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THE PEDAGOGICAL STRUGGLE OF MATHEMATICS EDUCATION FOR THE DEAF DURING THE LATE NINETEEN CENTURY: MENTAL ARITHMETIC AND CONCEPTUAL UNDERSTANDING

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Long before the founding of the first professional organization comprised of teachers of mathematics, The National Council of Teachers of Mathematics (NCTM), in 1920, to promote the teaching of mathematics and called for reforms to improve mathematics curriculum across schools in the United States, the struggles and, for some, reforms of mathematics education for deaf children in the country continued as their teachers struggled in finding a perfect one-size-fit-all instructional methodology for teaching mathematics, swinging back and forth between the traditional and modern paradigms of teaching during the late nineteenth century. Such struggles have shown that the evolution of curricular instruction had undergone numerous alterations, innovative pedagogies, controversies on methods and learned concepts, and inquires and illustrations in curriculum development.[1] Little has been known about mathematics curriculum and instruction for the deaf since the establishment of the American Asylum for the Deaf and Dumb in Hartford, Connecticut in 1817, and successive schools for the deaf in the country to serve deaf and hard of hearing children whose primary means of relating to the world is visual and who share a language that is visually received and produced.[2]

Two educational forces called for changes in curriculum and instruction in mathematics for the deaf during the late nineteenth century. First, the founding and implementation of a college for the deaf in Washington, D.C. in 1864 brought some calls from college professors and classroom teachers for increased expectation and standardization in the mathematics curriculum for pre-college deaf students across the country. Secondly, the development of vocational training programs for the deaf to meet the need for job specialization for industrial companies, especially manual trades as the country was progressing along the industrial movement, leaving behind agricultural dependence for economic growth.[3] Both forces called for increased mastery in arithmetic and later algebra before graduates enter either the college or the workplace. The struggle concerning how to teach the deaf mathematics effectively and prepare them for the college and the workplace world continued as teachers of the deaf brought to the field different perspectives and, in turn, criticisms of teaching and learning arithmetic in the field of deaf education. Manual training for industrial trades at the time called for changes in school curriculum in order to infuse necessary skills and knowledge for deaf pupils to acquire and develop before they entered the college or the workplace.[4] In this

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[1] Herbert Kliebard, *The Struggle for the American Curriculum, 1893-1958,* 3rd ed. (New York: Routledge, 2004).

[2] Harlan L. Lane, Robert Hoffmeister, and Ben Bahan, *Journey into the Deaf-World* (San Diego, CA: Dawnsign Press, 1996). Also see Paddy Ladd, *Understanding Deaf Culture: In Search of Deafhood* (Tonawanda, NY: Multilingual Matters Ltd, 2003).

[3] Chris Kurz, *Mathematics Education for the Deaf in the United States: A Historical Analysis of the Nineteenth Century*, Diss. Lawrence, KS: University of Kasnas, 2006.

[4] Convention of American Instructors of the Deaf, *The Proceedings of the Fifteenth Meeting of the*

paper, the pedagogical struggle between mental arithmetic and conceptual understanding during the last half of the nineteenth century was examined. By using primary sources such as *American Annals of the Deaf* and proceedings of past conventions for American Instructors of the Deaf, this historical research study brings to light a new understanding of the struggle in mathematics instruction and curricula for the deaf during the late nineteenth century, especially the struggle between the teaching of mental arithmetic and conceptual understanding.

Before the pedagogical struggle was analyzed in depth, this paper starts with the state of mathematical education for the general population in the country during the nineteenth century, following by potential problems associated with mathematics teaching and learning for deaf children.

Mathematical Education During the Early Nineteenth Century

Before the beginning of the nineteenth century, pupils were expected to discipline their mind by rote; therefore, arithmetic must be learned by memorization. In 1821, a young Harvard graduate, Warren Colburn (1793-1833), wrote an earth-shattering book on teaching arithmetic, *First Lessons in Arithmetic on the Plan of Pestalozzi, with Some Improvements*,[5] addressing his concern on teaching mathematics through memorization and promoting teaching mathematics with understanding. He believed that teachers should partake of the constructive approach where children would learn arithmetic with understanding and discovery. In his book, Colburn advocated and defended understanding and discovery, or conceptual knowledge, as the best avenue to learning mathematics, let alone arithmetic. Therein he launched an eternal debate between memorization and understanding in instruction and curriculum, let alone in mathematics learning.

In his address delivered before the American Institute of Instruction in Boston, August 1830, Colburn produced a window to the old system of teaching arithmetic: "At least, fewer made any considerable progress in it. Very few females pretended to study it at all, and the number of either sex that advanced much beyond the four primary rules was very inconsiderable. And the learner was seldom found who could give a satisfactory reason for any operation which he performed. The study of it used to be put off to a very late period. Scholars under twelve or thirteen years of age were not considered capable of learning it, and generally they were not capable. Many persons were obliged to leave school before they were old enough to commence the study of it."[6] He later claimed that with the old system, the learner was presented with a rule which specified the procedures to solve problems without explanations. Larger numbers were often used in such problems which could be overwhelming and mechanical for the learner to follow the rule. Although his ideas were a breakthrough in mathematical instruction and later became widely accepted, but not until after the nineteenth century, Colburn received on-going criticism during his time, a sign that the educational sphere was not ready for reform in

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Convention of American Instructors of the Deaf at Columbus, Ohio, July-August 1898, (Washington, D.C.: Government Printing Office, 1899): 85.
[5] Boston: Cummings, Hilliard, & Co. 1821.

[6] Warren Colburn, "Teaching of Arithmetic," Address delivered before the American Institute of Instruction in Boston, August 1830. Reprinted in *Elementary School Teacher*, 12 (1912).

school mathematics. Mental discipline, procedural knowledge, learning by rote or drill, was the popular and widely accepted method in education by the end of the nineteenth century.

The title of Colburn's first book showed his preference for Pestalozzi's pedagogies in teaching mathematics, which emphasize understanding and discovery over the traditional rule method that stresses memorization and drill. Colburn studied Pestalozzi's method of teaching young children and believed that his method would work with teaching mathematics to young children as well. Young children, as young as five or six years old, were encouraged to discover and formulate concepts of numbers and operations. Colburn's book suggested that young children should start with practical problems with which they could use manipulatives such as buttons, beans, and sticks, to make combinations until they fully understand the operations. Once the concepts were formulated and understood, children would learn the abstract numbers and signs where they were expected to develop their own principles, which would approach the general principles as established in the academics of mathematics.[7] Editors Bidwell and Ciason asserted that Colburn's personal beliefs about learning mathematics were "first for its practical use, and second for the mental discipline value."[8] In sum, Colburn emphasized three related instructional principles: 1. Students developed their own methods in solving problems; 2. Teachers made suggestions for improvements, bringing students closer to the standard method; and 3. Students were asked to explain and give reason for their said steps in solving problems.

Since the publication of Colburn's *First Lessons* in 1821, which was widely distributed, teachers were having a difficult time trying to incorporate Pestalozzi's pedagogies in their classroom. It was a common practice that teachers relied on teaching training advices provided in the arithmetic textbooks at the time. Teachers, during the early and mid-eighteen century, often received no training in pedagogy. Instead, they either followed the directive or hints for teaching that could be found in textbooks or used the exact method that they learned when they were younger. For example, in Ray's *Intellecual Arithmetic*, published in the 1850s and widely used in schools at the time, teachers would read the following excerpt:

> Let the pupils be classified with reference to their attainments and abilities. The recitation should be short and spirited, every pupil being required to give undivided attention to the question before the class.
>
> Generally, while reciting the pupils should be permitted to have their books open before them-the test of having properly studied the lesson, being the readiness and accuracy with which the several questions are analyzed and answered.
>
> The explanations and operations termed ANALYSIS, are intended as Model Solutions, pointing out to the learner the manner in which the questions in the lesson are to be solved and explained.
>
> The pupil should be required to furnish a similar explanation to each of the succeeding questions, and to give, not only a correct answer, but also, the reason for the method by which he obtained it.[9]

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[7] Furr, "A Brief History of Mathematics Education in America": 1.
[8] Bidwell and Ciason, eds., *Readings in the History of Mathematics Education*: 24.
[9] Joseph Ray, *Ray's Intellectual Arithmetic* (New York: Clark, Austin, & Smith, 1854): 8.

