

**Different perspectives on mathematical modelling in
mathematics educational research
- categorising the TSG21 papers**

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What do we have?

- A global theory for the teaching and learning of mathematical modelling?
- A number of different perspective and approaches
- Fifteen interesting research papers accepted for TSG21

Six perspectives on research in modelling and applications

(Kaiser & Sriraman, 2006)

Perspective	Main approach	TSG21
<u>Realistic</u>	Modelling as applied problem solving in real contexts	2
<u>Contextual</u>	Modelling Eliciting Activity, modelling as problem solving	0
<u>Educational</u>	Modelling as a means and as a goal. Modelling competency	5
<u>Epistemo-logical</u>	Realistic Mathematics Education, Model of – Model for Mathematical praxeologies	3
<u>Cognitive</u>	The learning processes and difficulties involved in modelling	1
<u>Socio-critical</u>	The formatting power of modelling Reflection, critique and empowering	4

In Kaiser, Blomhøj and Sriraman (2006, p. 82) , we argued that it is possible on a general level to identify in the field of research

... a global theory for teaching and learning mathematical modelling, in the sense of a system of connected viewpoints covering all didactical levels: learning goals, fundamental reasons for pursuing these goals at different levels of the educational systems, tested ideas about how to support teacher's in implementing learning goals and recognised didactical challenges and dilemmas related to different ways of organising the teaching, theoretically and empirically based analyses of learning difficulties connected to modelling, and ideas about different ways of assessing students' learning in modelling activities and related pitfalls.

The realistic perspective

- The widespread use of mathematical modelling in scientific and technological disciplines
- Mathematical modelling as applied problem solving
- Emphasize authentic modelling, data and problem situation
- Use of technology as a resource for modelling
- Take the subject area of application seriously and view modelling as an interdisciplinary activity
- Pragmatic real life criteria for the modelling process
- Pollak (1969) is an basic reference

A paper within the realistic perspective

The paper by Djordje Kadijevich: Simple spreadsheet modelling by first-year business undergraduate students: Difficulties in the transition from real world problem statement to mathematical model

In this study the students are

- modelling the total balance for realistic business activities by means of spreadsheet
- collecting and working with real/realistic data
- using a heuristic for technology-supported modelling of real life situations
- using support for business decisions as criteria for success in their modelling work

Papers within the realistic perspective:

Simple spreadsheet modelling by first-year business undergraduate students: Difficulties in the transition from real world problem statement to mathematical model

Djordje Kadijevich, Serbia

Mathematical modelling: From classroom to the real world

Denise Ferreira Lambardo & Otavio R. Jacobini, Brazil

The contextual perspective

- The educational importance of mathematical problem solving in every day real life contexts
- Modelling eliciting activities (MEA) starting from meaningful problem situations
- Emphasize on authentic situations in teaching enabling the students to set up their own criteria for their modelling work
- Design principles for MEA are developed and tested in research and the clear didactical structuring distinguishes this perspective from the realistic
- The learning difficulties of modelling are understood within the psychology of problem solving.
- Lesh & Doerr (2003) is an key reference

[Table](#)

