Infusing Assessment into Mathematics Content Courses for Pre-Service Elementary School Teachers

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The present study aimed to explore the use of assessment in mathematics content courses for future elementary school teachers. Analysis of self assessment data on mathematical understanding and peer assessment data on oral mathematical presentation showed that pre-service teachers had a balanced understanding of procedural knowledge and problem solving. Conceptual understanding was not in the structure of pre-service teachers' mathematical knowledge. Understandings of conceptual knowledge, procedural knowledge, and problem solving had no meaningful effects on gains in mathematics performance. Aspects of oral mathematical presentation were associated with improved understanding of procedural knowledge and in particular conceptual knowledge. The present study calls for a conceptual approach to mathematical knowledge and sufficient mathematical problem solving in college-level mathematics content courses and in particular infusion of assessment into college-level mathematics education for pre-service teachers.

Objectives

We explored in the present analysis the use of assessment in mathematics content courses for future elementary school teachers. At most institutions in the United States, these courses are taught by mathematicians whose emphasis is on learning or relearning the mathematics rather than pedagogical implications of mathematical knowledge, usually as a direct result of the disconnection between mathematicians and mathematics educators. These courses are typically found to be difficult by a large percentage of students for whom mathematics is a major hurdle in their teacher preparation programs. One intervention strategy to tackle these concerns is to infuse assessment into mathematics content courses. As a project of collaboration between the departments of mathematics and education, we explored this idea in two ways: self assessment and peer assessment. We involved students in self assessment of their procedural knowledge, conceptual knowledge, and problem solving, by asking them to define the terms in their own language and identify from their own experiences each type of knowledge. Through the addition of a required oral mathematics presentation, we also engaged students in peer evaluation of other students by asking them to rate the quality of the presentation from multiple perspectives.

Targeting pre-service elementary teachers, we attempted to use the results of this intervention to explore (a) whether there is any relationship among understanding of procedural knowledge, conceptual knowledge, and problem solving, (b) whether there is any factor structure of understanding of procedural knowledge, conceptual knowledge, and problem solving, (c) whether self assessment of understanding of procedural knowledge, conceptual knowledge, and problem solving impacts gains in mathematics performance (in a pre- and post-test design), and (d) whether peer evaluation of oral mathematics presentations impacts understanding of procedural knowledge, conceptual knowledge, and problem solving.

Our study is closely related to the main focus of TSG 29. Among the five key issues identified in TSG 29, our study overlaps substantially with three of them. We addressed what kind of mathematics we could present to pre-service teachers by identifying the weak spots in their content knowledge sturcture (the second issue). We presented an innovative and creative approache of developing mathematical content knowledge of pre-service mathematics teachers by infusing assessment into mathematics content

courses (the fifth issue), and we discussed the impacts and effectiveness of this reform-based pre-service mathematical education of teachers (the fourth issue).

Theoretical Perspectives

Teachers' professional knowledge of subject matter (e.g., mathematics) consists of two overlapping domains: subject matter knowledge and pedagogical content knowledge (see Borko et al., 1992). We focused on the former in the present study, because we worked with pre-service elementary teachers in mathematics subject matter courses. The literature on subject matter knowledge of pre-service teachers suggests that they typically enter teacher education programs with narrow conceptions of mathematics as a set of rules and conventions (e.g., Ball, 1990a; Quinn, 1997; Taylor, 2002; Wilson & Ball, 2002). Specifically, pre-service teachers do not have adequate conceptual understanding of fundamental K-12 (kindergarten to grade 12) mathematics concepts (Frykholm, 1999a). Indeed, conceptual understanding is difficult when pre-service teachers view mathematics as a collection of concrete procedures (see Wilson, 1994). This limitation, together with the traditional teaching models that they have experienced in pre-college and college mathematics courses, becomes obstacles to their acquisition of reform-based philosophies and practices in mathematics education (Frykholm, 1999b).

Mathematics education researchers have pinpointed specific deficiencies in pre-service teachers' subject matter knowledge. For example, Ball (1990b) found that division is one of the most difficult conceptions for pre-service teachers who hold a narrow understanding of division as forming a certain number of equal parts. Arguing that the limited conception of function seriously affects pre-service mathematics teachers' pedagogical thinking, Even (1993) reported that pre-service secondary school teachers do not have a modern conception of function as they fail to appreciate the arbitrary nature of functions and the importance and origin of the univalence requirement. Rowland, Martyn, Barber, and Heal (2001) identified understanding of reasoning and proof as the weakest area in pre-service mathematics teachers' pedagogical thinking, with misconceptions as the nature of mathematical errors.