Basically the excerpt indicates that analysis should be the main avenue to learning mathematics. The learning process becomes important as opposed to memorization of facts. Furthermore, pupils at the time were expected to solve mathematical problems on their own and discuss and justify their strategies used to arrive at their answer with the class. However, not all teachers followed the recommendations as found in most arithmetic textbooks at the time. The teacher's beliefs about how and how much mathematics should be taught had a great impact on the pupils' knowledge and skills in mathematics. When mathematics is perceived as a tool or sets of skills, pupils would learn mathematics by rote and drill. Otherwise, when mathematics is perceived as a connected body of knowledge that is integrated into one's personal experience and mental improvement, pupils would learn mathematics by concept understanding and discovery with analytical processes.

During the expansion of common school movements and the transition from urban free schools into public-funded common schools during the 1830s and 1840s, advocates of such movements called for common instruction and curriculum, from which the youth would receive the same body of knowledge and learn a common social and political ideology to lead the new nation and solve social problems as a people.[10] During the decades of the common school movement, the states established their boards of education to regulate instruction and curriculum, in order to prepare youth for jobs as demanded by the growing number of businesses and economics. Learning arithmetic, especially the rules, became a mandate for all the youth, not just small groups of children. In 1844, Horace Mann, secretary of the Board of Education in Massachusetts, and a strong advocate for teaching of the deaf and dumb, visited Europe to thoroughly study their educational system.[11] In his seventh annual report to the Board of Education, he described his observation of arithmetic lessons in Europe and compared them to the one in the United States. He noted the main differences between their mode of teaching arithmetic and the ones in the United States:

> "…consist in their beginning earlier, continuing the practice in the elements much longer, requiring a more thorough analysis of all questions, and in not separating the processes, or rule, more by an understanding of the subject. It often happens to our children that while engaged in one rule, they forget a preceding. Hence many of our best teachers have frequent reviews. But there, as I stated above, the youngest classes of children were taught addition, subtraction, multiplication and division promiscuously. And so it was in the later stages. The mind was constantly carried along, and the practice enlarged in more than one direction. It is a difference which results from teaching, in the one case, from a book; and in the other, from the head. In the latter case the teacher sees what each pupil most needs, and if he finds any one halting or failing on a particular class of questions, plies him with questions of that kind until his deficiencies are supplied."[12]

Mann noticed the difference in teaching approaches: one from a book and another from the head. He obviously was impressed with the latter, which supports individualized learning and dialogue to learning.

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[10] J. Spring, *The American School: 1642-1990*, 2d ed. (New York: Longman, 1990).

[11] Horace Mann visited schools for the deaf in Prussia, Saxony, and Holland. In his seventh annual report, he declared the schools in these countries superior to those in the United States, because there deaf pupils are taught how to speak eloquently and the ones in United States cannot.

[12] Horace Mann, *Seventh Annual Report of the Board of Education; Together with the Seven Annual Report of the Secretary of the Board*, (Boston: Dutton and Wentworth, 1844): 104.

Since mathematics education did not begin as a discipline in the mid-1800s, pedagogical instruction of arithmetic varied among teachers. There was no established system of teaching arithmetic other than the textbooks from which teachers and their students used to learn the four operations and their applications. Practical applications in the area of navigation, commercial trades, and vocational occupations, were the main topics among mathematical problems in the 1800s. During that time, students only learned elementary level arithmetic where they became apt in computations using basic operations with real numbers, often with large numbers. Students were expected to be drilled until mastered in mathematical concepts related to arithmetic. The most popular avenue to learning arithmetic and higher branches of mathematics during the nineteenth century was drill and rote, for it was often believed that mathematics was a tool for reason, exercising the rational faculty and promoting mental discipline.[13]

Problems Associated with Mathematical Learning In Deaf Children

In 1849, Jared A. Ayres of the American School for the Deaf and Dumb in Hartford wrote of the advantages of mathematical education of the deaf:

> In the elementary branches of mathematical study, the deaf and dumb commonly make early and rapid progress. In the simplicity and precision of such studies, their minds, yet cramped and hindered in their free exercise, seem to rejoice. …the fundamental rules and principles of mathematical science may be taught with ease and success. Beyond this, the deaf-mute does not advance successfully until he is familiar with language. … When, however, language is secured and the mind has learned to flow in its more precise and accurate channel, mathematical reasoning is no longer, as it has been, a difficult and uncertain effort. In this stage of advancement, the prosecution of close mathematical studies is one of the greatest auxiliaries toward a complete education which the deaf and dumb can have."[14]

Ayres conceded that deaf children learned basic arithmetic with ease, but when they went up in the branches of mathematical knowledge, they encountered difficulties when language had become complex. They had been greatly neglected at their institutions: "While their opening minds are grasping eagerly in every direction for the useful and the beautiful," Ayers wrote, "it would be hardly expected that they should pause voluntarily to survey the dry fields of mathematical science; yet, for want of the discipline which they might here attain, they suffer a mental inconvenience and disqualification all their lives."[15]

In the ninth meeting of the Convention of American Instructors of the Deaf, hosted at the Ohio Institution for the Deaf and Dumb, Columbus, Ohio, during August 17 – 22, 1878, John A. Jacobs of the Kentucky Institution opened a discussion on the methods of teaching arithmetic by commenting that:

> Deaf mutes, as a rule, have the same capacities for gaining the idea of numbers or combinations of numbers, that hearing persons have. It only requires a little longer process in the way of development, a little more care, and a little more patience in the beginning. I think that after the first steps are taken, and careful attention to all the processes, that nothing is left in the dark, nothing is left behind that is not explained, that they can progress in the study of arithmetic,

[13] Daniel H. Calhoun, *The Intelligence of a Person* (Princeton, NJ: Princeton University Press, 19773).
[14] Jared A. Ayres, "A Complete Education for the Deaf and Dumb," *American Annals for the Deaf and Dumb* 2, no.1 (1849): 28.
[15] Ibid., 28-29.

> at least as well as ordinary children, as surely, at any rate, if not as rapidly. … There is one important feature of the whole subject of arithmetic. I wish that no child should be allowed to take one step, not even to write a figure, nor to express a number by signs, that he does not understand. Teachers sometimes allow their pupils to write great row of figures that they do not comprehend, the pupils don't know what those figures stand for as individual figures, as expressive of a certain number they don't understand at all.[16]

Jacobs clearly stressed the importance of the pupil being able to understand what a number signifies in terms of its magnitude. First and foremost, Jacobs stressed that pupils should understand number magnitudes before they proceed to the four rules of operations. He expressed his disappointment when he received older pupils, having been in school for four or five years, who did not know the value of a given number after they worked with addition, subtraction, and multiplication. "For instance," he recalled, "they could write down 1864 x 1684 and could multiply it all up and get the correct answer, and yet they couldn't tell what their answer was in signs, or write it out in words; that is, they didn't know what the value of it was, I had to correct it."[17]

Since mathematics and language are sometimes inseparable, especially when it comes to teaching arithmetic, Richard S. Storrs of American School for the Deaf and Dumb in 1880 conceded that some of the brightest pupils in language remained backward in arithmetic.[18] In 1882, David Greenberger, Principal of the New York Institution for Improved Instruction of Deaf-Mutes, conceded that learning arithmetic for deaf pupils could be difficult due to the necessity of possessing abstract thinking; however, noting that arithmetic was the most effective branch of education existing for the development of reasoning facilities. "The difficulty is augmented by the fact that it has to be commenced at an early stage of the course, when the pupil's vocabulary is quite limited, and he is not able to think in spoken or written language to any great extent. Hence," Greenberger wrote, "we cannot strictly follow any of the numerous methods which have been devised for teaching this branch to hearing children, but have to modify these systems to suit the peculiar condition of our pupils."[19] He felt that the methods used in common schools were rendered ineffective for deaf pupils due to language difficulties and abstract concepts.

