Instructional Practices to Facilitate Prospective Mathematics Teachers' Learning of Problem Solving for Teaching

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Introduction

Problem solving is considered central to school mathematics. It is highlighted in reform documents as a key factor of change in mathematics education (NCTM 1989, 1991, 2000). As NCTM (2000, p. 52) states,

> Instructional programs should enable all students to build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving.

Similarly, Kilpatrick, Swafford, and Findell (2001, p. 420) explained,

> Studies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics. Problem-solving ability is enhanced when students have opportunities to solve problems themselves and to see problems being solved. Further, problem solving can provide the site for learning new concepts and for practicing learned skills.

Thus, problem solving is important as a way of doing, learning and teaching mathematics. However, whether or how such ways of viewing problem solving get implemented in the classroom will depend on the teacher. In addition, Schoenfeld (1985) has identified four aspects of students' problem solving that can be used to guide instruction – resources, heuristics, control, and beliefs. This means that teachers have to play a central role in helping students choose resources, implement heuristics, control their problem solving actions, and develop useful beliefs.

If problem solving should be taught to students, then it should be taught to prospective teachers who are likely to enter teacher preparation programs without having been taught it in an explicit way. If it is to form a basis of teaching mathematics, then prospective teachers should understand it from a pedagogical perspective. There are studies, discussed later, that raise issues about prospective teachers' understanding of problem solving and ability as problem solvers that could affect what and how they implement problem solving in their teaching. It therefore seems to be important that teacher education includes learning opportunities explicitly focused on problem solving. In this paper, I draw on studies, including my own work, that include instructional practices to facilitate prospective teachers' learning of problem solving and problem-solving pedagogy in order to highlight the nature of these practices and the learning that results from them and to discuss key characteristics of the practices that have implications for how we prepare prospective teachers to use problem solving in their teaching. The paper, then, is based on a review of research literature and a report of a study I conducted to identify (i) prospective mathematics teachers' knowledge and ability of problem solving; (ii) instructional approaches to facilitate their learning of problem solving; and (iii) implications for teacher education.

Prospective Teachers' Knowledge of Problem Solving

Studies focusing explicitly on prospective teachers' knowledge of problem solving are a scarcity in the research literature, regardless of whether routine or non-routine problems are considered. What are available deals mainly with the prospective teachers' ability or strategies in solving word problems, as in these examples. Schmidt and Bednarz (1995) examined modes of problem solving that 131 prospective elementary and secondary teachers used in arithmetical and algebraic word problems to identify the resistance and eventual difficulties that arose in the shift from one type of approach to the other. The majority of them confined themselves to algebra even when dealing with arithmetical problems. The prospective elementary teachers appeared to be the best prepared for addressing both fields, i.e., used arithmetic for arithmetic problems and algebra for algebra problems. van Dooren, Verschaffel, and Onghena (2003) investigated the arithmetic and algebraic word problem-solving skills and strategies of 97 prospective elementary and secondary teachers, both at the beginning and at the end of their teacher preparation. They found that the prospective secondary teachers clearly preferred algebra, even for solving very easy problems for which arithmetic was more appropriate. About half of the prospective elementary teachers adaptively switched between arithmetic and algebra, while the other half experienced serious difficulties with algebra. Contreras &Martínez-Cruz (2001) examined 68 prospective elementary teachers' solution processes to a word problem involving division of fractions in which the numerical answer to the division did not provide the appropriate solution to the problem when the realities of context of the problem were considered. They found that the participants did not always base their responses on realistic considerations of the context situation. Only 28% of their responses contained a realistic solution to the given problem. Contreras and Martinez-Cruz (2003) also examined 139 prospective elementary teachers' solution processes to additive word problems for which the solution was one more or one less than that produced by simply adding or subtracting the given numbers. They found that about 91% of the prospective teachers' responses contained incorrect solutions to the problems based on their failure to interpret correctly the solution produced by addition or subtraction of the two numbers given in each word problem. Simon (1990), in his investigation of 33 prospective elementary teachers' knowledge of division found that they failed to connect their understandings of division and the semantic features of the word problems to the procedures that they employed to divide. Finally, Verschaffel, De Corte, and Borghart (1996) investigated 332 prospective teachers' conceptions and beliefs about the role of real-world knowledge in arithmetic word problem solving. For each of the 14 word problems, the prospective teachers were first asked to solve the problem themselves, and then to evaluate four different answers given by students. The results revealed a strong overall tendency among the participants to exclude real-world knowledge and realistic considerations from their own spontaneous solutions of school word problems as well as from their appreciations of the students' answers.

