

Different perspectives on mathematical modelling in educational research - categorising the TSG21 papers

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Introduction

We do have a large and growing collection of didactical research on mathematical modelling and this research seems even to have had a serious impact on the practices of mathematics teaching. Together with the introduction of information technology, the introduction of mathematical modelling and applications is probably one of the most prominent common features in mathematics curricula around the world in the last couple of decades. Especially at the secondary level, curricula reforms in many western countries have emphasised modelling and applications as an important element in an up-to-date mathematics curricula in general education. Didactical research has undoubtedly played an important role in this development. Fundamental goals in the teaching of mathematical modelling and reasons for pursuing these goals developed and analysed in research can be pinpointed in the guidelines for mathematics teaching in many countries. Also, the general understanding of the model concept and of a modelling process expressed in many mathematics curricula are clearly influenced by didactical research. Evidence can be found in the ICME-14 study volume (Blum et al, 2007) and in (Haines et al, 2006).

However, it is still a pending question to what degree the many developmental modelling projects carried out and analysed in research have actually influenced the practices of teaching mathematical modelling. How mathematical modelling and applications are organised in curricula and especially how these parts of the curricula are assessed show only minimum influence from research.

However, influencing practices of mathematics teaching is not the only criteria for progress in the didactical research on mathematical modelling. It is also relevant to try evaluating the coherency of the theories developed. In the editorial *Towards a didactical theory for mathematical modelling* of the ZDM issue, no.2 vol. 38, we argued that it is possible on a general level to identify in the field of research

... a global theory for teaching and learning mathematical modelling, in the sense of a system of connected viewpoints covering all didactical levels: learning goals, fundamental reasons for pursuing these goals at different levels of the educational systems, tested ideas about how to support teacher's in implementing learning goals and recognised didactical challenges and dilemmas related to different ways of organising the teaching, theoretically and empirically based analyses of learning difficulties connected to modelling, and ideas about different ways of assessing students' learning in modelling activities and related pitfalls. (Kaiser, Blomhøj and Sriraman, 2006, p. 82)

However, this “global theory” is not based on a single strong research paradigm. On the contrary, in fact, it is possible to identify a number of different approaches and perspectives in mathematics education research on the teaching and learning of modelling. This is exactly the reason for choosing *Conceptualizations of mathematical modelling in different theoretical frameworks and for different purposes* as one of themes for the Topic Study Group on Mathematical applications and modelling in the teaching and learning of mathematics at ICME-11 (TSG21). We want to show the variation in the field but also to provide a

background for in-depth discussions for the theoretical basis of the different approach in the field.

Kaiser & Sriraman (2006) reports about the historical development of different research perspectives and identify seven main perspectives describing the current trends in the research field. These perspectives are not at all mutually exclusive nor do they cover the entire research area. Nevertheless, they have all distinctive perspectives of research on the teaching and learning of mathematical modelling, and they have developed in different research milieus over a long period of time and have all resulted in a considerable number of research publications. The main rationale for developing a categorisation of research perspectives is of course to deepen our mutual understanding of the individual perspective and to recognise similarities and differences amongst these. The idea is not to try to judge their relevance or relative importance.

Five of the research perspectives pinpointed by Kaiser & Sriraman (2006) are – according to my analyses – represented among the fifteen papers accepted for TSG21. Therefore, in this paper I take the opportunity to use this categorisation by briefly discuss the characteristics of each of the research perspectives and illustrate their particular focus by referring to the TSG21 papers, which I find represent a particular perspective. The aim is to provide a background for discussing the many interesting papers of TSG21 in relation to their theoretical foundation. Hopefully the categorisation can also facilitate discussions of similarities and differences among the perspectives. It goes without saying that the categorisation in itself should also be discussed and debated. At the end of the paper I summarise in form of a template the descriptions of the perspectives in a few words together with a list of the TSG-papers categorised under the individual perspectives according to my analysis.

The realistic perspective

The realistic perspective on the teaching and learning of mathematical modelling takes its point of departure in the fact that mathematical modelling and models are being extensively used in very many different scientific and technological disciplines as an interdisciplinary form of problem solving. Therefore, in order to teach mathematical modelling in a form that will really be helpfully for students in their subsequent professions, one needs to study carefully authentic real life modelling, and on the basis thereof to design situations where students work with authentic modelling supported by relevant technology, and assess the model and its results against the reality. The main criteria for the students' learning is that they should be able to apply mathematics to solving real life problems. Pollak (1969) can be regarded as a prototype of the realistic perspective.

