things Up (Teacher Intervention 1)

Teacher (2): 3/3 m is 1 m, no doubt about it (Note: he emphasized the "m").

Minamiyama Group: Right!

Matsuura Group: No, absolutely not.

The teacher focuses on whether or not 3/3 m is 1 m for trying to find sharable ground of discussion.

Suzuki (3): Teacher, it is not related with the problem (Note: the original question), isn't it?

Suzuki (3) takes this to mean that other than the original question, anyone would think that 3/3 m = 1 m, or in other words that the "base is 1 m". This is also a counterargument and then, they start to include the original question in teacher's new question.

Matsuura Group: You take 2/3 from 2 m, right? So maybe Minamiyama's 3/3 m is 2 m.

Matsuura group still assumes that their answer is correct and for persuading Minamiyama group, they begin to follow Minamiyama's thinking that 3/3 m is 2 m if 2/3 from 2m but it just reflects on their interpretation. On the other hand, Minamiyama explanation continued to be expressed as a divisional procedure and failed to explain using the basis (meaning) of a quantitative fraction.

Minamiyama (4): (Pointing at the 2 m figure) This half is 1 m, and these two segments are 2/3.

Matsuura Group: No mentioned 2/3 of "1 m" in the original problem.

Minamiyama Group: It doesn't say create "2 m" tape. It just says is "from 2 m of tape".

Minamiyama Group: Since the original problem doesn't say to make this only from a 2 m tape, you can make it from 1 m as well.

The teacher reorganizes the conflicting arguments. Matsuura's Group sees the "base as 2 m", and the Either Way Is Fine Group sees "both 2 m and 1 m can be the base". In order to summarize their viewpoints, the teacher questions Minamiyama Group as follows.

Teacher (3): Minamiyama, if your answer is 2/3 m, then we would like to say that the base is 1 m. This is the reason why 3/3 m is 1 m, and the base is 1 m, according to what you are trying to say, right, Minamiyama?

This questioning clarifies the ground of Minamiyama's inference as opposed to the aforementioned Matsuura Group's ground.

Scene 4: Sharing the Argument

Teacher (4): Well, this is a problem, isn't it?

Matsuura Group: Since Minamiyama has left 1 m over, doesn't that mean he really remain 1 and 2/3?

Matsuura Group: So Minamiyama does not take 1, but 1/6.

Teacher (5): No. Minamiyama's answer works when he's only using this (1 m). The remaining 1 m is irrelevant for him.

Many of Matsuura Group still reflect on their interpretation. Matsuura Group which "divides 2 m", interprets Minamiyama's unit quantity 1/3 m as 1/6 of 2 m. Accepting the teacher's statement that "the remaining 1 m is irrelevant", Suzuki stated that she is moving (to Minamiyama Group) and started to talk.

Suzuki (4): If the original problem involves making 2/3 of a 2 m tape, then Matsuura's side is right, I mean, I think Matsuura's argument is easier to understand. Since you're supposed to create 2/3 m from a 2 m piece of tape then it must be 2/3 m. So you ignore the 1 m, and this 3/3 m is also 1 m. Since you are going "from", you've got to deal with both "from" and "m". If there wasn't this "m", and if "from" was "of", then I would agree with Matsuura. (Repeating while reviewing the figure) This 2/3 m means that the base is 1 m. If there wasn't an "m", then you could use any amount of "m" as the base, but since there is an "m", then 1 m must be the base.

Matsuura Group: If the problem is "create 2/3 from a 2 m tape", or "create 2/3 m of a 2 m tape"?

Matsuura Group and Minamiyama Group: The first one, "create 2/3 from a 2 m tape", is Matsuura Group but what about the second one?

Matsuura Group: 2 m might be the base, but since its 2/3 m, 1 m might be the base, too.

Teacher (6): So the second one would be strange and contradicting.

