# A HYBRID MODEL FOR MATHEMATICS TEACHER EDUCATION: FACE-TO-FACE AND AT-A-DISTANCE

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Abstract: This is an empirical report of a developing hybrid model of pre-service education that is being offered to a diverse group of forty-six prospective teachers resident on four Pacific islands that are part of the Hawaiian chain. The model for preparing these pre-service teachers includes a mixture of face-to-face instruction interlaced with the use innovative computer technology that allows the instructors and students to interact with one another using audio, video, and whiteboards for sharing materials and writings. Issues that are addressed in this paper pertaining to the focus of the study group are: (a) What kind of mathematics could we present to pre-service teachers when physical contact time is restricted 24 hours per term? (b) What links or relationships could be forged using this mixed model between the kind of mathematics offered and the role of mathematical experiences of the pre-service teachers? (c) Would the use of the innovative and creative computer approaches support the development of mathematical content knowledge of pre-service teachers?

## Introduction

During the academic year 2005/2006 (as practiced in North America), the Institute for Teacher Education (ITE) at the University of Hawai‘i at Manoa (UHM) campus in Honolulu completed a pilot project designed to offer their two-year teacher certification program to potential elementary teachers resident on the four neighbour islands of the state. Prior to the creation of this pilot project, students from islands other than O‘ahu had to move to and live in Honolulu for nine months each year in order to complete the teacher education program. The pilot program was offered on the candidates' home islands thereby eliminating the need for the students to move to Honolulu. The creation of the pilot program meant that many candidates—many of them were mothers of young and growing families, or practicing teacher aides, or individuals in financial challenging circumstances—were now able to stay at home, maintain their jobs, and have secure financial support and still complete the requirements to become a certified teacher in Hawai‘i. The pilot program was successful and received enthusiastic support from the candidates and personnel involved with the teaching in the program. The decision was made to seek a full complement of candidates from four neighbour Hawaiian islands—Maui, Kaua‘i, Moloka‘i, and the Big Island—to undertake the full two-year certification program that involved all elements of ITE program as offered at the main UHM campus in Honolulu.

## The Statewide Program: the Scope, Setting and Site of the Program

This paper describes the sites and form the program used and adopted, the mathematical content and methods of instruction offered, and the nature of the students who comprised the first full complement of candidates who took part in the program beginning May 2006, and who graduated two years later in May 2008. This was the first time the new 2-year Cohorted Statewide Program was available to students from neighbour islands.

The ITE program is structured to operate as a cohort program; i.e., approximately 20 teacher candidates progress as a group through the various aspects of the program. They take all their courses together in a fixed pattern that is laid out in advance of the beginning of the program. The initial advertisement for the Statewide program garnered a record number of qualified applicants, so many that a decision was made to create two cohorts of students, one centered on Maui to include 26 candidates from Maui and the Big Island, and the other centered on Kaua‘i to include 20 prospective teachers from Kaua‘i and Moloka‘i. The program includes, among others, two mathematics education courses. The focus of this paper is on the two mathematics education courses during the fall 2006, and spring 2007 terms of the six-term, two-year program. A full description of the program can be found at http://www.hawaii.edu/coe/departments/outreach/documents/BrochureBED-ELEM7-07update.pdf. The author of this paper offered the two mathematics education courses to the 26 students centered on Maui. A colleague, Dr. Joe Zilliox, also a professor of mathematics education in the UHM College of Education, taught the 20 students centered on Kaua‘i. The two of us jointly developed and planned these new *online hybrid* courses, the first time the Elementary Mathematics I and II courses had been offered in this fashion. In the description of the courses that follows, I will focus on the Maui cohort that I taught but it should be understood that the conceptualization and design of their courses was a cooperative and collaborative effort of the two professors, and the pattern of implementation was parallel for the two sites, Maui and Kaua‘i.