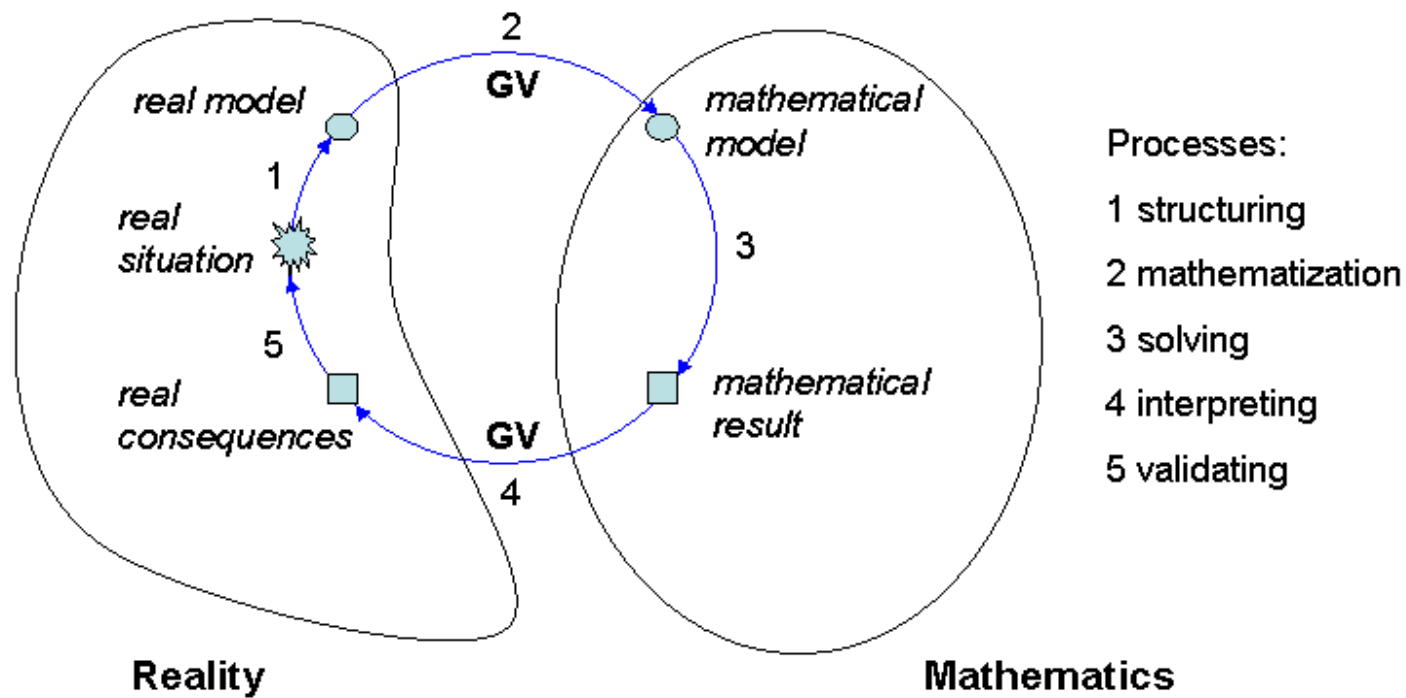
The educational perspective

- focuses on the integration of mathematical modelling in mathematics teaching
- studies classical didactical questions concerning why, what and how mathematical modelling
- emphasises the notions of a mathematical model, a mathematical modelling process and mathematical modelling competency
- see modelling as a goal in it itself – developing mathematical modelling competency – as well as a means for learning mathematics
- see reflections and critique in relation to mathematical models and applications of models as and an important element in modelling competency
- Niss, Blum & Galbraith (2007) gives a nice overview

A paper within the educational perspective

The paper by Rudolf vom Hofe, Alexander Jordan, et al.
On the development of mathematical modelling competencies
– *The PALMA longitudinal study*. This study

- investigates the development of students modelling competency from grade 5 to 10
- is based on a particular model of a mathematical modelling process (Blum & Leiss, 2005)
- analyses the students' difficulties in the modelling process especially those related to the mathematisation and the interpretation processes
- and pinpoint particular basic mathematical beliefs (GV), held by the students, as the cause for the difficulties
- intends to develop teaching material including modelling tasks that could help overcome these difficulties



The modelling process. The students' basis mathematical beliefs (GVs) are activated in particular in relation to processes of mathematization and interpretation. Blum & Leiss (2005)

Papers within the educational perspective:

On the development of mathematical modelling competencies – The PALMA longitudinal study

Rudolf vom Hofe, Alexander Jordan, Thomas Hafner, Pascal Stölting, Wener Blum & Reinhard Pekrun, Germany

A comparative study on mathematical modelling competences with German and Chinese students

Matthias Ludwig, Germany & Binyan Xu, China

The teachers' tensions in the practice of mathematical modelling

Andréia Maria Pereira de Oliveira & Jonei Cerqueira Barbosa, Brazil

Papers within the educational perspective:

Mathematical modelling in a European context – A European network-project

Stefanie Meier, Germany

Differential equations as a tool for mathematical modelling in physics and mathematics courses

Ruth Rodríguez Gallegos, France/Mexico

The epistemological perspective

- Under the epistemological perspective mathematical modelling is subordinated the development of more general theories on the teaching and learning of mathematics.
- Modelling is seen as a means for learning mathematics and not as an education end
- Two very different examples of such theories are the Realistic Mathematic Education theory (RME) and the theory of mathematical praxeologies developed by Chevallard
- The critical perspective on the modelling process and applications of mathematical are not in focus with the epistemological perspective.