Ever since Suzuki (4)'s statement, the semantic interpretation of each group was not the same. On the other hand, the statement of Matsuura Group here has been influenced by the teacher's specification of the base amounts and the group now shares Suzuki's statement. Matsuura Group should now focus on "from" and "of" while considering questions that they come up with themselves in order to review the points they presented themselves. This awareness of contradiction then causes some members of Matsuura Group to begin sharing Minamiyama's idea that the base quantity for the case of 2/3 m is 1 m, indicating that they are considering joining Minamiyama Group. Teacher (6) mentioned that "create 2/3m from a 2 m tape" do not contradict but "create 2/3m of a 2 m tape". It already implicated for the people who have appropriate knowledge that Matsuura group is inappropriate but they do not well understand teacher's saying even if they felt it as strange in Japanese.

In order to articulate this state where ideas have changed, the teacher asks the students to move their named magnets (for the second time). The results are 16 students in Matsuura Group, 20 students in Minamiyama Group, no student in the Either Way Is Fine Group and 2 students in the Undecided Group.

Matsuura Group: two parts of the three equal segments of the 2 m	Either Way Is Fine	Undecided	Minamiyama Group: 2/3m
16 students	No student	2 students	20 students

Scene 5: Stirring Things Up (Teacher Intervention 2)

In order to stir things up again, the teacher asked the students to forget the original question and whether or not "3/3 m is 1 m" temporarily. Some of Matsuura group are still the opinion that "it is three equally divided parts of 1 m or 2 m". This opinion indicates that some students are still caught up in the idea of divisional fractions. The teacher asks "can we change tracks?" and continued as follows.

Scene 6: Stirring Things Up (Teacher Intervention 3)

Teacher (7): If we have 0.5 m, then do we indicate what the length is?

Matsuura Group and Minamiyama Group: Yes, it's the same as 50 cm.

Teacher (8): Can we express this as a fraction? (Detailed discussion omitted) So is it the same as 1/2 m, or is it different?

Minamiyama Group: It's the same.

Matsuura Group: Wow! (Note: this is taken to mean that they are realizing their contradiction.)

Matsuura Group: It's different.

Suzuki (5): If 0.5 m is the same as 1/2, then what is 1/2?

Teacher (9): And if I asked you to express 1/2 m as a decimal of m, what would that be? (Note:

he added ‘m’.)

Matsuura Group: 0.5. (Note: it still lost ‘m’.)

Matsuura Group: It might be 1/2 of 2 m.

Some members of Matsuura Group now think that “if 1/2 m is an invariant then maybe 3/3 m is 1 m”. However even now, some members of Matsuura Group still recognize the fact that 0.5 m = 1/2 m, but do assert 1/2m≠0.5m because ‘1/2 m is 1/2 of 2m’ is also true. These members insisted that their own explanation on the original problem is correct and therefore account their thinking on divisional fractions with a division target of 2 m as the base. Minamiyama continued his explanation.

Minamiyama (5): 0.5 m = 1/2 m and 1/2 m = 0.5 m are the same thing, all you’re doing is reversing the order. So I think you can say that 3/3 m is 1 m. But if the base changes, I’m not sure if you can still say that 1/2 m = 0.5 m.

It is evident now that Minamiyama himself recognized the way of thinking of Matsuura Group. It looks that both groups had now reached a state where they recognized the thinking of the other group. At the same time, Minamiyama himself is dealing with the problem of how to find “what unassailable ground of discussion can be shared” with Matsuura Group which is still fixated on divisional fractions. Time runs out at this point and the teacher returns to the argument at hand about “if ‘m’ is affixed on 2/3 m, whether or not 1 m is the base”, asked the students to move their magnets for the third time. At this point, Matsuura Group has 14 students, Minamiyama Group has 23 students, the Either Way Is Fine Group has no student and the Undecided Group has 1 student.

Matsuura Group: two parts of the three equal segments of the 2 m	Either Way Is Fine	Undecided	Minamiyama Group: 2/3m
14 students	No student	1 students	23 students

Second class (15 min)

Scene 7: The Next day

The lesson began with the review on the explanation of what had been discussed in the previous lesson for students who were absent yesterday. The question of “from” or “of” is examined once again, with the aim of articulating the difference between interpretations that determine whether one is a member of either Matsuura Group or Minamiyama Group. However the discussion between the groups is not as heated as it was yesterday. The teacher noted the mood of the classroom and started the guiding instruction for concluding.