In addition to the above studies that focused on word problems, two studies, Taplin (1996) and Leung (1994), dealt with problem-solving approaches, more generally, and problem-posing processes, respectively. Taplin (1996) explored the approaches to problem solving used by 40 prospective elementary teachers and found that they preferred to work with a narrow range of strategies, predominantly verbal and numerical. They tended to select a method of approach and not change from that through the tutorial, implying inflexibility in their choice or management of problem-solving strategies. Leung (1994) analyzed problem-posing processes (i.e., posing a sequence of problems in each problem-solving activity) of eight prospective elementary teachers with differing levels of mathematics knowledge. Findings showed that those with high mathematics knowledge systematically manipulated given conditions to make problems and used solutions to prior posed problems as new pieces of information to pose subsequent problems.

Those with low mathematics knowledge posed problems that might not be solved mathematically and the mathematics problems posed were not necessarily related in structure. My work (Chapman, 2005) also examined prospective secondary mathematics teachers' knowledge of problems and problem solving as part of a study to investigate an instructional approach to enhance participants' knowledge of problem solving. The study indicated that in relation to their initial knowledge, most of the participants made sense of problems in terms of the traditional, routine problems they had experienced, directly or indirectly, prior to entering the teacher education program. They also understood these problems as genuine problems that required thought and logic to arrive at a solution. They understood the problem-solving process in a way consistent with the traditional classroom approach of dealing with routine problems.

These studies imply concerns about how prospective teachers of mathematics may conceptualize problem solving and engage in it. They provide evidence that prospective teachers are likely to need help in their development and understanding of problem solving from the perspectives of a learner and a teacher.

Instructional Practices for Problem Solving in Teacher Education

Based on a review of current research in mathematics teacher education for this paper, there seems to be also a scarcity of published studies that explicitly address instructional practices for problem solving in teacher education. In this section, I focus on five studies that include some form of open or non-routine problems that were intended to play a role in the development of prospective teachers' knowledge of non-routine problem solving and problem-solving pedagogy. Two of these studies are explicitly about problem solving in terms of their stated goals. None defined problem solving, but the implication is that it is associated with a way of thinking involved in solving non-routine problems in the learning of mathematics. These studies provide some basis for instructional practices in helping prospective teachers to grow in their knowledge of problem solving for teaching. This aspect of these studies will be highlighted here in order to make explicit possible ways of engaging prospective teachers in problem solving. In particular, the focus will be on the approaches used in these studies to facilitate learning of problem solving; the goals relating to problem solving of these approaches; and the effect of the approaches on the prospective teachers' learning of problem solving and its pedagogy. This sample of studies described next deals with approaches to help prospective teachers understand: the nature of problems (Arbaugh & Brown, 2004); the problem solver and problem-solving pedagogy (Lee, 2005); the problem-solving process (Ebby, 2000; Roddick, Becker & Pence, 2000; Szydlik, Szydlik, & Benson, 2003). All of these studies directly or indirectly dealt with problem-solving pedagogy, e.g., how to facilitate students' learning of problem solving. But there is less focus on teaching through problem solving.