What is lacking in the literature is what subject matter knowledge structure, in terms of conceptual and procedural understanding of mathematical knowledge, pre-service teachers bring into their teacher education programs. In considering pre-service teachers' subject matter knowledge structure, we also added understanding of problem solving in mathematics. The National Council of Teachers of Mathematics (NCTM, 2000) stated that problem solving is not only a goal of learning mathematics but also a major means of doing so. More importantly, NCTM (2000) warns against treating problem solving as an isolated part of mathematics. These arguments are our reason to bring in problem solving when considering pre-service teachers' understanding of conceptual and procedural knowledge. Our intention was to investigate how pre-service elementary teachers configure among understanding of conceptual knowledge, procedural knowledge, and problem solving to form their factorial structure of mathematical knowledge as they enter teacher education programs (see the first two research questions discussed earlier). We believe that this work both fills in a major gap in the literature and contributes greatly to the ever-increasing attention to pre-service mathematics teachers' subject matter knowledge.

To better prepare pre-service elementary teachers, reform of college-level mathematics courses is needed given that the literature has clearly identified pre-service teachers' experiences in college mathematics courses as one main obstacle to their professional development (e.g., Ball, 1990a). Essentially, mathematics course reform at the college level must challenge pre-service teachers' perceptions of mathematics as rules and mathematics teaching as telling that have been reinforced as their way of thinking about mathematical knowledge and mathematics education for years.

We realized that there are multiple ways to pursue this reform. One of the most effective and natural ways is to relate to or emphasize the teaching aspect in college-level mathematics subject matter courses. The Mathematics Sciences Education Board (1996) suggested that "there is increased evidence that prospective teachers can learn about teaching mathematics from studying the practice of mathematics teaching" (p. 7). Frykholm (1999b) argued that creating opportunities for pre-service teachers to engage in conceptually based mathematics discourse throughout their teacher education programs is beneficial in developing understanding of not only mathematical content knowledge but also pedagogical content

knowledge. The thinking is simply that *all* courses in teacher education programs must proactively highlight various aspects of mathematics teaching and learning.

With a similar conviction, we intended to infuse assessment into subject matter courses in mathematics, and there is a reason to focus on assessment. Black and Wiliam (1998) stated that assessment, in particular formative one, is not well understood by in-service teachers (not to mention pre-service teachers) and has been a weak spot in their classroom practices. Our intention was to create awareness and appreciation for the notion of classroom assessment early in pre-service elementary teachers' education programs, instead of waiting until they take (pedagogical) method courses to get to know this aspect of teaching. The infusion of assessment is also a natural companion to the instruction of mathematics, because the quality of understanding and interpretation of mathematical knowledge is a real assessment matter. The opportunities for pre-service teachers to assess their own understanding of mathematical knowledge and that of their peers are quite beneficial in the early professional development of pre-service teachers (see McTighe & Wiggins, 2004). This novel approach of intervening with college-level mathematics course reform is also quite unusual in the literature.

Methods

The present intervention was carried out during the 2005-2006 academic year at one of the major universities in the Southern part of the United States. Every year, about 375 pre-service elementary (90%) and middle (10%) school teachers enroll in a mathematics content (subject matter) course designed for majors in elementary and middle level education programs based on the textbook entitled *Mathematical Reasoning for Elementary Teachers* (Long & DeTemple, 4th edition, 2005). As one major component of the course, an oral presentation is closely related to the notion of infusion of assessment. The goals of this exercise emphasize the quality of the understanding and interpretation of mathematics based on the NCTM standards. Pre-service elementary teachers were given a wide range of subjects and could choose the topics on which they were interested in presenting. Groups were then constructed based on the choices of topics. Each presentation was evaluated for (a) quality of the oral presentation as a presentation, (b) preciseness and mathematical correctness, (c) applicability (relating the material to what would be expected to be seen in the elementary classroom), (d) creativity (bringing something to the presentation other than what was in the article or book), and (e) group work (collaboration).