Designating the course as *online hybrid* was meant to convey the fact that the course was offered via a blend of face-to-face (f2f) and electronic online instruction. The f2f component of the instructional model took place roughly once per month on Maui, four sessions per term. Twenty students from Maui, joined by six students who flew in at the College's expenses from the Big Island, engaged in a full weekend of instruction that encompassed three different courses. The weekend of instruction included three 4-hour time slots [Saturday morning 8-12 and afternoon (1-5), and Sunday morning (9-1)] and one

3-hour time slot [Friday evening 5:30-8:30]. During two of the four weekends, the mathematics education course had two of the instructional slots (8 hours), and had one time slot during the other two weekends (3 or 4 hours, depending if one time slot was on Friday evening). In total there was 23 hours of f2f instruction.

The second instructional component, interlaced between the f2f sessions, was the once monthly, 2.5-hour *online* session that made use of the computer-based *Elluminate* (www.elluminate.com) conferencing software and *WebCT*. *Elluminate* provides online, interactive eLearning and Web conferencing technology services that support student learning. *WebCT* is an online proprietary virtual learning environment purchased by the University of Hawai'i for eLearning. When using *Elluminate*, students, sitting in the comfort of their own homes, connect to the Internet using their laptop computers that are required for enrolment in the program. Once connected, they can talk with one another and the instructor, write (type and use drawing instruments) on a whiteboard that is jointly shared by all participants, be distributed by the instructor into small groups where they can discuss mathematical problems and readings and engage in mathematical investigations. When the small group work is complete, the instructor electronically brings all students back to the electronic classroom where they debrief the experiences they had in the small group. The *Elluminate* sessions provided a total of 10 hours of instructional time. Together the f2f and *Elluminate* sessions meant the students were in class for a total of 34 hours. When the mathematics education course is offered on the main campus of UHM, the total instructional time is 37.5 hours. We used *WebCT* that is an online system through which students submitted and we marked and returned assignments, engaged in asynchronous discussions with each other and me, provided students with session agendas (f2f and *Elluminate*), instructions for course readings and mathematical investigations and problems, and a set of course notes in lieu of a commercially bought textbook. The active use of *WebCT* accounted for the additional 3.5 hours of instruction required by the course syllabus for the mathematics education courses.

The Students and the Mathematical Content

The above details provide information about the scope, setting, and site where the *online hybrid* courses were offered to the 46 prospective elementary teachers. Typically, the participants came to the program with two years of community college experience that usually included at most one college level mathematics course. Many of the students professed to be fearful of mathematics, and their initial reactions during f2f sessions provided support for that view of themselves. They were nervous about what this *math* course would be about and whether or not they'd be able to handle the material. Moreover, they lamented, they had to take two *math* courses so it was a significant portion of the total program. What then were the two courses all about? What kind of mathematics was involved? What instructional methods were used that might assist students in overcoming their initial nervousness about the course?

This Topic Study Group's (TSG29) call for papers included a distinction between academic mathematics and school mathematics. The focus in the two courses I offered on Maui was on school mathematics. Prior experience had taught me that many students would be nervous about the course so during the first f2f session I made a concerted effort to engage them in mathematical activities at which I could almost guarantee they would be successful. Because they are human, I knew they could seek out and identify patterns. Much of school mathematics can be understood in terms of patterns. The initial tasks involved looking for patterns, and as expected all the students were successful.[1] Just as first impressions are

[1] The first task I used was to ask them to add the two digits of their age together, and keep doing so until they arrived at a single digit. I asked students to tell me their digit, and when they did I told them how old they were! Initial amazement gave way to questions about how I could do that. This led us to analyze the mathematics of situation so that they too could understand and perform this little bit of mathematical magic! Yes, one woman who said her single digit was 4 caught me. I was torn between saying, based on

important (though not full proof) when meeting new people, the students first impressions of whether or not they'd be able to 'do the math' in this course was key to changing their attitude about themselves as mathematics learners, and eventually teachers of elementary school mathematics. This focus on changing student attitudes was an overarching concern as the course was conceptualized, developed, and offered to the students. It led us to create activities, experiences, and challenges that we felt confident most students could succeed at but which at the same time provided a strong basis for them coming to understand conceptually, pedagogically, and procedurally the mathematics behind the tasks we set for them.