Kate D. Williams, of the Horace Mann School in Boston, presented on number work with language in 1890. She was concerned with deaf pupils' dependence on key words to which they could relate certain operations for solving word problems. She rationalized that it would be better if verbs were to be used instead, so that the pupils would give a full attention to solving the problem without eliciting key words. She wrote:

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[16] *Proceedings of the Ninth Meeting of the Convention of American Instructors of the Deaf* (1879): 274.

[17] Ibid., 275.

[18] Richard S. Storrs, "Methods of Deaf-Mute Teaching—II," *American Annals of the Deaf and Dumb* 25, no.4 (1880): 236-237.

[19] David Greenberger, "Arithmetic," *American Annals of the Deaf and Dumb* 27, no.1 (1882): 12. Greenberger claimed that "of all the branches of knowledge, which we teach our pupils none is more important than arithmetic. A deaf-mute—or a hearing person—who enters one of the lower walks of life after leaving school may not have any immediate use for his knowledge of history, geography, etc., but I cannot conceive of any circumstance in which he may live and not feel the need of a familiarity with the principles and rules of common arithmetic. Besides, no other branch is better adapted to the development of the reasoning faculties of children than this. It is therefore well worthy of the special attention of instructors." [p.12].

"I depend on the verbs from the start, and try to avoid little key-words, which a child quickly learns to associate with each of the four rules. 'All' is a key-word suggesting addition, when perhaps the question as a while is not understood. 'Left' suggests subtraction, and so on."[20]

In an attempt to address the enigma of arithmetic development for deaf-mutes where they easily learned geography, history, mechanics, physiological, and chemistry with rapidity and keen interest while learning arithmetic could be a struggle, Weston G. Jenkins, Principal of the New Jersey School for the Deaf-Mutes, suggested that there was a delay of English acquisition in the development of arithmetic knowledge, and teachers needed to make a correct diagnosis in order to find a remedy for the obstacle.[21] He wrote, "For one thing, the lack of a true mother-tongue probably has more to do with this deficiency than is usually considered."[22] He made a comment about his observations of eight-year-old children that they incorporated mathematical concepts in their daily conversations without any introduction from older individuals.[23] Deaf-mutes who were not exposed to daily discussions or accessible informal mathematical information at home or in neighborhoods were thus deprived of their potential for grasping mathematical concepts early in their life.

Professor Amos Draper of the National Deaf-Mute College in Washington, D.C., became concerned about the mathematical skills of deaf-mutes, calling for increased opportunities to study arithmetic, outlining disappointed statistics:

> As to the College records, they show that, taking the whole number of students since the foundation in 1864, only 26, or *twelve-and-a-half per cent.*, have sustained the entrance examination in arithmetic; of applicants since the standard has approached its present condition, that is during the last eight years, only 11, or *seven per cent.*, have endured the same test; while of 23 who presented themselves at the last examination for admission, not one was able to answer satisfactory all the questions asked them.[24]

The examinations for enrollment in the National Deaf-Mute College covered arithmetic, algebra to quadratic equations, English grammar, history, geography, philosophy, Latin, and science.[25] Draper did point out that the last part of the examination might be severe with mathematical problems. In the last part, the deaf applicants had to answer six problems, as follows:

1. Write the tables of long measure and of cubic measure, and state for what each is used.

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[20] Kate D. Williams, "Number Work with Language," in *The Proceedings of the 12th meeting of the Convention of American Instructors of the Deaf* (Washington, D.C.: Government Printing Office, 1890): 67.

[21] Weston G. Jenkins, "The Teaching of Arithmetic," *American Annals of the Deaf* 37, no.1 (1892): 9-14.

[22] Ibid., 9.

[23] Jenkins made no mention that the children on which he made an observation were deaf-mutes. It can be safely said that he meant hearing children due to his reference to a "true mother-tongue." Examples of conversations are: "It is three miles to Manalapan and five miles to Freehold: if it was one mile more it would be twice as far." "Stella picked six quarts of raspberries and she gets a cent and a quarter a quart. She earned – pause – seven cents and a half. But we have no half cents. I wonder if she got seven or eight cents." "Dan is fourteen: when I am as old as he is, he will be twenty." "I can easily earn ten cents picking blackberries: you get two cents a quart, and I can pick five quarts mighty quick." Jenkins: 10.

[24] Ibid.,253.

[25] Edward M. Gallaudet, *Eighth Annual Report of the Columbia Institution for the Deaf and Dumb* (Washington, D.C.: n.p., 1865), 7.

2. Find the length of a field which has an area of 144 sq. yds, and is 28 ft. broad?
3. How does a decimal fraction differ from a common fraction?
4. A note of $500 was dated June 10, and bore interest at 5 *per cent*; Oct. 3 a payment of $140 was made. What was due on the note one year from its date?
5. If 5/8 of a ship cost $9,875, what would 7/8 of it cost? [Solve this question first by proportion and then analysis.]
6. What is the square root of the fifth power of 3?[26]

With the mastery in the English language and the knowledge of mathematics, one is able to solve the above mathematical problems. Needless to say, Draper believed that institutions for the deaf should restructure their mathematics curriculum to involve more instruction in arithmetical terms and accuracy in handling such questions, meaning deaf children should be taught how to analyze arithmetical language within questions and perform the operations accordingly. He further reasoned that the fault was based in several areas of mathematical instruction: "that they had been taught without a text book; that they had never been dwelling on a subject, but passed rapidly through; that they had long since been given higher branches; that they had never studied arithmetic at all, or at least the recollection of it had passed from them."[27]

At the 10th meeting of the Convention of American Instructors of the Deaf in Jacksonville, Illinois in 1882, Professor Swiler of the National Deaf-Mute College, claimed that there is no difference in mathematical abilities and development between deaf and hearing children:

> I can see no reason why mutes may not distinguish themselves in any branch of mathematical knowledge to as eminent a degree as any speaking girl or boy. There are reasons which render the acquisition of speech difficult. While it is exceedingly difficult to understand all the expressions of our intricate and complex language, I can see no reason why deaf-mutes may not get as clear and plain knowledge of mathematics, both pure and applied, as other students. With respect to the acquisition of a knowledge of numbers in the concrete, there is no doubt that their education can be developed to an indefinite extent; and with reference to the application of abstract numbers, they seem to handle them at least as well as other children. … I trust that this subject of arithmetic will receive proper attention, in order that the charge that we give imperfect instruction in this science may be met, and that henceforth time enough be devoted to give our pupils a sufficient knowledge of numbers, so that they may be enabled to perform the most difficult operations in arithmetic.[28]

Whether there is a significant difference in mathematical abilities and development between deaf and hearing children has been debated for years, there are numerous factors to consider when exploring into this area: family background and values in education, mental capabilities of deaf children, curriculum and instruction where the deaf child receive formal schooling, incidental learning in school and home, and many more.