Each oral presentation was accompanied by a peer mentor assessment assignment. This means that each student was given a score (in a scale of 1 to 5) on each of the five presentation criteria shown above. This was how we collected peer assessment data that we used to address our research question (d). As another infusion of assessment into mathematics content (subject matter) courses, self assessment on the understanding of conceptual knowledge, procedural knowledge, and problem solving was also implemented in the course. Each student defined in their own words conceptual knowledge, procedural knowledge, and problem solving (the definition part) and also provided a mathematical example for each type of understanding (the identification part). Each student's response was evaluated by a three-member team (a mathematician, a mathematics educator, and a doctoral student in mathematics) and was given a score (in a scale of 1 to 3) on the quality of his or her definition and identification of each type of understanding. As a result, each student had a score on definition and a score on identification for each type of understanding. For each student, the scores could be used separately or combined to indicate the overall quality of each type of understanding. This was how we collected self assessment data that we used to address our research questions (a) to (c).

Our first two research questions (a) and (b) addressed the relationship among understandings of conceptual knowledge, procedural knowledge, and problem solving (the self assessment component). We used combined scores of definition and identification to examine this relationship through a correlation analysis, with correlation coefficient as a measure of the association among understandings of conceptual knowledge, procedural knowledge, and problem solving, providing some initial ideas about this relationship. We proceeded with a factor analysis (again using combined scores) to discern the factor structure among understandings of conceptual knowledge, procedural knowledge, and problem solving. We used the Varimax rotation method to reduce overlapping among factor loadings for a clear factor structure, providing insights into how understandings of conceptual knowledge, procedural knowledge,

and problem solving were configured among pre-service elementary teachers. To address our research question (c), we took the regression approach to analysis of covariance to investigate the impact of understandings of conceptual knowledge, procedural knowledge, and problem solving on gains in mathematics achievement during the course. Here we considered both combined scores and separate scores on definition and identification. The mathematics achievement test was made by instructors which is very standard and topical to this level of mathematics courses well-aligned with the content of the courses.

Note that, so far, all statistical analyses discussed above were based on self assessment data and mathematics achievement (testing) data collected during the course. Finally, peer assessment data were brought in to match up with self assessment data (combined scores between definition and identification) to study our research question (d), the relationship between quality of oral presentation and understandings of conceptual knowledge, procedural knowledge, and problem solving. We used this analysis to investigate the effects of oral presentation on each type of understanding (conceptual knowledge, procedural knowledge, and problem solving).

Results on Research Question (a)

As mentioned above, we used correlation coefficients to measure the relationship among quality of understanding in terms of procedural knowledge, conceptual knowledge, and problem solving. We found that understanding of procedural knowledge was statistically significantly related to understanding of problem solving among pre-service elementary teachers. Although pre-service elementary teachers were able to see a connection between procedural knowledge and problem solving, the magnitude of this correlation (0.17) is considered small in terms of effect size in behavioral sciences. Thus, we concluded that pre-service elementary teachers could loosely connect between understanding of procedural knowledge and understanding of problem solving.

On the other hand, understanding of conceptual knowledge had no connection at all with understanding of either procedural knowledge or problem solving among pre-service elementary teachers. There were clear indications in our data that the lack of understanding was more serious regarding conceptual knowledge than either procedural knowledge or problem solving. We concluded that this disconnection was related to the confusion among pre-service elementary teachers about what exactly conceptual knowledge meant to them.

Results on Research Question (b)

Both of our conclusions were supported by a factor analysis that examined the factor structure of understanding of procedural knowledge, conceptual knowledge, and problem solving among pre-service elementary teachers. We disaggregated definition and identification of each type of knowledge in this factor analysis. The results were a clean factor structure that clearly indicated two factors. Procedural knowledge (definition and identification) and definition of conceptual knowledge loaded on the first factor, which can be clearly labeled as procedure dominated factor. Problem solving (definition and identification) and identification of conceptual knowledge loaded on the second factor, which can be clearly labeled as problem solving dominated factor. Therefore, pre-service elementary teachers clearly separated procedural knowledge from problem solving, and they did not demonstrate any unique factor dominantly of conceptual knowledge.

The other essential finding from factor analysis, which also supported our conclusions, was that the two factors were equally important judging from the proportion of variance explained by each factor. This clean balance between procedure (dominated) knowledge and problem solving (dominated) knowledge was another convincing indication that (a) pre-service elementary teachers had a balanced understanding of procedural knowledge and problem solving as separate factors in the learning of mathematics and (b) conceptual knowledge was not in the picture of pre-service elementary teachers' understanding of mathematical knowledge in general—their understanding of conceptual knowledge was a subset of their understanding of procedural knowledge and problem solving.