Teacher (10): The class seems to be in Minamiyama’s direction. Matsuura Group, do you have anything to add to the discussion?

Matsuura Group: It says “from” a 2 m tape, right? If it said “from a 1 m tape”, or if it didn’t say “from” (“of 1m”), then Minamiyama would be right, but it does say “from”, so 2 m is the base.

Minamiyama Group: 2 m is larger than 1 m, right? So we can just forget 1 m of the 2 m for the moment, and take 2/3 m from 1 m, for instance.

Minamiyama Group: Just ignore where it says “from”.

Teacher (11): So that you are saying, just “create a 2/3 m tape” is the same as the original question.

The teacher asked the student Suzuki to explain the answer by focusing on the original problem from the children regarding “how the exercise changes depending on whether m is affixed or not.”

Suzuki (6): For instance, you have a blackboard and you have 2/3 of a blackboard. We say this is 2/3. For instance, if you have a blackboard eraser, you could say 2/3 of this blackboard eraser. Understand?

Teacher (12): I know what you’re trying to say. I really do understand.

Suzuki (7): You can go with anything whatever. But it says 2/3 m. Since it has an “m” on it, that “m” must be the base. We studied that it was determined by the distance from the Equator to the North Pole divided by some tens of millions, right? Before they standardized it that way, “1 m” was not always equal, right? If you use 2 m as the base, you back then against the determination. Anyway, since m has 1 m as the base. This is the difference between when you have a given base and when you don’t.

The teacher then asked other students to explain how they understand what Suzuki had explained in their own words and summarized the discussion as follows:

Teacher (13): Suzuki wants to say that since there is a unit affixed, the base is already completely settled. So that’s why she feels she has to join the Minamiyama Group.

Teacher (14): We haven’t heard from the Matsuura Group at all lately. Can we end this discussion now, then? Since “m” is affixed, the base is “1 m”, but since we are dealing with 2/3, we can change the base accordingly. It’s as simple as that, isn’t it? Is this fine with everyone? (The class ends at this point.)

The teacher ends the class by using Suzuki’s statement as the basis of bringing the discussion to a conclusion. Although there are no longer any counterarguments from Matsuura Group, some of the members remain unconvinced. The teacher continued by reformulating the lesson using fractions of various sizes of quantity such as more than 1 m as a subject matter in order to convince those who still remain in doubt.

This overview includes excerpts focusing on the following phenomena which are recognized as part of the argument process.

a) At the beginning, Matsuura Group failed to share Minamiyama’s way of thinking and interpreting it wrongly. This prevented connection between the two groups.

b) Minamiyama explanation without using the unit “m” in his words is unsuitable for persuading Matsuura Group.

c) Suzuki’s statements are consistent from the beginning but still fail to fully persuade Matsuura Group.

d) The teacher intervention for trying to conclude the contradiction with respect to quantity was effective in persuading the class.

e) As both sides interacted, they shift from a state of misunderstanding each other to a state of shared understanding.

f) Controversially, there were some students in Matsuura Group who developed the hard core on insisting that their believed conclusion is true which enable them to insist 0.5m=1/2m but 1/2m $\neq$0.5m.

g) Some students were convinced, but others were not completely convinced by the efforts of the persuasion.

The flow of the class overview is shown in figure 3 on the next page.

2.2 Analyzing argumentations between groups

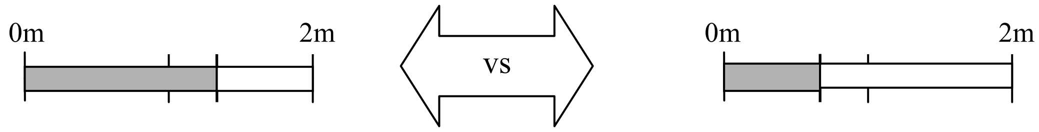
Japanese problems solving approach usually goes in a whole classroom work based on individual solution and the position of each individual opinion (mathematical ideas) is recognized on categorized solution (positioned discussion). A Teacher usually conducts argumentation (mutual assertion) in a whole classroom with categorized solutions after solving individually. The difference of individual ideas will be communicated in relation to categorized solutions which are presented by students. On this context, here, we analyze the example via categorized groups which are shown on figure 3.