Often physicists and sometime also researchers from other natural sciences argue that what we call mathematical modelling in their subject area should be thought of as physics modelling (or just physics – because modelling is what physicists do all the time – they say) or biological modelling. Of course as mathematics education we are focusing on the general elements in the teaching and learning and not the differences of modelling in different areas. Should we need to defend ourselves, we could argue that so far not much educational attention or research has been directed towards (mathematical) modelling outside mathematics education research. However, the realistic perspective is really taking the subject area of the application of mathematics very seriously and actually sees modelling as an interdisciplinary problem solving activity.

The paper by Rodríguez (TSG21) is an example of how the way we conceptualise a mathematical modelling process may be heavily influenced by the subject area in which the modelling takes place. In this work modelling is used as a didactical means for supporting the students' learning of mathematics and physics in a calculus and in a physics course, respectively. In physics the modelling process was conceptualised as illustrated in figure 1. However, since the learning goal in this project is to support the students' learning of mathematics and physics by means of mathematical modelling and not to model real life situations, in my mind, this paper does not belong to the realistic perspective.

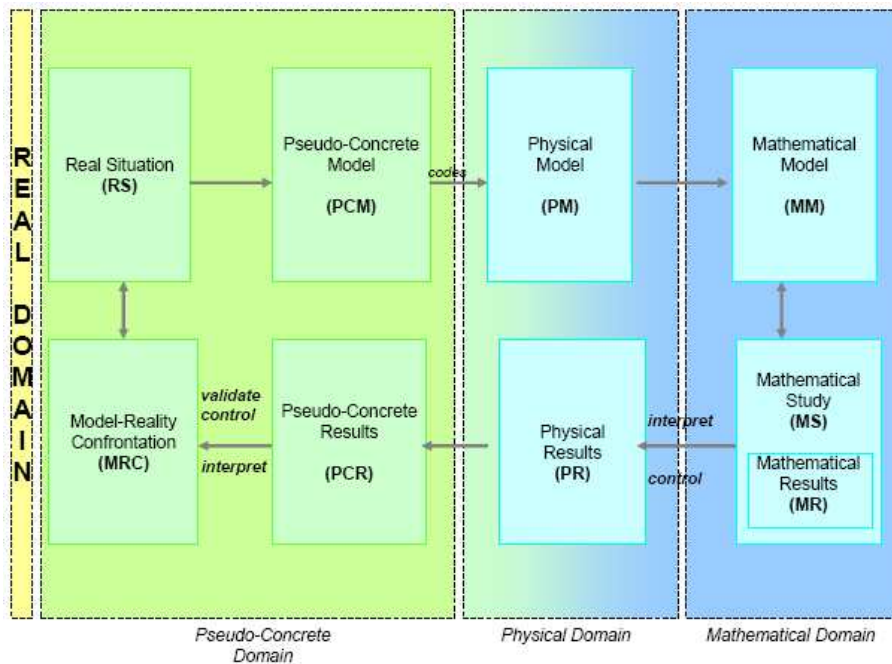


Figure 1: Modeling process in a physics course (Rodríguez, TSG21).

The paper by Kadujevich (TSG21) is my example of the realistic perspective. Here the author analyses the experiences from a developmental project for undergraduate business students. The students were to make and analyse a total balance model in the form of a spreadsheet for a business activity of their own choice. The use of technology in the form of a spreadsheet is an important and integrated element in this approach. The success criteria for the students' modelling work was to apply the model for deciding whether or not their business activity was profitable and to make suggestions on how to make more profit. This is I think characteristic for the realistic perspective. I take it also as a characteristic element in this approach that Kadujevich in his design builds on the heuristic for technology-supported modelling of real life situations developed by Galbraith & Stillman (2006).

In the paper by Lambardo & Jacobini (TSG21) the authors are reporting from their developmental work with teaching Linear Programming and mathematical modelling to students employed in various business and industries, and who are taking a college degree. Working in pairs the students were challenged to find problems from their own working life that could be addressed by means of mathematical modelling and Linear Programming. The experiment involved both a mathematics course and a course in data processing in which the students was introduced to software for optimisation. The clear connection to the students working life created a strong motivation for learning the "mathematics behind" and for learning how to use the software in order to reach an optimal solution to a LP problem but the students did not, by themselves, engage in reflections about model assumptions, the stability

of their optimal solution or the general validity of the model, and the possible implementation of the model results in real life. So authenticity and close connection to real life experiences do not necessarily support the development of relevant reflections and critique. I consider this study to the realistic perspective. However, the students' modelling work is clearly structured for the purpose for supporting their learning of Linear Programming as an applied mathematics discipline. So the study could also be counted as belonging to the educational perspective.