Pedagogical Struggle

In his 1882 article on mathematical education for the deaf, S. Tefft Walker of the Illinois Institute for the Deaf and Dumb in Jacksonville described some practices that, he suspected, tended to slow a class's progress:

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[26] Ibid., 252.
[27] Ibid., 253.
[28] *Proceedings of the 10th Meeting of the Convention of American Instructors of the Deaf, August 26-30, 1882* (Jacksonville, IL: n.p., 1883): 88-90.

> I. The practice is to begin addition during the latter half of the first year. Once begun, rapid strides in the handling of abstract numbers are made; the pupils are delighted with the novel enigmas they are solving, and the teacher himself, pleased with their seeming progress and the effect produced upon visitors, loses sight of the prime object of arithmetic, and goes on doling out the abstractions that are so pleasing to his class…
> II. The second objection is the almost universal tendency to let pupils use large numbers too early in the course—numbers that instead of being within the grasp of their intelligence are such as are seen only in census reports or in political majorities…
> III. The third objection is the paucity of objective apparatus to be found in the school-rooms of primary teachers. This is the more serious because of the inexpensiveness of such paraphernalia…
> IV. Neglect to give sufficient drill in the use of oft-recurring phrases, a complete knowledge of which is necessary to an understanding of some of the simplest classes of arithmetical problem; e.g., "as many as," "half as many as," "twice as many as," "three more than," "five less than," "as high as," "as old as," "as far again." etc. These expressions are among the most common of childhood, and are usually used by children of from four to five years of age with perfect ease…
> V. …the custom of teaching addition, subtraction, and division, in the order named, teach the four rules simultaneously.[29]

In short, Walker outlined five objections: early exposure to abstract concepts, early exposure to larger numbers, insufficient practical apparatus, lack of drill with common English phrases that are mathematically pertinent, and separable, but subsequent operations.

During the late nineteenth century, there were insoluble struggles between the two tents: the traditional and the modern paradigms of teaching and learning mathematics to the deaf. Some teachers and administrators elected to build a tent in between the two extremes in an attempt to combine some principles from each tent. The issues were as follows: (a) mental arithmetic and conceptual understanding; (b) use of finger counting or signs for calculations; (c) teacher as a facilitator of information or teacher as a source of information (d) textbook and teacher as a means of information; (e) learning language first, arithmetic second or both subjects simultaneously; (f) teaching the four rules separately or the rules altogether; (g) teaching the concrete before the abstract or teaching both areas simultaneously; and (h) use of language form in arithmetic and use of numerical form in arithmetic; and. In this paper, the first issue was examined in depth.

Mental Arithmetic and Conceptual Understanding

In the field of education, there were two shifting movements, one which called for intimate relationships with one's environment through object-lessons, nature-study and child-study, all of which emphasize conceptual understanding as a learning tool, and another for memorization through rote and drill.

John Robinson Keep of the American School for the Deaf and Dumb wrote, in 1854, an article, "Addition, How It May Be Taught," noting that teachers should take great care to start associating numbers with objects, first teaching counting, then addition.[30] After the pupil mastered the counting process via writing, then he should be encouraged to substitute counting with memorizing. Keep believed that the deaf pupil

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[29] S. Tefft Walker, "The Teaching of Numbers," *American Annals of the Deaf* 27, no.4 (1882): 224-226.
[30] John Robinson Keep, "Addition, How It May be Taught," *American Annals of the Deaf and Dumb* 6, no.2 (1854).

should learn addition facts from experimentation and observation and should then construct an addition table for the purpose of memorization.

In terms of classroom environment for promoting student learning in mathematics, Thomas Gallaudet, son of Thomas H. Gallaudet who founded the American School for the Deaf, in 1857 made suggestions for the first year classroom:

> Upon the wall of the room opposite to the teacher's and over the pupil's large slates, might be displayed the addition table, upon which the class could be frequently drilled with great facility. In connection with this table the class should have a small arithmetical treatise, unfolding the principles of numeration and addition and nothing else. During the first year they should be thoroughly drilled in adding figures, that they could perform operations without counting their fingers.[31]

Deaf pupils in the first-year classroom would learn numeration and addition operations through drill and rote, and Gallaudet urged that teachers help reduce their pupils' habit of counting fingers while performing addition operations.

In 1858, George M. McLure of the Kentucky Institution for the Deaf and Dumb proposed some ideas on the teaching of fractions in an article entitled "How May Fractions in Arithmetic Be Best Taught to the Deaf and Dumb."[32] He concluded:

> When the pupil has become familiar with all this, he will be prepared to generalize the rules he has learned, and to apply them to all fractions, of whatever denominations. He will at least understand what is the intent and meaning of the several operations, and will have formed habits of thinking clearly and closely on arithmetical subjects. He may be expected afterwards to proceed rapidly enough to make up for all the delay occasioned by laying thoroughly the foundation, and will have the increased pleasure of seeing the way clearly, instead of groping blindly he knows not whither.[33]

McLure's conclusion showed his support for Warren Colburn's conceptual understanding instruction, from which pupils would construct knowledge through conceptual understanding instead of drill-and-rote. McLure suggested that deaf pupils should do the drill-and-rote practice only after they understand the concept and its implications, and then their computational rate would increase significantly.

In 1866, J. Scott Hutton, Principal of the Institution for the Deaf and Dumb in Halifax, Nova Scotia in Canada, published a mathematical textbook for the deaf and dumb, entitled *Elementary Arithmetical Exercises, Chiefly on the Provincial Currencies*.[34] Contrary to McLure's conceptual understanding instruction, which might require lengthy explanation and discussion, J. Scott Hutton preferred the instructor's modeling method, when an instructor models his thought process on the blackboard as he solves a mathematical problem; and thereafter, his pupils would imitate his process to solve similar problems from the textbook. To become skilled arithmeticians, Hutton suggested, deaf pupils would employ drill-and-rote practice until they attained mastery with high accuracy.

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[31] Thomas Gallaudet, "School Room Arrangement," *American Annals of the Deaf and Dumb, 3*, no.2 (1857): 74.

[32] "How May Fractions in Arithmetic Be Best Taught to the Deaf and Dumb," *American Annals of the Deaf and Dumb* 14, no.4 (1858).

[33] Ibid., 253.

[34] Printed by James Bowes and Sons in Halifax.

In his "Arithmetic for Deaf-Mutes" article, Richard Storrs in 1871 cautioned against teaching arithmetic as mere mental gymnastics without having the pupils understand arithmetic concepts at the same time.[35] Furthermore, "numbers should be chiefly considered by the deaf-mute, in their relations to life and language," he wrote.[36] The very first steps in teaching numbers, in the concrete sense, should be to teach the name of the number as an adjective, in terms of its word form, and associate it with as many concrete items in deaf pupils' daily lives as possible. In the same article, Richard Storrs suggested that the teacher spend less time on memorizing rules and that the pupil make his own rules, for "this would tend to cultivate sharpness of thought."[37] In this case, the deaf pupil was encouraged to discover or create mathematical concepts and rules with support of visual illustration, so the pupil's visual memory would be optimally stimulated for success in gaining and retaining principles.