As dialectic, figure 3 has following discussion structures:

Make a 2/3 m piece of tape from a 2 m piece of tape

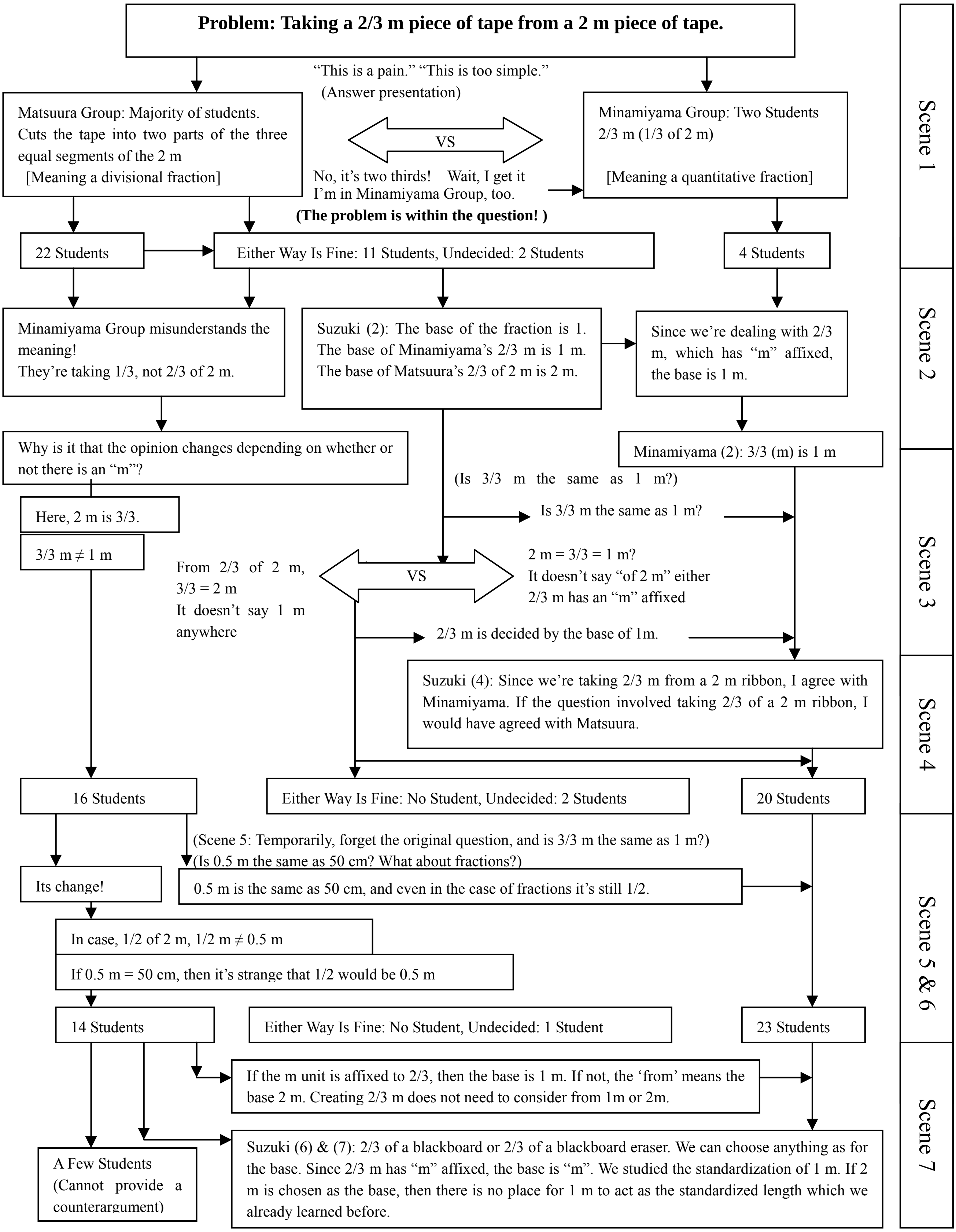
Thesis (Antithesis from other side)
Matsuura Group: two parts of
the three equal segments of the 2 m