(a) Nature of Problems (Arbaugh & Brown, 2004)

Although this study is not explicitly about problem solving, the focus on tasks in the form of problems makes it relevant to problem solving, which begins with the selection of worthwhile problems. The goal of this study was to help prospective teachers understand the relationship between a task and the kind of thinking that task required of students. The tasks consisted of mathematical problems, both routine and non-routine, categorized based on "level of cognitive demand," i.e., the type of thinking that the task required of students. For example, the category that can be associated with genuine problem solving is "higher-level demands (doing mathematics)" with the following characteristics: require complex and non-algorithmic thinking; demand self-monitoring or self-regulation of one's own cognitive processes; require students to access relevant knowledge and experiences and make appropriate use of them in working through the task; require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions; require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required (Arbaugh & Brown, 2004, p. 30).

The instructional approach consisted of a task-sorting activity. The researchers developed a set of high school-based mathematical tasks to us in the task-sorting activity in order to help the teachers to learn about the levels of cognitive demands. They used the task-sorting activity with six different groups of prospective high school mathematics teachers.

The effect of the approach was that it was successful in helping the prospective teachers learn the levels of cognitive demand criteria. Each group left their methods class with the ability to categorize tasks according to the criteria, and overall the prospective teachers had been able to communicate the importance of considering the cognitive level demanded of the tasks. They learned to look past surface characteristics of the individual problems and analyze them on the basis of the types of thinking required by students. They built knowledge about the activities that had enabled them to reach a deeper understanding of worthwhile mathematical tasks and the relationship between those tasks and students' learning.

(b) Problem-solver and Problem-Solving Pedagogy (Lee, 2005)

This is one of the studies that focused explicitly on problem solving with an approach that allowed participants to learn about students as problem solvers and about problem-solving pedagogy. The goal of the approach was to help the prospective teachers interpret and develop in their role of facilitating students' mathematical problem solving with a technology tool.

The approach consisted of a cycle of "planning–experience–reflection" repeated twice during an undergraduate course to allow the prospective teachers to change their strategies when working with two different groups of students. The prospective teachers enacted the six phases of the cycle by: (1) Individually solving the open-ended problem using a java applet and discussing the problem with peers and the teacher educator/researcher. (2) Developing anticipatory ideas and planning a learning trajectory for students. (3) Interacting with two students as they solved the same problem with a java applet. (4) Discussing the experience with peers, reflecting, and planning a revised learning trajectory for different students. (5) Interacting with two different students as they solved the same problem with a java applet. (6) Reflecting on their role in facilitating students' problem solving with technology and their understanding of what the students understood about the problem. Several prompts in Phase 2 of the cycle helped the prospective teachers think about students' learning trajectory by considering possible solution strategies, difficulties students may have, and questions that might be asked to help students overcome those difficulties. In Phase 4, the prospective teachers were asked to reflect on their interactions with students, students' understanding and problem solving, and changes or improvements desirable for the next group of students. In Phase 6, they were prompted to compare the two experiences and to reflect on what may have caused any similarities or differences in their interactions with the students and how the students solved the problem.

The effects of the approach, based on a study of three prospective teachers, was that the planning–experience–reflection cycle provided opportunities for them to begin to struggle with issues of facilitating students' problem solving and to make their struggle an open and reflective activity used as an opportunity to improve their practice. Six themes emerged from the cases, i.e., the prospective teachers: (1) Used their own mathematical problem-solving approaches to influence their pedagogical decisions. (2) Desired to ask questions that can guide students in their solution strategies without ''giving it all away.'' (3) Recognized their own struggle in facilitating students' problem solving and seem focused on improving aspects of their interactions with students. (4) Assumed the role of an explainer for part of each facilitation phase. (5) Made pedagogical decisions to use representations in the java applet to promote students' mathematical thinking or focus their attention on specific aspects of the problem. (6) Used the technology tools in ways consistent with the nature of their interactions and perceived role with students.

(c) Problem-Solving Process (Roddick, Becker & Pence, 2000)

This is another study that explicitly focused on problem solving with an approach that allowed participants to learn about the problem-solving process and some related pedagogical processes. The goal of the approach was to influence prospective teachers' problem solving, problem posing, modelling, and beliefs about the role of problem solving in teaching mathematics. The authors organized two courses aimed at: improving prospective teachers' problem-solving abilities, their learning of ways to assess problem solving, broadening their views of problem solving and mathematics, and enhancing their understanding of equity issues in teaching mathematics.