In 1879, William L. Bird, instructor at the American Asylum, was a strong advocate for rote and drill in order to prevent mistakes, rather than correct them. The motto of Bird was "one step at a time and complete mastery of each and every step as far as taken."[38] His justifications for drill in arithmetical operations were as follows:

> As the mind memorizes by frequent repetitions, which are so much the better if made understandingly, and as very lasting impressions are produced when short intervals of time come between these repetitions, the pupil can be so well drilled that it becomes more like play than work to him; he finds it a pleasure instead of a task to perform operations in figures.
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> Nor is it a waste of time to train a class so highly as here indicated; on the contrary, it is a positive saving. Fewer mistakes are made and less time is lost in future operations. The pupil, feeling himself perfect so far, is encouraged and confident. He walks firmly along, instead of stumbling and soon falling hopelessly in the rear. It is with him as with a colt: both must be trained to know and use their strength, which, while being fully exercised, must not be overtasked, or both will balk. Slow pupils especially need attention in this respect. If they once fall behind and lose confidence in themselves, how can they overtake their quicker fellows?
>
> Again, rapid and accurate operations in figures allow more of the attention to be given to other points in the work, such as the steps to be taken in solving a problem, the meaning of the modifying words given, and like considerations. The mind is not troubled by a fear of making mistakes with the figures; its force is spent on perceiving the method required for the solution, or, to speak algebraically, on making the statement, not on working it out.[39]

Bird felt that, through memorization and drill, deaf pupils would eventually master the basic facts and become accurate and rapid arithmeticians. In this case, the pupils would save time in calculating operations and proceed with other advanced works should they continue schooling or work in the business world. Urging that the pupil should be drilled in addition until mastery, Bird wrote, "Not till he is able to repeat in every instance, without hesitation or error, … and the teacher has had an opportunity to find out his

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35 *American Annals of the Deaf and Dumb* 16, no.3 (1871): 144-160. The paper was originally prepared for a meeting for the instructors at the American Asylum in which the subject under discussion was titled, "Our Arithmetical Course of Instruction."
36 Ibid., 144.
37 Ibid., 153.
38 William L. Bird, "Preparatory Drill in Figures," *American Annals of the Deaf and Dumb* 24, no.1 (1879): 1.
39 Ibid., 1-2.

particular deficiencies. He must keep striking at the knots, which is the best way in dealing with logs—and blockheads."[40]

In 1882, David Greenberger proposed that during the first two or three years only mental arithmetic be allowed and that the children learn the fundamental operations for all combinations under 10. He wrote of mental arithmetic: "I devote so much time and attention to mental arithmetic because it is more frequently applied in real life than written arithmetic. Pencil and paper are not always conveniently at hand, and the mental processes do not occupy so much time as the written ones. Hence, the former are preferred in business transactions."[41]

Albert L. E. Crouter, principal of the Pennyslvania Institution for the Deaf and Dumb, wrote in a circular of instructions to parents and guardians of deaf children in 1885: "We see the benefits of home instruction among our new pupils. Many of them can write nicely the names of many objects, some of them can work addition, and I feel that this field can be made productive of much greater good if properly and persistently looked after."[42] He further advised parents:

> What has been said concerning the instruction of our child in language, whether spoken or written, applies with equal force to the study of numbers. Teach him simple numbers as soon as he is able to write. In doing so, make use of objects. Marbles are very convenient. With these teach him to count orally, in signs, or in writing, up to twenty. Treat each number by itself, counting out the number of marbles each time. Then let your child count them out, or let him pick up a number for you to count. You may next teach him to add, being careful to use only the smallest numbers at first, as *one* and *one*, *two* and *two*, *three* and *one*, *three* and *two*, etc. Use the marbles till he understands the process, and then let him add mentally. After he has been made thoroughly familiar with small combinations, it will be safe to try larger ones. Do not force the child under any circumstances; rather than do so, make no attempt in this direction at all, for it will only end in his conceiving a distaste for numbers which will be very hard to overcome.[43]

Object manipulation, Crouter believed, would assist deaf children to acquire knowledge in numeration and notation. Parents were encouraged to use objects like marbles or pebbles to help teach their deaf children how to count. It was Crouter's hope that deaf children would arrive at his school with basic knowledge of numeration and notation.

In discussing the students at the college during the year 1887, Dr. Edward M. Gallaudet, President of the National Deaf-Mute College in Washington, D.C. and brother of Thomas Gallaudet, stated that the pupils who did not do well in it had been taught by rote and were not taught the principles upon which the rules were based. He further stated that mathematics was no pleasure for them, but was a great bore and a trial and a terror.[44] He however did not make any recommendation except that signs should be used in the classroom for recitation and calculations.

In 1887, Jonathan H. Eddy of the New York Institution for Improved Instruction quoted Francis Bacon when discussing the importance of instilling logical reasoning in deaf children's minds by teaching arithmetic: "If a man's wits be wandering, let him

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[40] Ibid., 5.
[41] David Greenberger, "Arithmetic," *American Annals of the Deaf and Dumb* 27, no.1 (1882): 13.
[42] A.L.E. Crouter, "Instructions to Parents," in circular of Pennsylvania School for the Deaf (1885): 28.
[43] Ibid., 29-30.
[44] *Thirtieth Annual Report of the Columbia Instruction for the Deaf and Dumb to the Secretary of the Interior* (Washington, D.C.: Government Printing Office, 1887): 7-9.

study arithmetic."[45] He felt deaf children needed to learn arithmetic in a logical way, instead of by rote, to prepare and construct their brain faculties to deal practical situations appropriately. In addition, Eddy affirmed that Froebel's "Knowledge precedes name"[46] concept was important in teaching arithmetic to the deaf child:

> Every principle or process should be introduced by such simple exercises that the idea is made plain before it is technically named. In this way all the simple principles of arithmetic and the four simple operations can be made to suggest themselves. If there are three books on one desk and two on another, ask a pupil how many he sees; he will count, and answer five books. Thus he adds without knowing it, and, by means of a multitude of such simple exercises, in time the sums of different numbers will become fixed in his memory. Would any pupil thus taught write, 'Three books and four desks make seven *books?*' If you say so yourself, he would at once challenge the correctness of your statement. So the principle of addition, that only like numbers can be added, is brought out of his own mind.[47]

Instead of introducing terms of arithmetic, which he believed would trap deaf students into a limited imagination, he believed the teacher should act it out by role-play, hypothetical situations, and hands-on applications, rather than rote and drill.

In the same year, Charles N. Haskins, an instructor at the Ohio Institution, noted the revival of the use of mental arithmetic in institutions for the deaf, although there was a blanket banishment to employing mental arithmetic in public classrooms at the time. After discouraging teaching experiences with students who had no sense of mental arithmetic and took many hours to compute simple operations of arithmetic, he one day decided to throw "aside the book for the first two or three months of the term, and devoted the hour for arithmetic to drilling the pupils mentally in the fundamental rules involving numbers from one to one hundred, in the following manner, with the satisfactory result of accomplishing twice as much during the year twice as easily and twice as well as the year before."[48] He continued with the description of his method:

> I commenced with addition—beginning with zero, and adding or counting by twos to one hundred; thus, 0, 2, 4, 6, 8, 10, etc. When the class had all become quite proficient in this, they were required to begin with 100, subtracting or counting backward by two; thus, 100, 98, 96, 94, 92, 90, etc., to zero; then counting by two from one to one hundred in the same manner. Thus all the combinations of the twos in addition and subtraction were exhausted. Then multiplication and division by twos from zero to one hundred were taken up in the same manner; thus, 2 time 0 is 0; 2 X 1 is 2; 2, 4; 3, 6; 4, 8; 5, 10; etc. 100 divided by 2 is 50; 98, 49; 96, 48; 94, 47; etc. After this the class was given a thorough practice in skipping about from one number to another.
> Thus all the combinations and relations of one number after another can be exhausted until the pupil can mentally compute the largest numbers with ease, accuracy, and dispatch. By this method the pupil will himself perceive the relationship existing between the fundamental principles—a thing that comes rather by the slow process of assimilation through practice than by any explanation the teacher can give; he not only becomes more rapid and accurate in his computations, but by perceiving the relationship of things he also vastly improves his solutions. After a thorough drill has been given in the above manner the pupil should be taken into what is usually called the 'Properties of Numbers.' He should be taught the divisors, common divisors, greatest common divisors, multiples, common multiples, and least common multiples of numbers,

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[45] Jonathon H. Eddy, "Arithmetic in the Education of the Deaf," *American Annals of the Deaf* 32, no.1 (1887): 94.
[46] Ibid., 94.
[47] Ibid., 95.
[48] C.N. Haskins, "Mental Arithmetic," *American Annals of the Deaf* 32, no.2 (1887): 157-158.