Results on Research Question (c)

Could pre-service elementary teachers' understanding of procedural knowledge, conceptual knowledge, and problem solving impact their mathematics performance? We examined this issue first by

testing combined scores between definition and identification of each type of understanding in relation to gains in mathematics performance (from pre test to post test). We found that, *individually*, the quality of understanding of procedural knowledge and conceptual knowledge had no statistically significant effects on gains in mathematics performance.

On the other hand, a better understanding of problem solving was statistically significantly related to a higher level of gains in mathematics performance. Pre-service elementary teachers with one standard deviation higher in the quality of understanding of problem solving showed more gains of about 16% of a standard deviation in mathematics performance. To make statistical results like this easier to understand, we translated all effects into common language using score points. In the current case, 1 score point increase (in a scale of 1 to 3) in understanding of problem solving would be associated with an increase of 1.16 score points (in a scale of 0 to 25) in gains in mathematics performance. Obviously, such a magnitude is practically small though statistically significant. We concluded that the effects of understanding of problem solving on gains in mathematics performance lacked practical importance.

Our conclusion became evident when we examined understanding of problem solving *collectively* with understanding of procedural knowledge and conceptual knowledge. Understanding of problem solving ceased being statistically significant, and as a matter of fact, none of the three types of mathematical knowledge turned out to be statistically significant. Therefore, understanding of procedural knowledge, conceptual knowledge, and problem solving were not associated with gains in mathematics performance. There was one more piece of evidence that supported this conclusion—the proportions of variance accounted for by these models. These proportions were trivial—even the joint model with the presence of all types of understanding accounted for only 9% of the total variance. Obviously, mathematics performance did not have much to do with understanding of procedural knowledge, conceptual knowledge, and problem solving.

So far, we have examined overall understanding of procedural knowledge, conceptual knowledge, and problem solving. Because overall insignificance may not nullify significance associated with specific aspects of each type of understanding, we used definition scores and identification scores separately to examine whether definition and identification were statistically significantly related to gains in mathematics performance. We found that, *individually*, pre-service elementary teachers who could define procedural knowledge better showed a higher level of gains in mathematics performance (i.e., 1 score point increase in definition of procedural knowledge would be associated with an increase of 0.92 score points in gains in mathematics performance). We also found that pre-service elementary teachers who could identify problem solving better displayed a higher level of gains in mathematics performance (i.e., 1 score point increase in identification of problem solving would be associated with an increase of 0.97 score points in gains in mathematics performance).

Again, magnitudes as reported above are practically unimportant though statistically significant. We concluded that aspects of each type of understanding were not meaningfully related to gains in mathematics performance. This conclusion remained once we examined aspects of the three types of understanding *collectively*. Although pre-service elementary teachers who could identify problem solving better continued to show a higher level of gains in mathematics performance, magnitude of effect was trivial (i.e., 1 score point increase in identification of problem solving would be associated with an increase of 0.87 score points in gains in mathematics performance). Once again, the proportions of variance explained by these models offered one more piece of evidence that mathematics performance did not have much to do with aspects of understanding of procedural knowledge, conceptual knowledge, and problem solving. Therefore, we concluded that, overall, understanding of procedural knowledge, conceptual knowledge, and problem solving had no meaningful effects on gains in mathematics performance.

Results on Research Question (d)

We went on to examine whether peer evaluation of oral mathematics presentations impacted understanding of procedural knowledge, conceptual knowledge, and problem solving. For understanding of procedural knowledge, pre-service elementary teachers who showed better group work demonstrated better understanding of procedural knowledge. This effect remained when all aspects of the presentation

were examined *collectively*. As we controlled other aspects of the presentation, 1 score point increase (in a scale of 1 to 5) in group work would be associated with an increase of 0.31 score points (in a scale of 1 to 3) in understanding of procedural knowledge. We considered this magnitude a moderate effect size which indicates that better group work was associated with improved understanding of procedural knowledge.