Thesis (Antithesis from other side)
Minamiyama Group: 2/3m



Synthesis
2/3m means 2/3 of 1m. 1m was standardized

Figure 3. Class Flow



Arguments by students were controlled by the teacher. He intervened in the following ways for completing dialectic;

First, he gave opportunity of assertion from both sides (Scene 1),

Second, he allowed Suzuki who understands both sides to explain,

Third, he gave a counter example 3/3m in relation to the original question (Scene 3),

Fourth, he gave generalized counter example 1/2m (Scene 6), and

Fifth, he supported Suzuki who explained the necessity to fix '1m' as a unit for conclusion.

Finally, because the teacher supported Suzuki and there was no new counter argument, Matsuura Group did not give their comment further more. Teacher set remedial instruction about the quantitative fraction more than 1 m such as 3/2 m: Students already learned those concepts in grade 4.

Figure 3 showed that students shifted their position of argument depending on the changes of ideas through persuasion by others. From the changes of their position, we can identify following types of students in Matsuura Group:

<u>Type A: Students who changed immediately at Scene 1</u>

Although they knew both the procedure of division and the meaning of the quantity on fractions, they did not used well or understand the meaning of the quantity fraction at the beginning. They applied the division procedure at the original question and some of them immediately remembered the quantity from Minamiyama's answer and recognized the difference between 'from' and 'of' by themselves. Even if they moved 'either way is fine' but did not moved to Minamiyama Group at Scence 1, it means that they well understand both the procedure of division and the meaning of the quantity on fractions. They well understood that the confusion originated from the difference between 'of' and 'from' and that why they can understood both position.

<u>Type B: Students who changed to Minamiyama Group until Scene 6</u>

They knew the divisional procedure and did not well understand the quantity on fractions at the beginning. They applied the division procedure at the original question. They did not change their position when they meet Minamiyama's answer. It means their understanding of the quantity on fractions is not enough to distinguish the difference of 'from' and 'of' in this moment. From the contradiction of 3/3 m = 2m and additional contradiction of 2/2m = 2m which deduced from their divisional procedure at original question, they gradually recognized the difference of 'from' and 'of' and until Scene 6, they understand the meaning of the quantity on fractions. Then, they moved to Minamiyama Group. We can say that their understandings of the meaning of the quantity on fractions and the quantity itself were different depending on when they had changed.

<u>Type C: Students who did not changed until Scene 6.</u>

They knew the divisional procedure and did not understand the quantity on fractions, and finally did not understand the relationship among quantity. In their case, their divisional procedure is stronger enough to disregard the contradiction and change the meaning of the quantity. For their persuasion, they have to use their divisional procedure on original question and they must support it. Then, their dialectic enhanced their procedure as the hard core which should be kept.

From the view point of the meaning and procedure, Type A students used well divisional procedure and understood the quantity on fraction at beginning. Type B students used well divisional procedure but did not understand the quantity on fraction at beginning and finally learned the quantity on fraction from the contradictions until Scene 6. Type C students used divisional procedure but they did not understand the contradiction and failed to conceive the meaning of the quantity on fraction. Some of them rejected to conceive the quantity on fraction for asserting their conclusion to be true. In their case, their procedure is functioning like a meaning as the base to explain why their conclusion is true. They had to choose wrong assumption for deducing conclusion to be true. Those categorization from the viewpoint of the procedural knowledge and the conceptual knowledge which were observed in the process of argumentation is summarized in following table;

In the process, they applied	Type A	Type B	Type C
the appropriate procedure of division	Kept	Kept	Kept
the appropriate meaning of the quantity	Kept	non → having	non → non

2.3 Getting others' perspectives; the ways of persuasion and the moments of conviction

The four principle of hermeneutic effort are "Understanding," "Getting others' perspectives (the assumption of the position of others)," "Instruction from experience (self-understanding)," and "The hermeneutic circle." Here, I illustrate the difficulty of getting other's perspective for knowing the ways of persuasion and the moments of conviction.