The contextual perspective

This perspective has developed primarily on American ground, and is based on extensive research on problems solving and the role of word problems – called contextual modelling in mathematics teaching. In the last decay the modelling eliciting perspective has been developed further, within this perspective, by deepening the philosophical background as well its connection to general physiological learning theories.

Foremost focusing this research perspective focuses on developing and testing designs for modelling eliciting activities, which are guided by six principles: (1) the reality principle – the situation should appear meaningful to the students, and connect to their former experiences; (2) the model construction principle – the situation should create a need for the students to develop significant mathematical constructs; (3) the self-evaluation principle – the situation should allow students to assess their elicited models; (4) the construct documentation principle – the situation and context should require the students to express their thinking while solving the problem; (5) the construct generalization principle – it should be possible to generalize the elicited model to other similar situations; and (6) the simplicity principle – the problem situation should be simple. (Lesh & Doerr, 2003)

It is the clear focus on the didactical design of modelling eliciting activities where situations are carefully structured to support the students' learning that distinguishes this perspective from the realistic perspective. It could be argued that the contextual perspective could therefore be thought of as part of the educational perspective described below. However, the modelling eliciting perspective insists on seeing mathematical modelling as a special type of problem solving, and therefore the psychological aspects of problem solving is taken as a basis for understanding the learning difficulties related to mathematical modelling and for teaching mathematical modelling under the contextual perspective.

Maybe, surprisingly – Mexico being close to the US – we did not for TSG21 receive any papers within the contextual perspective.

Educational perspective on mathematical modelling

The main idea of this perspective is to integrate mathematical modelling in the teaching of mathematics. Accordingly goals and related justifications for teaching mathematical modelling at various educational levels, the way of organising mathematical modelling activities in different types of mathematic curricula, the problems related to the implementation of modelling in teaching practices, and the problems related to assessing students modelling activities have all been dealt with under this research perspective. Niss (1987, 1988) and Blum & Niss (1991) are classical references to this research perspective, which has been quite dominant in Western Europe in the last three decays. Defining and discussing the basic notions in the field such as: model, modelling, the modelling cycle or modelling cycles, modelling competency and applications, and the meaning of these notions in relation to mathematics teaching at different educational levels is an important element in

research under the educational perspective. The introduction to the ICME-14 study volume gives an overview of the concept clarifications and the history of the field. (Niss, Blum & Galbraith, 2007)

In my interpretation (Blomhøj, 2004), the three main arguments for teaching mathematical modelling as an integrated element in mathematics in general education can be identified in research under the educational perspective.

- (1) Mathematical modelling bridges the gap between students' real life experiences and mathematics. It motivates the students' learning of mathematics, gives direct cognitive support for the students' conceptions, and it places mathematics in the culture as a means of describing and understanding real life situations.
- (2) In the development of highly technological societies, competences for setting up, analysing, and criticising mathematical models are of crucial importance. This is the case both from an individual perspective in relation to opportunities and challenges in education and work-life, and from a societal perspective in relation to the need for an adequately educated workforce.
- (3) Mathematical models of different kinds and complexity are playing important roles in the function and forming of societies based on high technologies. Therefore, the development of an expert as well as layman competence to critique mathematical models and the way models and model results are used in decision making are becoming imperatives for the maintaining and further development of democracy.

The third argument is also part of the basis of the socio-critical perspective dealt with below, and here it is developed further. However, it is important to recognise that a critical perspective on mathematical modelling and the use of mathematical models in society are also included in the educational perspective.

It is within the educational perspective that we find most of the TSG21 papers, and in papers the research reflects modelling both as a means for learning mathematics and as an educational goal. Therefore, I do not distinguish between these two types of research in my listing of the TSG21 papers.