> and required to find them within the scope of the numbers he has been drilled upon in the fundamental rules as quick as a flash. Roots and powers of numbers should also be taught at this point; in fact every combinations and relationship of numbers within a proper limit, commensurate with the development and advancement of the pupils, should be exhausted and continually reiterated until it becomes a second nature—until *two times thirty-seven* is as easy as *two times seven;* or the square or cube of seventeen is almost if not quote a matter of intuition.[49]

Haskins pointed out the importance of employing mental arithmetic so that deaf children would develop a habit of computing and remembering numbers as the foundation of knowledge and mental habit for practical use both to save time inside the classroom and outside the school. He related the process of mental arithmetic in the classroom for the deaf to the blind pupils' study of arithmetic: "I have seen them rapidly and accurately compute long problems in partial payments involving half a dozen dates or more."[50] Furthermore, Haskins stressed that numbers that were used in computations should be within the deaf student's comprehension. Otherwise, he would be labeled "a mental murderer," working in the dark, without understanding.[51]

In 1888, the Minnesota School for the Deaf established a committee to review their curriculum.[52] The committee of the Minnesota School recommended more attention to mental work, child, the banishment of finger counting, and a full written analysis of word problems.

In 1890, George M. McClure echoed C.N. Haskin's call for mental arithmetic when he gave a paper presentation at the 12th meeting of the Convention of American Instructors of the Deaf:

> The first step in teaching is to develop an idea of the value of numbers, and to do this, objects are indispensable. The first year, pupils should have no other exercise in arithmetic beyond drills in writing numbers up to ten, and in counting. The second year I would begin teaching in earnest. Teaching the class to write the numbers up to 20 and to make the Arabic symbols to correspond, and with objects before me, I would construct the addition and subtraction tables with the class, letting them make the discoveries, while I guided the operation. I would drill the pupils thoroughly on these table, giving meanwhile practical problems, and before passing on, I would make sure of three things—that the pupil could do his work with accuracy, with a reasonable degree of rapidity, and that he could state it in correct, if simple form. I would teach the tables of the other two rules in the same manner, and the principles themselves on the same plan, using objects at first with all of them, drilling unsparingly, and insisting on the three points of accuracy, rapidity, and correct form of expression.[53]

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[49] Ibid., 158-9.

[50] Ibid., 159. He recounted an incident: "I once heard a college professor, in an address to a thousand students, ask the students before him how many of them were qualified to take a position as accountant—able to add a whole column of figures at once—at a salary of two thousand dollars a year. Of course no one was ready to take the position. Is this not an eligible field of usefulness and profit for deaf-mutes, and would not the course we have indicated fit them to enter it? I believe it would, in addition to the benefit of the mental drill, and its practical use in other walks of life." [p.161].

[51] Ibid., 160.

[52] J.L. Smith, L.C. Tuck, and Geo. Layton, "A Course in Arithmetic," *American Annals of the Deaf* 33, no.2 (1888): 197-199. The editor of the *American Annals of the Deaf* noted that the Teachers' Association of the Minnesota School had had several meetings during the past year and drawn the report of a nine or ten years' course in arithmetic.

[53] G.M. McClure, "Primary Arithmetic," in *The Proceedings of the 12th meeting of the Convention of American Instructors of the Deaf* (Washington, D.C.: Government Printing Office, 1890): 71-72.

He believed that every pupil should master all four rules with three conditions: accuracy, rapidity, and clear and correct form of expression in printed English. He penned: "After the tables are taught, drill on them should be incessant. A plan that I have found very successful is, to write out the combinations in, say addition, on the board, range the class in front of it, and day after day practice on it, pointing here and there as rapidly as I can get an answer. . . . After the four rules have been taught, there should be a review extending over a period of several months, during which every part of the foundation should be tested, and the weak spots, if any be found, strengthened. The dry bones, so called, of arithmetic, need not be so very dry, if the teacher will only throw a little life into his instruction, and set the pupils to figuring out such live problems as the cost of the clothes he wears and the food he eats."[54] Like Haskins, McClure called for the old-fashioned method of learning arithmetic, that is, by mental drills, to promote accuracy and rapidity. However, McClure encouraged his pupils to discover the basic four rules with appropriate guidelines and use of practical problems that were connected to their daily life.

In regards to the primary teaching of numbers and their combinations, Weston Jenkins of New Jersey's State Institution for the Deaf and Dumb, in 1892, criticized the mental discipline approach, by which students are to drill themselves in learning addition, subtraction, multiplication and division. He cautioned "it may teach adroitness; it may be useful as a training for games of jugglery; but whatever its usefulness may be, it is of no use worth mentioning in teaching arithmetic to deaf-mutes."[55] Additionally, he urged that mathematical concepts must be accessible to the sense of sight, for it is the only primary channel means of acquiring arithmetical knowledge for deaf-mutes.

Combining conceptual understanding through discovery approach and mental arithmetic after discovery, William A. Caldwell of California School for the Deaf, Berkley, in 1893 wrote:

> The pupil should be familiar with the yard-stick, the weights, and the measures. He should compile his own tables from actual experiment. He should *know* that eight quarts make a peck, because he has poured that many quarts of sand into a peck measure, and has discovered that they filled it. He should be prepared to prove that there are one hundred twenty-eight cubic feet in a cord, not by the book, but by showing with a foot-rule the dimensions of a cord. These facts once discovered by him, they should of course be learned by heart; and this is where the text-book should come in, but preferably the text-book which he has prepared himself. He should be required to study and memorize the facts which he has discovered.[56]

Based on his paper, he advocated the method of discovery, one of several themes in the modern paradigm of teaching mathematics, that teachers guide their pupils in discovering mathematical concepts through hands-on activities and that the pupils would develop a repertoire of mathematical facts, thus leading to their own textbooks.