A paper within the epistemological perspective

The paper by Mette Andresen, *Reinforced bonds between modelling and reflecting* is within this perspective

- The paper presents and discuss a “model” for balancing the instrumental aspects for upper secondary students’ work with modelling and applications supported by the use of CAS calculators with related reflections
- The model is based on the four levels of mathematising from RME: situational, referential, general and formal and four level of reflection in mathematical activity – from doing mathematics to epistemological reflections
- The main research interest is to understand and describe the nature of the mathematical activities and related reflections involved in CAS-supported mathematical modelling.

Papers within the epistemological perspective

Reinforced bonds between modelling and reflecting

Mette Andresen, Denmark

Applying mathematics: Metamatics or Mathematism?

Allan Tarp, Denmark

Prograph Diagrams - a new old system for teaching
functional modelling

Hans-Stefan Siller, Austria

The cognitive perspective

- Within the cognitive perspective the main interest is to understand which cognitive functions are involved in individual students' mathematical modelling activities.
- Individual students' cognitive processes and their individual routes through the modelling are analysed.
- The aim is to identify (types of) individual cognitive barriers - be it mathematical, psychological barriers or cognitive barriers related to modelling.
- This perspective is closely related to the educational goal of developing modelling competency, and it could also be considered as basic research on the learning of modelling competency within the educational.
- Boromeo Ferri (2006) is a good example of research within the cognitive perspective.

A paper within the cognitive perspective

The paper by Patrica Camarena Gallardo: *Mathematical models in the context of sciences* is the only TSG paper within this perspective.

- The study is framed within a theory called “Mathematics in the sciences context” develop by the author
- The paper investigates the cognitive skills involved in the modelling of engineering problems.
- The cognitive elements identified are mainly related to the (1) mathematical conceptions that are needed in the mathematisation process, (2) more general cognitive skills in engineering related to the different phases of the modelling process, and (3) specific cognitive elements concerning particular types of engineering models and problems.

[Table](#)

The socio-critical perspective

- takes as its point of departure that mathematics through modelling and applications of models is a strong formatting power in modern societies. Therefore, mathematics education has a responsibility to support the development of reflections and critique in relation to mathematical modelling and applications in all students.
- sees mathematics education, and especially the teaching of mathematical modelling and applications as having a potential for empowering students as autonomous and independent citizens in society – mathemacy
- emphasizes the need of a dialogical learning theory to support these educational goals
- Skovsmose (1994, 2005) and D’Ambrosio (1999) are key references for the socio-critical perspective.

A paper within the socio-critical perspective

The paper by Ademir Donizeti Caldeira, *Mathematical modelling and environmental education* is a clear example.

- The paper describes and analyses a particular form of teaching practice implemented in teacher education, called Participatory Environmental Diagnosis (PED).
- Groups of teacher take as their point of departure modelling activities their local district and identify serious environmental problems in that district.
- Mathematical modelling seems naturally to become part of the teacher students' work describing, analysing and making forecast for the particular problems.
- The object for students' reflections and critique are the environmental problems and the societal handling of these problems, and mathematical modelling is a means for such reflections, rather than an end in itself.

Reflections and critique in the context of modelling

– internal reflections

- Reflections related to the different phases in the modelling process. For example in:
 - the formulation of the problem
 - creating the object of the modelling process
 - idealisations and abstractions made in the systematisation
 - in mathematisation process
- Reflections in relation to knowledge base or theoretical foundation of the modelling process
- Reflections concerning the empirical basis for estimation of model parameters and validation of the model
- Reflections concerning the documentation of the modelling process

Reflections and critique in the context of application

– external reflections

- Reflections related to the context of the application.
Is the model and its results used
 - in a scientific investigation
 - in the development of a technology design
 - in a particular societal or business decision process
 - in general societal or technological
- Reflection on the models relevance and validity in relation to the context of the application
- Reflections related to the function of the model
 - reformulation of the original problem(s)
 - changes in the discourse to pro and contra the model
 - limitation of actions considered to the domain of the model
 - delimitation of the group of people included in the process
 - the possible consequences of the application

Papers within the socio-critical perspective

Mathematical modelling, the socio-critical perspective and the reflexive discussion

Jonei Cerqueira Barbosa, Brazil

Formatting real data in mathematical modelling projects

Jussara de Loiola Araújo, Brazil

Mathematical modelling and environmental education

Ademir Donizeti Caldeira, Brazil

Mathematical models in the Chilean education

Maria Aravena Días & Carlos Caamaño Espinoza, Chile

[Table](#)