In the approaches used in two courses, the prospective secondary teachers were provided with rich and varied problem-solving experiences. They spent significant time on topics such as: what is a problem; examination of problem solving in traditional and innovative curricula; equity issues in problem solving and its assessment; assessment of problem solving and use of technology. Students used a model for reflecting on one's problem solving (i.e., that of Mason, Burton, & Stacey, 1985) and concentrated on specializing, generalizing, and justifying their work. Both courses included substantial in-class time working in groups on problems and giving presentations and justifications to the class.

The effect of the approach was to impact changes of the participants' beliefs and practice to various degrees. The participants fell on a continuum ranging from not much discernible implementation to substantial integration of problem solving in their teaching, as in the case of one participant, described in detail. She experienced considerable growth in her views of problem solving and its role in instruction and incorporated such learning into her teaching. The case study demonstrated the changes that can occur in beliefs and instruction as a result of an intensive year-long course that immerses prospective teachers in being reflective problem solvers themselves.

(d) Problem-Solving Process (Szydlik, Szydlik, & Benson, 2003)

This study involves problem solving in an indirect way as part of a mathematics content course for prospective elementary teachers that was designed to provide participants with authentic mathematical experiences and to foster autonomous mathematical behaviors, i.e., behaviors that involve sense-making rather than memorization or appeals to authority. The implied goal of the approach in relation to problem solving was to help the prospective teachers to become autonomous problem solvers by promoting community autonomy rather than autonomy of individuals.

The approach involved engaging the prospective teachers in authentic mathematical behaviors arising out of community work on a set of demanding problems, each of which had an underlying mathematical structure that formed a part of the course content. The problems generally allowed for a variety of problem-solving strategies with a mathematical structure that can be discovered by collecting data, solving a smaller version of the problem, considering several specific cases, or by logical considerations. During a typical class meeting, the prospective teachers worked for 20 to 30 minutes on such a problem in small groups of three or four. The class then convened in a large semicircle for a discussion of their findings, strategies, solutions and arguments. In these discussions, the course instructor emphasized the necessity of mathematical justification; complete solutions required logical arguments. The class was designated as the mathematical authority. The instructor declined to give the final word on the correctness or completeness of any solution and there was no text. The instructor also provided almost no assistance in the problem solving aspect of the course and no answers were provided for problems. The only way for the class to understand a problem was to figure it out. The only way to know they were correct was to find a convincing argument. However the spirit of the class was consistently one of community inquiry. On six occasions throughout the semester, the prospective teachers were asked to produce written reports that focused on their mathematical thinking by describing the problem, discussing the strategies they used to work on the problem (including those that did and did not lead to a solution), providing a solution, and, finally, arguing that their solution was complete and valid. In some cases these reports were produced as group projects. These reports provided opportunities for further reflection and discussion.

The effect of the approach was that a classroom focusing on problem solving using a variety of strategies, reflection on the process of problem solving, and engagement in the process of exploration, conjecture, and argument can help prospective teacher develop mathematical beliefs that are consistent with autonomous behavior. The community work on the problems made the process less frustrating for the prospective teachers, allowed them to see the ways in which their peers did mathematics, and showed them that problems could be solved in more than one way, i.e., a broadening in the acceptable methods of solving problems. The participants' beliefs became more supportive of autonomous behaviors during the course.

(e) Problem-Solving Process (Ebby, 2000)

This study involves problem solving in an indirect way as part of a methods course in a teacher education program that aimed to integrate fieldwork and coursework. One goal of the approach studied that relates to problem solving is helping prospective teachers to draw on their experiences as learners of problemsolving in developing a conception of their teaching role. Thus the approach provided opportunities for them to learn about problem-solving and mathematics pedagogy.