In 1895, Effie Johnston, of the Illinois School for the Deaf in Jacksonville, gave a presentation on number work.[57] In her presentation, she discussed the importance of

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[54] Ibid., 73.
[55] Weston G. Jenkins, "The Teaching of Arithmetic," *American Annals of the Deaf* 37, no.1 (1892): 11.
[56] W.A. Caldwell, "Text-Books," in *The Proceedings of the Thirteenth Meeting of the Convention of American Instructors of the Deaf, Chicago, IL, July 17-24, 1893* (Washington, D.C.: Government Printing Office, 1893): 111.
[57] Effie Johnson, "Number Work," in *The Proceedings of the Fourteenth Meeting of the Convention of*

numbers in daily life, the fact that it is everywhere and in every step of human progress, let alone practical life activities in the workplace, home, and school. She supported the notion that learning should be active: "a normal child is active and loves to be doing something, that he learns words relating to activity more readily than other words, and that his greatest interest is aroused when he is the actor. Therefore, the more we can make the children the actors in daily lessons, the greater their interest will be."[58] She suggested that teachers make their number work lessons look like play with various classroom objects in order to grasp pupils' attention and interest. Furthermore, she also suggested that deaf pupils act out arithmetical problems with objects to solve for unseen facts by discovery. The activity in concrete form allows deaf pupils to discover mathematics by using objects and acting out the problems. She wrote:

> The pupils, observing these conditions, think and reason for themselves, forming their own conclusions. They are actively engaged in the free use of their own powers. They see, handle, experiment and discover for themselves, and are not memorizing numerical facts which the teacher is giving forth from his store-house of knowledge. They are teaching themselves, the teacher guiding and supplying names and technical language when it is needed. They are developing their reasoning powers, forming correct modes of judging, finding out that there is something unknown, something which they must discover which depends upon existing conditions for facts already known, and it gives them confidence in their own powers of observation and discovery. Jerome Allen says: "The joy of discovery is a most powerful mind incentive," and "Curiosity is an incentive. We are all extremely curious to know things hidden from us, for men are but children of a large growth."[59]

Her motto, which followed the modern paradigm, was "Principles and process first and mechanical rapidity will follow when they see the necessity for economy of time."[60] Ellie Johnston made recommendations for teachers that they should include, but were not limited to, the following: restraining from repeating problems, lest lazy members would memorize instead use their reasoning skills; repeating arithmetical processes and language; using real objects in lessons; inventing new conditions for arithmetical problems; using as much illustration as possible with classroom materials, such as books, pencils, pupils' articles of clothing; and keeping a monitor on the pupils' understanding.

According to Superintendent David C. Dudley of the Colorado School for the Deaf in Colorado Springs, there were two parts to arithmetic: "the mechanical work and the application of mechanical work in the solving of problems."[61] Deaf pupils, he believed, should focus on the former in their early years and on the latter after their reasoning minds were developed and trained in mental arithmetic with fluency. For deaf pupils to train in mental arithmetic, he wrote:

> During these first years, the pupil should commit to memory the multiplication table and the tables in denominate numbers. We should learn to add accurately long columns of figures and to subtract and divide without error. There is nothing so exasperating to a teacher of an advanced grade as to

American Instructors of the Deaf, 1895 (Washington, D.C.: Government Printing Office, 1895): 114-122.

[58] Ibid., 115.

[59] Ibid., 116.

[60] Ibid., 116.

[61] D.C. Dudley, "Arithmetic," *The Proceedings of the Fourteenth Meeting of the Convention of American Instructors of the Deaf, 1895*: 329.

> find his pupils capable so far as judgment is concerned, but incapable of accurate mechanical work.[62]

At the end of the fourteenth meeting of the Convention of American Instructors for the Deaf in 1895, William H. DeMotte of Indiana School for the Deaf cautioned against mental arithmetic as the educational objective:

> The pupil may seize the facts and perform the operation more rapidly, indeed, and more satisfactorily to himself, in this way. But allow me to say, rapidity in the performance of a process is an incident of the end, not the end at which we aim. I have observed a certain securing and holding of the educational result, by the use of words—in the language of the question, 'making a language lesson' of it. You can force a pupil to commit to memory the fact 4 x 5 = 20, and he will be no better educated by it. But if you first lead him, by actual count of objects, to recognize a group of four objects, and then that five of these groups taken together are twenty, you are giving him not merely the fact, but what is of infinitely greater value, the mental exercise involved in discovering the fact, with the gratification and skill following. The 'rapid mode of operation' in the question is a resultant of the clear perception attained from the drill. If not secured at once, just go over the drill again. A mechanical rapidity may be secured to a very limited extent by memorizing. 'Four times five are twenty,' *memorized*, will serve only in case of multiplication. Four groups, of five individuals each, *recognized*, will make the pupil to group and handle four, five, twenty, wherever they may occur.
>
> The effort to learn develops the mind. We have quit memorizing the alphabet and long columns of words. By *use* we expect the pupil to learn. Why should we continue the task of memorizing the tables in arithmetic? You want a pupil to respond at once, 'twenty,' when you ask '4 x 5,' but it would much better be because he recognizes the composition of twenty—4 groups of 5 each—than because he mechanically recollects the formula, 4 x 5 = 20.
>
> It is better for him to recognize a bushel, a peck, a quart, and know that the first is a measure composed of four of the second, each of which is a group of eight of the third, than to unreasoningly memorize 'the table' as given in the book…
>
> After the mental drill is secured, the language may be dropped—signs substituted and *rapidity* cultivated. As, after you have taught him to write a complete sentence in reply to a question and you are sure he knows it, you allow him to write abbreviated answers, so in arithmetic he may be permitted to use figures and signs, as:-- 4 x 5 = 20, and .06 x 75 x 2½ = 11.25…
>
> I seriously caution the teacher against attempting to secure speed at the expense of thoroughness. One thing at a time, and each in its logical order.[63]

Memorization, DeMotte suggested, interfered with an understanding of the concept. He asked that deaf pupils learn the four rules thoroughly, with explanations and applications. With that method, rapidity and accuracy would come.

Affirming McClure's stance on mental arithmetic, James L. Smith, principal of the Minnesota School for the Deaf, in 1897, felt that the preponderance of time should be given to mental work to increase rapidity with accuracy.[64] In his article, "Co-operation," he stressed the importance of rapidity and accuracy in mental operations when it came to small numbers and fractions of small denominators.[65] According to him, the duty of teacher should ensure that the pupils acquire rapidity and accuracy in mental operations. Under his traditional rule, a committee of the Teachers' Association in the Minnesota

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[62] Ibid., 330.
[63] Convention of American Instructors of the Deaf, "Question Box," in *The Proceedings of the Fourteenth Meeting of the Convention of American Instructors for the Deaf, 1895,* (Washington, D.C.: Government Printing Office, 1895): x1ii-x1v.
[64] James L. Smith, "Co-operation," *American Annals of the Deaf* 42, no.5 (1897): 325-335.
[65] Ibid.

School for the Deaf called for the use of memorization in school by rote. They argued that it would assist pupils in the acquisition of language in the most effective way, as it was thought that the cultivation of the memory had direct association with the acquisition of education. Hence, they advocated learning lessons by rote and drill.[66]

An avid reader of works by Rousseau, Pestalozzi and Froebel, Edward S. Tillinghast of California School for the Deaf in Berkeley, in 1898, criticized the instruction in schools for the deaf at the time, claiming that it was all but defective, for it dealt with abstractions excessively and from the teacher's point of view, not the other way around.[67] In his "The Correlation of Instruction and Environment" article, he emphasized the importance of environment on instruction and on the education of the deaf: "Education is the process of bringing life into the fullest possible correspondence with environment."[68] Any method of instruction, Tillinghast cautioned, not based upon life experience, pupils' elevated and excited responses, would result in mechanical or careless thinking with no interest, such as the memorizing of arithmetic rules.[69] He asserted "overwhelming evidence proved that the tendency of such methods was towards mental stagnation and dyspepsia, rather than to such healthy growth as proceeded in spite of them. It is seen that unanalyzed truth cannot profitably be boxed up in words and packed away in memory's chambers for future use, because the mind, in its efforts to reach and maintain unity in its processes and their results, tends continually to throw out all such uncorrelated elements of knowledge."[70] In this case, he condemned the educational movement that promoted memory by rote and drill, as seen at the Minnesota School for the Deaf. Instruction should be tied to pupils' knowledge and experience through object- and nature-study and brought to the next level, where the call for analysis and reasoning is reached, and where pupils are able to construct associated thoughts socially and individually. Furthermore, Tillinghast criticized Dudley's paper on "Arithmetic":