The paper by vom Hofe et al. (TSG21) is within the educational perspective. However, the research has a double focus. On the one hand, mathematical modelling is seen as a means to challenge and develop the students' mathematical understanding and especially their basic mathematical beliefs (Grundvorstellungen, GV), and, on the other hand, mathematical modelling is seen as an educational goal in its own right. The research is based on comprehensive data material from a longitudinal study yearly assessing grade 5 to 10 students' performance solving mathematical modelling tasks. The findings concerning the development of the students' modelling competency from grade to grade in the three different school branches in the German system are presented and discussed. However, the data are also intended for pinpointing weak spots in the students' mathematical understanding and beliefs (their GVs) at the different grade levels and the various school branches, with the intention of forming a basis for designing teaching material that would help overcome identified learning difficulties in the future. The connection between the students' mathematical beliefs (GVs) and their performance in modelling task is illustrated in figure 2.

It is in the processes of mathematization and interpretation that students' basis mathematical beliefs (GVs) can be unveiled.

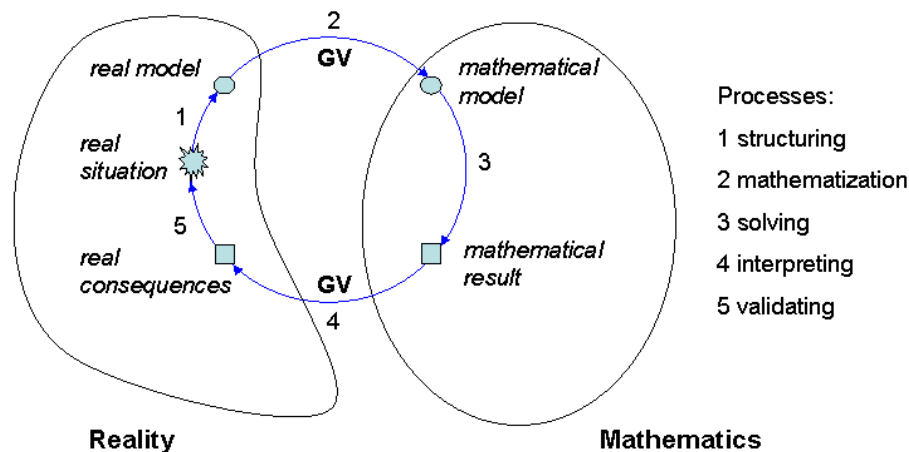


Figure 2: The modelling process. The students' basis mathematical beliefs (GVs) are activated in particular in relation to processes of mathematization and interpretation.

Ludwig & Xu (TSG21) report on a comparative study on the development of mathematical modelling competency in upper secondary students in Germany and China. Building on the conceptualisation of the mathematical modelling process by Blum & Leiss (2005), the authors define five levels of mathematical modelling competency, which they use to measure the students performance on different modelling task in the two different countries. This research lies within the educational perspective with a clear focus on mathematical modelling competency as an educational goal.

The paper by Meier (TSG21) reports on a comprehensive developmental European project supported by the EU, where mathematics teachers, primarily for the secondary level, and mathematics education researchers from eleven countries work together to develop and test mathematical modelling tasks. One of the main research questions in the project is “What is a good modelling task”, and so far a template for assessing modelling task with respect to particular learning objectives has been developed in the project. In the paper this template, which is intended to be a tool for teachers for selecting and reflecting on modelling tasks, is explained and illustrated through the analysis of a particular task. The project clearly lies within the educational perspective and the research characterising good modelling tasks tries to take both types of goals into account.

Oliveira & Barbosa (TSG21) have investigated tensions that elementary teachers experience when teaching mathematical modelling. This research also seems to be within the educational perspective. However, it is not told in the paper whether the teaching was focusing on modelling as a means for learning mathematics or as a goal. The tensions that the teachers experienced might be different in the two cases, so this something worth discussing further.

The paper by Rodríguez already mentioned also belongs as fare as can see, to the educational perspective and her research also seems to have a dual focus on mathematical modelling as a goal as well as an means for learning mathematics and, in her case, also physics. The paper by Kadjevich too could be considered to belong to the educational perspective. However I think that the references to real life problems and the integrated use of technology are so prominent in the paper that is more adequately placed under the realistic perspective.

The epistemological perspective

Under the epistemological perspective mathematical modelling is subordinated the development of more general theories on the teaching and learning of mathematics. Two very different examples of such theories are the Realistic Mathematic Education theory (RME) (see Treffers (1987), and Gravemeijer & Doorman (1999)) and the theory of mathematical praxeologies developed by Chevallard (see Garcia et al, 2006).

