Two mathematical reforms in their context in twentieth century France : similarities and differences.

First, I have to apologize not being with you in Monterrey and to thank Renaud d'Enfert who has accepted to read my paper.

This talk is focused on two reforms which were undertaken in France during the twentieth century: the first one, at the very beginning of the century, in 1902, and, the second one, in the second part of the sixties. Both changed dramatically mathematical curricula, influenced -both times- by new goals and organization of education system and new epistemological views on mathematics. We shall present, in this paper, similarities and differences induced by these two different social and epistemological contexts.

These two moments were moments of deep reorganisation of the structures and the goals of instruction - secondary instruction for 1902, primary and secondary instruction for the sixties – taking into account new goals and new audiences.

1902 reform concerned the *lycées*, that is the type of school intended for the learned and social élite – including the scientific one - which provided, during all the nineteenth century, first and foremost, classical and humanist education. Mathematics education was on the fringe of this secondary training, postponed to the very final year of the *lycée*. And that, even for the scientific élite who continues their studies in *Grandes écoles* like the *Ecole* polytechnique where mathematics was so essential. This monopoly of classical humanities in the *lycées* became more and more untenable for the economic and political élites of the French Third Republic who came to initiate in 1899 a comprehensive inquiry all over the country to discuss the educational question of this time: Which training, for which elite in a modern country? What modernity, what humanities does the country need?
As for mathematics and science, which were always considered together, different positions were maintained, and sometimes complementary values were argued for: first, cultural values, as part of new modern humanities – so called the "scientific humanities"- together

with living languages and French modern literature ; second, utilitarian values, mathematics seen as an applied and practical subject, its applications being another part of the modernity.
As a consequence of this enquiry, 1902 reform was undertaken. One of the considerable impact of the reform was the end of the monopoly of classical humanities and the development of "modern" subjects as languages, science and mathematics.

In the sixties, important institutional educational reforms took place in France to allow the country to assume new economical stakes of post-second-world-war decades. Primary instruction became for all children the first stage of an extended school attendance in secondary system; "middle school" had then new aims and new publics which differed from precedent periods, providing education to children whose educational and social future was as different as long and general studies, practical studies or apprenticeship. At the same time, the "new math Refom" was presented as a response to modernity, as an attempt to meet new demands issued from new economical, social and cultural stakes. The fundamental new achievements of mathematics - in fields as diverse as structures, logic, optimisation, calculators and numerical analysis, statistics, computer science – caused agencies for economic development and government to assert a key role of mathematics in the industrialized society ; they wanted to promote a reform of the contents and the methods of mathematics instruction to make it more efficient.

In both cases, in 1902 and in the sixties, it has been taken for granted that mathematical instruction was a decisive part of modernity. But the rhetoric which linked mathematics and modernity was very different at the two moments.

In 1902, mathematicians and reformers stressed epistemological conceptions of geometry, which was the main topic in the *lycées* , as a physical science. Mathematics was a privileged tool for physics and had to be tied at physical reality. The considerable impact on mathematical education can be resumed by theses two quotations, the first one from Hadamard, the other from Borel:
* "In dealing with geometry as a physical science – what it's truly is – we shall make disappear what its teaching offers as artificial and tiresome."
* "We have to introduce more life and more sense of reality in our mathematics education".

In the sixties, the epistemology is radically different. Owing to the notion of structure, which has been promoted at the heart of mathematical research and activity, mathematics was thought as the common and universal language for the understanding of real, of natural but also human and social real. As a reservoir of abstract forms and structures, which were the new tools mathematics has to provide to comprehend the real and participate of modernity, mathematics had no more any privileged ties with physics and physical world. This epistemological conception was shared by all reformers, mathematicians as political deciders. The impact it had on mathematical education was even far more considerable than in 1902. Mathematical curricula, from primary school, had to take advantage of this new universality and efficiency of mathematics due to consideration of structures.

We shall now focus on a part of the curricula for which these epistemological differences led to divergent mathematical syllabus. We shall consider mathematics taught to children from thirteen to fourteen years old.

Regarding geometry teaching, it was emphasized in 1902 reform that it should "be essentially concrete". In a pedagogical conference to promote new curricula, Henri Poincaré advised to make use, as often as possible, of moving instruments (instrument mobiles) to define geometrical notions as line, circle, plane, parallels etc. And he added it was much more philosophical than it could appear, geometry being the study of the group of solid bodies movements.

The reform broke away from the classic principles of euclidian presentation grounded on abstraction and systematically distrusting intuition. The goal of geometry teaching became to class and precise notions acquired in every day life, to infer from it more complex and hidden notions and to show their applications to practical problems. To explain geometrical results, teachers had to call for experience and admit as an experimental truth all that seams obvious to the children.

Instructions for the new math reform curricula could not be more contradictory. First of all, it is said that mathematics is a deductive science and is not an experimental one. A logical presentation of the different mathematical notions has to be privileged in order to eliminate all that could rely on intuition and could be obvious for some pupils and not for others. Second, mathematics is a theory which has to reassemble scattered knowledge under a same structure. Were excluded from the syllabus notions which do not comply with this scheme.

Pupils of 13-14 years old were then invited to learn methodically deductive reasoning and to clearly differentiate concrete world from its mathematical model. The first geometrical model

taught to pupils was affine geometry, taught one year before the euclidian model for it is more simple on a mathematical point of view. The fact that it is, at the same time, the more irrelevant to the real world and every day life, did not appear first as a problem.

As to pedagogical methods which were respectively promoted by the reformers in 1902 and in the sixties, and it will be our last point, they were far less opposed that the differences in the curricula might suggested.

In 1902, as we have seen, teaching – at this level - had to be concrete, had to be based on intuition. These pedagogical principles, which where all new at that time for the *lycées*, came from methods used in the primary school system intended for lower classes. These schools gave a key role to mathematics and science education which were taught according to practical aims, and did focus on applications. This origin has not been easy to assume considering it was social élite who studied in *lycées* and it raised a question that Emil Borel stated in this words: "Don't we risk diminishing this great educative value [of secondary instruction] when making mathematics education more practical and less theoretical?". As for Borel, the ' answer was clearly: no.

Although the new math reform considered mathematics as a deductive science and not an experimental one, the reformers claimed an active pedagogy and a previous role of concrete situations in the learning process of axioms, abstract and formal definitions and properties. We have to said that, here too, there has been a huge difference between the model which was promoted and the realities of mathematics teaching in the classrooms.

We hope that examining the two reforms concurrently has permitted to stress the importance of the historical context and to show the way its different components - social, epistemological, pedagogical – are linked altogether.

Thank you.