The approach involved inviting the prospective teachers to participate in mathematical inquiry through individual, cooperative, and whole-class problem solving. Each week the professor assigned a non-routine problem-of-the-week and encouraged the prospective teachers to work together on the solutions outside of class. In class, the professor asked for volunteers to share their solutions and strategies with the rest of the class. These problem-solving activities were designed to invite prospective teachers to learn mathematics in a non-threatening environment and to reconsider what it means to learn, teach, and know mathematics. By focusing on solution process over answer, the professor was also modeling an alternative pedagogy for them.

The effect of the approach based on the three participants studied was to provide them with a different experience. One participant experienced what it was like to be an active agent in her own mathematical learning through her engagement in problem solving. Another discovered that others had a diversity of approaches to mathematical problems and that their understandings were often different from hers. Another developed a new definition of the nature of mathematics as a result of her engagement in mathematical problem solving in the methods course.

The Author's Multi-goal Approach

This section deals with a study I have conducted that explicitly deals with problem solving as its primary focus. It is an extension of the instructional approach reported in Chapman, 2005. It incorporated what was learned then to further enhance the approach. Thus the goal of this follow-up study was to investigate the extended approach to help prospective teachers to understand problem solving as a mathematical and pedagogical process, i.e., as mathematical thinking and a method of instruction, respectively. This is a multi-goal approach because it explicitly deals with: development of understanding of problems, problem solver, problem-solving process, problem-solving pedagogy and problem solving as inquiry-based teaching. What follows is an abbreviated report on the study to highlight the nature and effect of the instructional approach.

Theoretical Perspective of Approach: The approach is framed in the work of Dewey (1933) with a focus on inquiry, reflection and social interactions; cognitive guided instructions (Carpenter et al., 1999) with a focus on understanding students' thinking and strategies to inform instruction; and narrative inquiry (Polkinghorne, 1988), with a focus of using personal experiences to understand self. An inquiry perspective framed in social constructivism formed a basis of the learning activities used in this study.

Description of the Approach: The activities were organized in three stages: individual reflection, inquiry activities and final reflections.

Individual-reflection: The first stage focused on individual self-reflection on problems and problem solving in order to create awareness of each of the prospective teacher's initial conceptions and knowledge. The participants were required to respond to a list of questions and prompts in sequence that included: What is a problem? Choose a grade and make a mathematics problem that would be a problem for those students. What did you think of to make the problem? Why is it a problem? Is it a 'good' math problem? Why? What process do you go through when you solve a problem? If possible, represent the process with a flowchart.

Inquiry activities: The second stage consisted of inquiry activities intended to extend the prospective teachers' initial conceptions and knowledge. The prospective teachers worked on all problems in these activities without the facilitator's intervention. These activities included:

(1) Comparing different types of problems without solving them in order to explore the nature of problems used in teaching mathematics and the goal of these problems in learning mathematics. The prospective teachers were provided with a list of different categories of problems influenced by Charles and Lester (1982). They were asked to compare and contrast the problems and to draw conclusions about problems in learning/teaching mathematics.

(2) Writing and unpacking narratives of their experiences in order to examine the cognitive and affective components of the behaviors involved in solving a non-routine problem. The prospective teachers were required to write narratives of their experiences solving a problem that was assigned to them. The narrative had to be a temporal account not only of the mental and physical activities they engaged in to solve the problem, but the emotional aspects of the experience. They later analyzed the narrative in terms of the affective aspect of the experience.

(3) Investigating others (e.g., peers and secondary school students) solving non-routine problems to explore the thinking of others compared to their own. The prospective teachers were required to solve an assigned problem and make notes of the thought process. They then worked in pairs, took turns to observe each other solve the problem while thinking aloud, and compared their thought processes. They selected a non-routine problem appropriate for a secondary school student, first solved it, and then used it to observe the student solving it while thinking aloud. They also solved a problem as a group in order to explore the collaborative and cooperative problem-solving experience.

(4) Developing a model for non-routine problem solving, representing it as a flowchart, and applying it to solving a non-routine problem in order to evaluate it.