The paper by Andresen (TSG21) is the closest we get to a study with a particular reference to one for these two general theories. In this paper the author presents and discusses a model (in a different sense) for teaching mathematical modelling, which is based on her combining the four level model of mathematical activities developed in RME and a four level model of mathematical reflections from the philosophy of mathematics. The teaching model is illustrated by a number of questions, which refer to specific mathematical tasks, and which are intended to prompt students' reflections on each of the four different reflection levels. The model is intended as a tool for upper secondary teachers to balance, on the one hand, the instrumental aspects of the students' work with solving problems related to modelling and models using advanced CAS calculators and related reflections on the modelling process and use of the model results, on the other hand. The model has not yet been tested in teaching practice. In my view, this research falls under the epistemological perspective, since the main research interest seems to be to understand and describe the nature of the mathematical activities and related reflections involved in CAS-supported mathematical modelling.

Of course, one also finds research within mathematical modelling where the aim is to support the development of theories of other types and nature. The papers by Tarp, and I think also, the paper by Siller are examples hereof.

Tarp (TSG21), in his paper, analyses the epistemological basis of mathematical concepts in arithmetic, algebra and analysis. He argues that traditional mathematics teaching, which he labels "pastoral metamatism" disregards important aspects of the epistemology of the concepts, and that this lead to serious learning difficulties when the concepts are activated in modelling reality. The basic claim in the paper is that fundamental mathematical concepts should be re-invented in mathematics teaching through work on modelling real phenomena, without losing important aspects of the concepts' epistemology.

The work by Siller (TSG21) analyses the potentials of a particular type of software, Prograph, as a tool for teaching functional modelling, I would also consider this investigation as belonging to the epistemological perspective. The aim here is to develop and investigate a technological tool for supporting the students' modelling of stochastic phenomena, and the main focus seems to be on how certain mathematical concepts and relations can be represented in a particular algorithmic form so as to be represented in Prograph.

The cognitive perspective

Within the cognitive perspective the main interest is to understand which cognitive functions are involved in individual students' mathematical modelling activities. To that end students' particular modelling processes are analysed and they are interviewed with the purpose of reconstructing their individual routes through the modelling process in specific modelling situations. The aim is to identify (types of) individual cognitive barriers in the modelling processes.

This perspective is of course closely related to the educational perspective and the goal of developing mathematical modelling competency. However it could also be considered as basic research on the learning of modelling competency, while research under the educational perspective rather might be characterised as applied science. The work by Boromeo Ferri (2006) is a good example of research within the cognitive perspective on mathematical modelling.

I only find one TSG21 paper clearly belonging to the cognitive perspective. That is the paper by Camarena (TSG21). She investigates the cognitive skills involved in the modelling of an engineering problem. In this work a conceptualisation of a mathematical modelling process in engineering is used to structure the analysis. The cognitive elements are mainly related to the (1) mathematical conceptions that are needed in the mathematisation process, (2) more general cognitive skills in engineering related to the different phases of the modelling process, and (3) specific cognitive elements concerning particular types of engineering models and problems. I regard this research as belonging primarily to the cognitive perspective. However, the main objective behind the research is to place the mathematical modelling competency as an important part of mathematics in a science and engineering context. Therefore the research also has an educational perspective in relation to modelling competency as a goal.

The socio-critical perspective

Mathematical models of different kinds and complexity are playing important and growing roles in the functioning and forming of societies both in developing countries and in more developed countries. Mathematical models are used to define and describe social and economic inequality; both micro and macro economics are built on mathematical models of different kinds – e.g. interest rates in micro loans and mortgages for financing real estate, population and epidemic forecasts and control policies are based on mathematical models, while taxation and election systems are prescribed by mathematical models, and healthcare data and crime rates are described and discussed by means of statistical models. These and many more important aspects of societal life are being transformed and formatted through mathematical modelling and applications of mathematical models. Therefore the development of an expert as well as a layman competence in the general populace to critique mathematical models and the ways in which they are used in decision making, is becoming, imperative for the developing and maintaining of societies based on equality and democracy. Skovsmose has analysed the formatting power of mathematical modelling in detail and discussed its consequences for mathematics education in several papers and books, see Skovsmose (2005, part 2). This analysis forms an important part of the basis of the social-critical perspective on mathematical modelling in mathematics education.