More effects appeared in the case of understanding of conceptual knowledge. Pre-service elementary teachers who showed better preciseness and mathematical correctness demonstrated better understanding of conceptual knowledge, and those who showed better creativity also demonstrated better understanding of conceptual knowledge. These effects remained statistically significant in the joint model. Once we controlled other aspects of the presentation, 1 score point increase in preciseness and mathematical correctness would be associated with an increase of 0.44 score points in understanding of conceptual knowledge, and 1 score point increase in creativity would be associated with an increase of 0.30 score points in understanding of conceptual knowledge. Finally, we found a negative effect associated with overall quality of the presentation, once we controlled other aspects of the presentation. Pre-service elementary teachers who had good understanding of conceptual knowledge appeared unprepared to make a sound oral mathematics presentation. We suspected that although some future teachers might be good at logical and conceptual thinking and reasoning (in connection with conceptual knowledge), they appeared unable to articulate mathematical knowledge or mathematical thinking in a well organized and understandable way. Given the magnitude of this effect (1 score point increase in overall quality of the presentation would be associated with a decrease of 0.57 score points in understanding of conceptual knowledge), such a problem surely calls for attention.

Peer evaluation of oral mathematics presentations appeared to have the least impact on understanding of problem solving. This conclusion could be appreciated from two perspectives. First, although there was a statistically significant individual effect that pre-service elementary teachers who were creative in presentation demonstrated improved understanding of problem solving, the magnitude was the smallest among all statistically significant variables. Second, once other aspects of the presentation were considered, creativity ceased being statistically significant.

Overall, we concluded that peer evaluation of oral mathematics presentations was indeed associated with improved understanding of procedural knowledge and in particular conceptual knowledge. Group work was associated with improved understanding of procedural knowledge, and preciseness and mathematical correctness and creativity were associated with improved understanding of conceptual knowledge.

Discussion

One of the most striking findings from the present study is that pre-service elementary teachers did not have a concept domain in their mathematical knowledge structure. It is easier to approach conceptual understanding from a procedural perspective (see Skemp, 1977). Our results also suggest that pre-service elementary teachers could approach conceptual understanding from the perspective of problem solving. It seems to us that, most likely, when conceptual knowledge was applied to problem solving by pre-service elementary teachers, knowing how overweighed knowing why. In both cases, pre-service elementary teachers demonstrated seriously inadequate appreciation of conceptual knowledge especially with regard to its difference to both procedural knowledge and problem solving. Results of the present analysis provide further support for the argument that reform on mathematics content (subject matter) courses at the college level must emphasize a conceptual approach to mathematical knowledge.

We were intrigued by the lack of relationship between understandings of conceptual knowledge, procedural knowledge, and problem solving and gains in mathematics achievement among pre-service elementary teachers. This disconnection indicates largely one thing to us—knowing what a procedure is does not necessarily mean knowing how it is applied or used. We believe that practice is very likely one of the major reasons. Although good arguments can be made against drills in mathematics (and we certain agree), we still believe that knowing mathematics requires considerable time commitment.

Finally and most importantly, results of the present study do suggest that the infusion of assessment into mathematics content (subject matter) courses holds great potential to strengthen

mathematical understanding among pre-service elementary teachers. We were particularly encouraged by the finding that the level of the oral mathematical presentation was associated with aspects of conceptual understanding or knowledge.

Educational Importance of the Study

The unique contribution of our study is twofold. First, our study represents a pioneering effort to understand the structure of subject matter knowledge among pre-service elementary teachers. Among long-standing concerns about their conceptual understanding of mathematics, we have demonstrated that pre-service elementary teachers in fact lacked a domain of conceptual knowledge. In contrast, we have demonstrated that pre-service elementary teachers had clear domains of procedural knowledge and problem solving. We conclude that pre-service elementary teachers' understanding of conceptual knowledge was part of or confused with their understanding of procedural knowledge and problem solving. This pioneering finding strongly suggests the need for postsecondary mathematics instructors to clearly separate different types of mathematical knowledge with an emphasis on the unique characteristics of conceptual knowledge in relation to procedural knowledge and problem solving.

The other unique contribution is that we have found that the infusion of assessment into mathematics content (subject matter) courses is a potentially effective way to strengthen mathematical understanding of pre-service elementary teachers. Particularly important in an educational sense is our finding that this educational practice (the infusion of assessment) was associated more with conceptual understanding than with either procedural understanding or problem solving. Therefore, our study has added another potentially effective alternative to postsecondary mathematics instructors who are concerned with conceptual understanding of pre-service elementary teachers. The infusion of assessment into mathematics content (subject matter) courses is a potentially effective way to improve conceptual understanding of pre-service elementary teachers.

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