We usually say the "logic of persuasion" states that "it is not possible to easily persuade people unless one does so with an understanding of their perspective first". This is a kind of ancients' dialectic which begins the reason: "if what you say is true…." Suzuki knew the idea of both groups and repeated the same explanation. But depending on students, understandings were different because they do not share the same reasoning.

In figure 3, even though the reasoning and understanding of each student are not the same we could categorize Matsuura Group into Type A, Type B and Type C. Suzuki's explanations did not change through the discussion but depending on their understanding their decisions were different. Type B students could not agree with Suzuki from the beginning because they do not have a sharable ground. As Hegel,G. described, the antithesis works positively and negatively. In the case of Type B, counter examples given by teachers supported them to develop (or remember) the meaning of the quantity. In the case of Type C, counter examples influenced them to develop the hard core for reasoning based on their conclusion. Both developments are different instructions from experience in this lesson. It means that counter examples given by teachers functioned positively for developing ground of discussion for Type B students but works negatively for Type C students.

However, in case, students can share the ground of discussion, they can share the ideas. In this lesson, the teacher prepared several strategies for developing the ground of discussion for the conviction. First, he gave students the opportunity to exchange the different answers. Second, he gave the opportunity to exchange their reasoning. Third, he gave counter examples. Fourth, he fixed the ground of reasoning by supporting Suzuki's explanation, especially, he posed different counter examples: 3/3m, 0.5m and 3/2m in the next lesson. 3/3 m is limited within fractions, 0.5m is related with decimal notations and 3/2m is the extension of divisional fraction to the quantity. Each counter example has their different roles for developing the concept of quantitative fractions. He posed them in the sequence from specific to general: students could not represent 3/3 by decimal notations. Divisional fraction divide whole and is not larger than 1.

Through these teaching strategies, students engaged in the hermeneutic cycle and were enable to develop others perspective as the ground for sharable discussion, and developed appropriate understanding.

ICME 11, Topic Study Group 26, Mexico.

Reference

Arcavi,A., Isoda,M.,(2007). Learning to listen: from historical sources to classroom practice. *Educational Studies in Mathematics*, Vol. 66, No. 2. pp. 111-129.

Brown, T. (1997). *Mathematics education and language: interpreting hermeneutics and post-structuralism*, Dordrecht : Kluwer.

Isoda, M., McCrae,B., Stacey, K. (2006). Cultural Awareness Arising From Internet Communication Between Japanese and Australian Classroom, Leung, G. K. S., Graf, K., Lopez-Real, F. J. edited. *Mathematics Education in Different Cultural Tradition*, USA: Springer. 397-408.

Jahnke, H. N., (1994). The historical dimension of mathematics understanding; Objectifying the subjective. Ponte, P. et al edited. *Proceedings of the International Conference for the Psychology of Mathematics Education*, 139-156.

Schubring, G. (2005). *Conflicts Between Generalization, Rigor, and Intuition*, USA: Springer.

Further References in Japanese;

Isoda, M. (2002). Hermeneutics for Humanizing Mathematics Education, Tsukuba Journal of Educational Study in Mathematics. University of Tsukuba, 23. 1-10

Isoda, M. (1994). Logic of Argumentation, Hokkaido University of Education edited, Education and Laguage, Tokyo Syoseki.

Isoda, M. edited (1996). Lesson Study for Problem Solving Approach at Elementary School Mathematics, Tokyo:Meijitosho.

Isoda, M. et al edited (1999). Lesson Study for Problem Solving Approach at Middle School Mathematics. Tokyo:Meijitosho.

Isoda, M. et al edited (2005). Lesson Study Aimed for Understanding at Elementary School Mathematics. Tokyo:Meijitosho.

Isoda, M. et al edited (2008). Lesson Study Aimed for Argumentation at Middle School Mathematics, Tokyo:Meijitosho.

Isoda, M. et al edited (2008). Lesson Study Aimed for Argumentation at Middle School Mathematics, Tokyo:Meijitosho.