The extensive use of mathematical modelling in society contributes to establishing mathematics as a language of power. Therefore, mathematics education, and especially the teaching of mathematical modelling and applications, possesses a potential for empowering students as autonomous and independent citizens in society. This phenomena is captured and analysed by D'Ambrosio (1999) using the concept *mathemacy* as a parallel to *literacy*. Moreover, the uncovering of the societal role and function of mathematical modelling in the teaching of mathematics can create an important motivation for learning mathematics and modelling among students. This seems to be especially evident in areas and countries with much poverty and inequality.

Empowering students to use mathematical modelling to reflect critically on societal issues and to criticize actual mathematical modelling processes and authentic applications of

mathematical models in real life situation, is therefore pinpointed as the predominant goal for teaching mathematical modelling under the social-cultural perspective.

This leads to teaching practices where students are working with mathematical modelling in relation to real life problems – often of a social, medical and environmental nature. In this work particular emphasis is placed on the development of a reflexive discourse related to the modelling process among the students. Quite a few of the TSG21 papers belong to the socio-critical perspective and this perspective seems to be developing fast in some Latin American countries.

The paper by Barbosa (TSG21) is an example of this approach. In this study the question is how to support the development of a reflexive discourse related to the students' mathematical modelling. The point of departure is the students' modelling in relation to an environmental problem concerning the level of water in a lake used for power production. From the findings it seems that in this educational setting it was possible to create a reflexive discourse among the students concerning the criteria for a good model in the given situation and in relation to the comparison of the different models developed by the students.

Araújo's (TSG21) paper is another example of research adhering to the social-critical perspective. Here the thesis of the formatting power of mathematics is addressed directly. The paper analyses and discusses to what extent undergraduate students working with modelling the yearly variation of rainfall in a particular region, based on authentic data, actually experience the formatting power of their own modelling.

Caldeira (TSG21) describes and analyses a particular form of teaching practice implemented in teacher education up till middle school. The approach is called Participatory Environmental Diagnosis (PED) and the main idea is that groups of teacher students should take as their point of departure their local district and identify a number of serious environmental problems in that district. In this practice, it seems as if mathematical modelling naturally becomes part of the teacher students' work when they describe, analyse and forecast the seriousness of the particular problems. In this work the objects of the teacher students' reflections and critique are the environmental problems and the societal handling of these problems, and here mathematical modelling becomes a means for such reflections, rather than an end in itself.

The paper by Aravena & Caamaño (TSG21) reports on a developmental project at secondary level where the focus was on developing the students' mathematical modelling competency through project work. The students are from a non-privileged area. They decided themselves about the theme of their modelling work and they chose the prevalence of breast cancer. The main educational focus in the project was on the formative impact on the students of working in project groups using mathematical modelling to analyse an important societal problem. I therefore consider this project as belonging to the socio-critical perspective rather than the educational perspective.

Generally, within the social-critical perspective on teaching mathematical modelling reflection and critique plays a dominant role. However, the object of these reflections and critique can be either (1) the modelling process or selected sub-processes, (2) actual applications of mathematical models or (3) societal issues where mathematical modelling is used as means for critique.

In the case of (2) the situation could be depicted as in figure 1. A mathematical model has been developed for intended applications under the influence of certain interests held by the acting subjects. The modelling process that lies behind the model could be theoretically and/or empirically well or less well or even bad founded. However, when the model is applied as part of a political and/or technological investigation, the model is often separated from the modelling process and from the possible critical reflections connected to the modelling process behind the model. The model is used for particular purposes that might be different from what the model was originally constructed for, and its new use may be guided by other interests. Therefore the process of applying a mathematical model in general tends to cause (a) a reformulation of the problem in hand in order for it to be adequate for investigation by means of mathematical modelling, (b) changes in the discourse about the problem in the direction towards pro and contra the model and possible adjustments of it, (c) a limitation of the possible actions taken into consideration to those that can easily be evaluated within the framework of the model, and (d) a delimitation of the group of people that can take part in the discussion and create a basis of critique. Awareness of and experiences with such general phenomena can provide a strong basis for the students' external reflections concerning the use of mathematical models and is also one of the main concerns within the social-critical perspective on the teaching of mathematical modelling and applications.