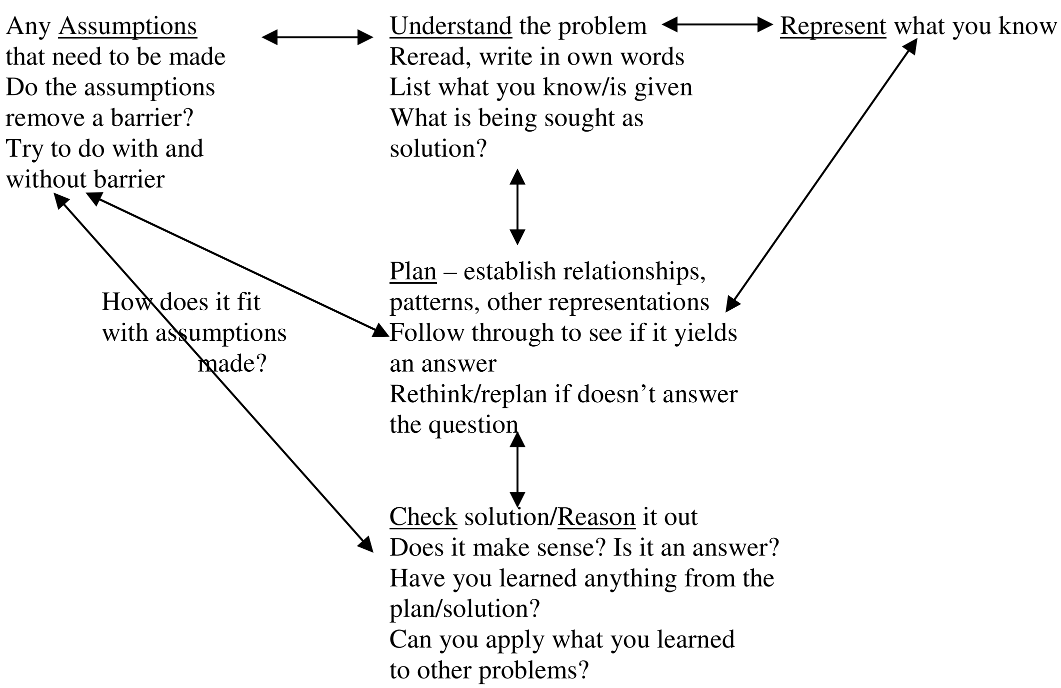
Final reflection: The third stage included activities that required the prospective teachers to engage in a final reflection by comparing their post-Stage 2 thinking with their pre-Stage 2 thinking; comparing their understanding of problems to theory (e.g., Charles & Lester, 1982; NCTM, 1989); comparing their problem-solving models and flowcharts with those from theory provided to them (e.g., Mason, Burton, & Stacey, 1982; Polya, 1954; Verschaffel, Greer, & de Corte, 2000); relating their problem-solving models to an inquiry instructional model for teaching secondary school mathematics; applying their knowledge to critique a current secondary school mathematics textbook approved for use in the Province; and preparing a lesson plan based on their inquiry instructional model.

Group reflection: Each of the three stages also required small group and whole-class interactions. This included: participants sharing and comparing their individual reflections in Stages 1 and 3 and their findings from the inquiry activities in Stage 2; preparing a model of problem-solving or inquiry-based teaching in small groups; and sharing and discussing small-groups' findings in a whole-class setting.

Research Method: The participants were 29 preservice secondary mathematics teachers in the second semester of their 2-year post-degree education program. This was their first course in mathematics education, so they had no instruction or theory on problem solving prior to this experience. They also were not taking any other mathematics education course in this semester. The reflection and inquiry activities served both research and learning purposes. Thus data consisted of copies of all of the participants' written work for all of the activities. There were also field notes of their group discussions and whole-class discussions. The analysis began with open-ended coding (Strauss & Corbin, 1998) of the data. The researcher and research assistants, working independently, coded the data. Coding included identifying significant statements about the participants' thinking of problems and problem solving and the changes in thinking resulting from the activities. The coded information was categorized based on common themes and frequency of occurrence to form the findings.