> … it says: "During these first years the pupil should commit to memory the multiplication table and the tables in denominate numbers. He should learn to add accurately long columns of figures, to subtract and divide without error.…I do not mean, of course, to devote these years exclusively to abstract work, but to make that predominate….Just here I wish to say that in the solution of every problem there are only two essentials: knowing what to do and how to do it. Why it is so done is a pleasure to know, but by no means an essential." The intimation seems to be that the mechanical parts should be mechanically memorized without bothering much as to the "why." It seems to me that even in the first steps in arithmetic nothing can be more fatal to later sound and rapid progress than by our methods to give the pupil a fair chance to conclude that it is a pleasure, "but by no means an essential," to know the *why* of his work.[71]

Concerned with the spread of the mental arithmetic paradigm, which supported learning through memorization during the first three years of schooling, Tillinghast imploringly

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[66] James L. Smith, "The Function of Memorizing in the Acquisition of Language," *American Annals of the Deaf* 44, no.4 (1899): 242-250.
[67] E.S. Tillinghast, "The Correlation of Instruction and Environment," *American Annals of the Deaf* 43, no.1 (1898): 22-32.
[68] Ibid., 22.
[69] Ibid., 23.
[70] Ibid., 23-24.
[71] E.S. Tillinghast, "The Correlation of Instruction and Environment.—II," *American Annals of the Deaf* 43, no.3 (1898): 225-226.

asked teachers to reconsider their position on the traditional paradigm which, he believed, was rendering learning dull and dry for the deaf, and to teach deaf pupils to understand the relationships between all four rules and their applications to the practical world. He believed learning and understanding the mathematical concepts thoroughly were crucial to building a strong foundation of knowledge upon which related new concepts could be best and effectively learned through association.

At the 15th meeting of the convention of American Instructors of the Deaf at Columbus Ohio, in 1898, William K. Argo, of Colorado School for the Deaf and Blind, replied to a question that was brought up at the end of sessions: "How can we make our pupils apply reason and common sense to the solution of arithmetical problems?"[72] Argo answered:

> In the first place, I should insist on a thorough mastery of the four processes as applied to whole things and to parts of things. Illustrating each process in the concrete, go then to the abstract and drill, drill, drill until the child does all kinds of work without conscious effort. Along with this there should go, taking them one by one, a drill in the language forms used in arithmetic until the child fully understands the meaning. All new language constructions should be placed on the large slate and carefully explained, then examples given to be worked one at a time before the class until the dullest pupil fully understands. Here the teacher who knows signs thoroughly has a great advantage, since he can know at once just what is in the mind of the pupil. The teacher should exercise the greatest care in all the work, both abstract and concrete, to use the correct form and to be sure that each step is perfectly understood in itself and in relation to what has gone before. In all problems have the pupil write enough to show that he has a clear idea of what he is doing, and always given enough problems to enable him to master the principle and to give him confidence in his ability to work them. Thoroughness must be the watchword from the beginning, and the exercises must be so varied as to prevent their being tiresome.
>
> With an enthusiastic teacher there will be enthusiastic pupils, and if pupils can be made to like the study of arithmetic, it will present little more difficulty than any other study. My experience has been that children do not like arithmetic because they do not understand, and they do not understand it because they were not properly trained in the elementary principles.[73]

In sum, his motto for deaf pupils was: learn numbers concretely, commit the four rules to memory through constant drill, gradually increase the use of language in arithmetic, continually review the concepts for thorough understanding and write solutions and steps in plain English to demonstrate knowledge and understanding.

In June, 1899, Principal Weston Jenkins, of New Jersey, presented a paper on the use and abuse of memory in education at the Sixth Summer Meeting, held at Northampton, Massachusetts.[74] He cautioned that memory, fragile as it can be, should be cultivated and used for good purpose and daily life applications. Oftentimes, pupils were asked to memorize information that was not usable or important, thus creating a case of memory abuse: "The phenomenon of 'cramming,' of rapidly acquiring facts and holding them for a brief time, for a definite purpose, is a curious one but not of special value to us as teachers of the deaf, except as a danger to be avoided. In some professions however, it

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[72] Convention of American Instructors of the Deaf, *The Proceedings of the Fifteenth Meeting of the Convention of American Instructors of the Deaf at Columbus, Ohio, July-August 1898* (1899: Government Printing Office: Washington): 253.
[73] Ibid., 253-254.
[74] The paper, titled "Use and Abuse of Memory in Education," was later printed in *The Associative Review* 2, no.1 (1900): 6-15.

serves a useful purpose, and it deserve careful study as a mental curiosity."[75] Jenkins urged that teachers of the deaf teach meaningful concepts, which their students could commit to memory for the benefit of rapidity and accuracy in order to save time.

The closing of the nineteenth century brought no consensus for instructing deaf children arithmetic; however, there was a consensus that mastery in arithmetic had become a necessity for deaf graduates as they prepared for the college or the industrial workplace.

Conclusion

Over the years from 1850 to 1900, the majority of the teachers of the deaf concurred that understanding practical mathematical applications was as important as the knowledge of arithmetical concepts for entering the college or securing a job in the workplace sphere as society moved from the agricultural-business sphere toward the industrial-business sphere. However, they were struggling with finding a more effective method to teach deaf pupils arithmetic and higher levels of mathematics as society was preparing for the inevitable Industrial movement, which required specialization in work trades and increased demand for workers to possess knowledge in arithmetic, let alone measurement. In terms of curriculum and instruction to prepare deaf students for the working sphere, the pedagogical pendulum had kept swinging back and forth since the foundation of the permanent American school for the deaf in 1817 as teachers of the deaf struggled to define effective pedagogies in teaching and learning. Again, this paper focuses on one specific pedagogical issue: mental arithmetic and conceptual understanding.

Questions, such as "how should arithmetic be best taught to the deaf?" and "should deaf students be drilled for rapidity and accuracy in arithmetical calculations, lest it may lead to bore and negative disposition to mathematics?", were on many teachers' minds as they shared their introspective thoughts on different areas in the field of deaf education and their teaching experiences and thoughts in *American Annals of the Deaf*, one of the oldest educational journals, and conventions for American Instructors of the Deaf where professionals in deaf education convened every 2-3 years.

Teachers for the deaf had to deal with in teaching arithmetic: mental arithmetic (i.e., Thomas Gallaudet, J. Scott Hutton, William L. Bird, George M. McClure, David C. Dudley, David Greenberger, Charles N. Haskins, teachers at Minnesota School, and James L. Smith) versus concept understanding (i.e., John Robinson Keep, George M. McCure, Richard Storrs, Albert L.E. Crouter, Weston Jenkins, William A. Caldwell, Effie Johnston, Edward S. Tillinghast, William K. Argo, and Jonathan H. Eddy). Some argued mental arithmetic could lead to accuracy and reduction in time spent on tasks. Others suggested that conceptual understanding could lead to appreciation for knowledge and relationships between concrete things and abstract ideas. Some felt that deaf children should use mental arithmetic first in order to develop automatic calculations and confidence before they understand the concept. Others felt that deaf children should understand what is covered first through visual illustrations, object teaching or discover activities before they go into drill and rote for accuracy, lest they become bore and drudgery. Some methods that might aid mental arithmetic and conceptual understanding as discussed in both *American Annals of the Deaf* and conventions for American

[75] Ibid., 9.

Instructors of the Deaf are (a) use of visual illustrations and object teaching, such as combination cards and concrete things, (b) use of signs for calculations and procedures, such as signs for figures and explanations, (c) teacher's content knowledge in deafness and mathematics, and (d) students' language and reasoning and analytical skills. These methods need to be examined to evaluate their support for student learning.