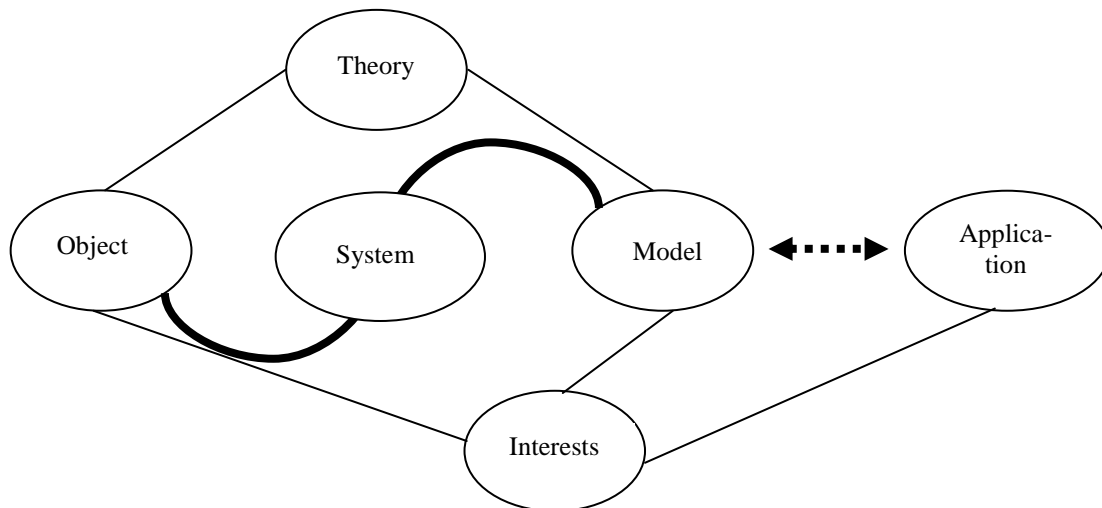


Figure 3: Application of a mathematical model to real life problem is a process more or less independent of the modelling process, and the application process tends to cause changes in the context of the real life problem as well in the model when it is adjusted to the problem.

As a closing remark I would like to repeat that the different perspectives on research in the teaching and learning of mathematical modelling, taken from Kaiser & Sriraman (2006) and used in this paper to categorise the TSG21 paper, should be thought of as tentative categories with considerable overlap open for interpretation and debate.

Template over different perspectives on the teaching and learning of mathematical modelling

Perspective	Aims	Background	Papers from TSG21	Research questions	Role of modelling cycle
Realistic	Pragmatic-utilitarian goals	Pollak (1969)	Kadijevich Lombardo & Jacobini	What conditions and support (in form of IT) are needed to model a particular real problem?	Used for analysing a real life practice or problem situation.
Contextual	Subject related and psychological goals	Lesh & Doerr (2003) Lesh & Caylor (2007)		How to design contexts for students' meaningful modelling activities?	The modelling process is not in focus - Modelling Eliciting Activities are
Educational – learning mathematics	Modelling as a means for learning mathematics	Niss (1987,1989) Blum & Niss (1991) Blum & Leiss (2005) Blum, Niss et al. (2006)	vom Hofe et al. Ludwig & Xu Meier Oliveira & Barbosa Rodríguez	How to challenge the students' mathematical conceptions (GVs) and to support their mathematical learning?	Used for designing and analysing modelling tasks with respect to particular intentions for the students' learning
Educational – learning modelling	Modelling competency as an educational goal			What is a good modelling task? What specific learning difficulties can be detected in relation the different phases in modelling	Used for defining mathematical modelling competency as a learning goal. Building heuristic for modelling
Epistemo-logical	Re-constructing mathematics through modelling, RME. Math. praxeologies	Freudenthal (1983) Traffers (1987) Chevallard	Andresen Tarp Siller	How can modelling be used to re-construct the function concept for learning?	Model of – model for Used to characterise a modelling praxeology
Cognitive	Analyses of cognitive processes involved in mathematical modelling	Piaget, Skemp Boromeo Ferri (2006)	Gamarena	What cognitive structures are involved in modelling competency and which cognitive skills are related to the different phases in the modelling cycle?	Used to structure the modelling process so as to identify the cognitive skills needed to model a given situation
Socio-critical	Critical and reflexive understanding of reality and the use of mathematical modelling	Skovsmose (1994, 2005) D'Ambrosio (1999)	Barbosa Araújo Caldeira Aravena & Caamaño	Uncover the formatting power of math. modelling How to create reflexive discourses among students	To structure the critique and reflections in relation to the modelling process and the application process

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