Findings of Effects of Approach: The approach was effective in expanding and deepening the participants' understanding of problems, problem solving, problem-solving pedagogy and inquiry-based teaching. Their thinking of problems shifted from predominantly traditional exercises or word problems to an understanding of characteristics that constitute worthwhile mathematics problems. Some of their descriptors of good problems are: they are of many different forms and types, challenging, needs one to use deductive or inductive reasoning to come to a solution, have one or more solutions and many approaches, interesting, not procedural and memorization of facts, challenging but solvable for grade level, applicable to real life problem, often requires more than one attempt to find a solution.

The participants' description of the problem-solving process was also enhanced, as reflected in their flowcharts of it, which showed the need to move back and forth, as opposed to their initial thinking of a linear path, to get to a solution. Following is a participant's problem-solving model.



The theoretical problem-solving models the participants examined in Stage 3 of the approach allowed them to validate, and in some cases refine parts of, their models, but not to change it, showing preference for their experience as a basis of validating what was more meaningful to them. Most of the participants viewed Polya's model to be closer to theirs and easier to follow and use in their teaching. One explained, "It flows more smoothly, it is cyclic, and non-exclusive in its processes. Although very generic in describing the steps, it reflects my cognitive steps in problem solving. As well, the steps I take in developing a plan varies from problem to problem." In general, the inquiry activities of Stage 2 of the approach allowed the participants to construct knowledge compatible with formal theory of problem solving. This allowed them to relate to the theoretical approaches they examined in Stage 3 of the approach in a more meaningful way.

In terms of problem-solving pedagogy, the thinking of the prospective teachers shifted to a student-centered approach. For example, they explained that they will have students work on a problem first, then share and discuss it in a whole-class discussion, or will have a whole-class discussion first to understand the problem, have students work in groups or individually, then share and discuss. They will pose questions or prompt the students when they (students) are stuck or in response to their (teachers) questions. They would use their problem-solving model to help to frame prompts and guide the whole-class discussions. This approach they viewed as being similar to inquiry-based teaching where they will start with an applied situation that embodied the mathematics concept being taught and allow students to unpack it with their (the teacher) guidance to understand the concept.

Overall, the study suggests that this three-stage approach framed in a social constructivist perspective and providing inquiry-based and self-reflective opportunities involving non-routine problem solving, followed by comparison to theory can help to deepen prospective secondary teachers' understanding and knowledge of problem solving in their teaching. However, whether they are able to enact this knowledge in their teaching has not been researched as yet. But holding such knowledge seems like an important first step in getting there.

Implications for Teacher Education

Based on the literature review and my study in the preceding sections of this paper, following are eight key characteristics of instructional practices identified as important to form a basis of preservice teachers' learning about problem solving. (1) Exploring others as problem solver: e.g., the prospective teacher works with a child or peer to observe, interview, and document information about this child or peer as a problem solver. (2) Exploring self as problem solver: i.e., inquiring into one's thinking, learning and instructional practices and developing ability to monitor and to control one's activities when solving problems. (3) Exploring nature/structure of problems. (4) Solving challenging problems individually and in small groups without external assistance, e.g., to develop awareness of strategies and skills for solving problems. (5) Posing problems. (6) Comparing self with others, e.g., peers, students, theorists. (7) Formulating instructional model for problem solving. (8) Exploring self as facilitator of problem solving, i.e., to develop an understanding of the teacher's role in facilitating students' problem solving.

If teachers are to hold knowledge to help them to teach about and through problem solving, they should be provided with experiences not only in solving problems, but also with all of these eight characteristics. It may also be necessary for these characteristics to be embodied in an integrated experience as opposed to being in isolation.

References

Arbaugh, F. & Brown, C.A. (2004). What makes a mathematical task worthwhile? Designing a learning tool for high school mathematics teachers. In R. R. Rubenstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics* (pp. 27-41). Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

Charles, R. & Lester, F. (1982). *Teaching problem solving: What why & How.* Palo Alto, CA: Dale Seymour Publications.

Chapman, O. (2005). Constructing pedagogical knowledge of problems solving: Preservice mathematics teachers. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th PME International Conference*, *2*, 225-232.

Contreras, J.N. & Martínez-Cruz, A.M. (2001). An investigation of preservice elementary teachers' solution processes to problematic story problems, In Van den Heuvel-Panhuizen, M. (Ed.), *Proceedings of the 25th PME International Conference*, 2, 289-296.

Contreras, J.N. & Martinez-Cruz, A.M. (2003). Preservice elementary teachers' solution processes to problematic addition and subtraction word problems involving ordinal numbers and their interpretations of solutions. In N. A. Pateman, B. J. Dougherty & J. T. Zillox (Eds.), *Proceedings of The 27th Conference of the International Group for the Psychology of Mathematics Education*. Honolulu: University of Hawaii, 2, 197-204.

Dewey, J. (1933). *How we think*. Boston: Heath.

Ebby, C.B. (2000). Learning to teach mathematics differently: The interaction between coursework and fieldwork for preservice teachers. *Journal of Mathematics Teacher Education, 3*(1), 69-97.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Lee, H. S. (2005). Facilitating students' problem solving in a technological context: prospective teachers' learning trajectory. *Journal of Mathematics Teacher Education,* 8:223–254.

Leung, S.S.(1994) On analysing problem-posing processes: A study of prospective elementary teachers differing in mathematics knowledge, In Ponte, J.P. & Matos, J.F. (Eds.), *Proceedings of the 18th PME International* Conference 3, 168-175.

Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. New York: Addison-Wesley

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Council of Teachers for Mathematics (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Polkinghorne, D. (1988). *Narrative knowing and the human science*. Albany: State University of New York Press.

Polya, G. (1954). *How to solve it.* New York: Anchor Books.

Roddick, C., Becker, J. R., & Pence, B. J. (2000). Capstone courses in problem solving for prospective secondary teachers: Effects of beliefs and teaching practices, *Proceedings of the 24th PME Conference,* Vol. 4, 97-104.

Schmidt, S. & Bednarz, N. (1995). The gap between arithmetical and algebraic reasonings in problem-solving among pre-service teachers, In Meira, L. & Carraher, D. (Eds.), *Proceedings of the 19th PME International Conference*, 2, 82-89.

Schoenfeld, A.H. (1985). *Mathematical problem solving*. Orlando: Academic Press.

Simon, M.A. (1990). Prospective elementary teachers' knowledge of division. In Booker, G.; Cobb, P.; Mendicuti, T. (Eds.), *Proceedings of the 14th PME International Conference*, *3*, 313-320.

Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2000). *Implementing standard-based mathematics instruction: a casebook for professional development*. New York: Teachers College Press.

Strauss, A. & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks, California: Sage.

Szydlik, J.E., Szydlik, S.D., & Benson, S. R. (2003). Exploring changes in preservice elementary teachers' mathematics beliefs. *Journal of Mathematics Teacher Education, 6*(3), 253-279.

Taplin, M. (1996). Pre-service teachers' problem solving strategies, In Puig, L. & Gutiérrez, A. (Eds.), *Proceedings of the 20th PME International Conference*, 4, 299-306.

Van Dooren, W., Verschaffel, L., & Onghena, P. (2003). Preservice teachers' preferred strategies for solving Arithmetic and Algebra word problems. *Journal of Mathematics Teacher Education*, 6(1), 27-52.

Verschaffel, L., Greer, B. & de Corte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets and Zietlinger Publishers.

Verschaffel, L., De Corte, E. & Borghart, I. (1996). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in arithmetic word problem solving, In Puig, L. & Gutiérrez, A. (Eds.), *Proceedings of the 20th PME International Conference*, 4, 387-394.