Instructional Methods and Mathematical Content: Electronic Aspects

Because of the *hybrid* nature of the course, three kinds of experiences were designed that recognized that approximately one third of the instructional time was electronic. The three experiences were mathematical problems, mathematical experiences, and the discussion of and at times implementation of ideas gained from reading a mathematics teacher journal.

All students were required to obtain a student membership in the National Council of Teachers of Mathematics (NCTM), and to select the NCTM journal *Teaching Children Mathematics* as their journal of choice. Each term, students were required to read five or six assigned articles, to discuss these articles in small groups during an *Elluminate* session, and write reflective 500 word papers about ideas in three of the articles, and how they might (or had) implement those ideas into the practicum setting. The students spent 2 days per week in assigned schools during the two fall and one spring term of the program, and were full time in an assigned school during the second spring term. They thus had many opportunities to try out ideas suggested in their journal readings, and to share their experience in writing and during *Elluminate* sessions. One example of two articles from which many students developed lessons which they taught in their practicum setting are contained in the January 2007 issue of *Teaching Children Mathematics* and were titled (1) ***The answer is 20. What is the Question?*** and (2) ***Solutions to the Cheerio Count Problem*.** Students were asked to discuss these articles during an *Elluminate* session, and subsequently write a reflective piece about the article. The reflection in many cases included a discussion about using the ideas from the article with children.

There was a common pattern developed for engaging students with the three mathematical problems and three mathematical experiences presented to the students during each of the two courses. The problems and experiences were posted on *WebCT*, typically ten days to two weeks prior to an *Elluminate* session. Students were asked to try the problem or engage in the experience on their own prior to meeting their classmates online. During the *Elluminate* session, the students were randomly assigned (by the software) to small groups of 4 or 5, to discuss what they had found out about the problem and had experiences with the exploration. They students were encouraged to **not** give answers to other students. They were encouraged to act like teachers; i.e., pose questions to their classmates, suggest lines of exploration, provide gentle hints, and so on, but to not just blurt out an answer. As the term progressed, the students got better and better at helping each other in supportive and constructive ways, and to avoid giving away answers. They did testify however that it was difficult not to *tell the answer*, and that they had to practice the ten-second rule many times. The final phase of a problem or exploration assignment differed for the two activities. In the case of the mathematical problem, each student had to submit via *WebCT* a solution to the problem plus a reflection on the process the student went through in solving the problem, including any discussions she had during an *Elluminate* session. In the case of the mathematical exploration, students were to bring their notes and scribblings to the subsequent f2f session where a class wide discussion was held regarding what students had discovered during their explorations, and any reflections they had about the process they went through in tackling the exploration.

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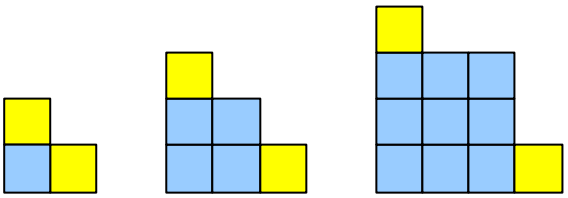
her appearance, that she was 40 or 49. A gentleman always picks the least age! I was wrong. She was 49, but she thanked me saying she was 40.

A couple of the mathematical problems are provided below, followed by a description of two of the mathematical explorations that were used. The *Generalize the Pattern* mathematical problem was the first one given to the students during the first course. Based on the conviction that students could identify patterns it seemed a good choice, one that most students could complete even if they didn't quite get to an algebraic statement for the generalized pattern. The second problem, the *Locker Problem*, presented a greater challenge in identifying the generalized pattern that occurs. During the *Elluminate* session, students assisted each other, shared drawings on the white board, used the application sharing aspect of *Elluminate* to display charts and graphs they had drawn, all of which enabled students to move forward and eventually solve the problem. They did say, however, that the second problem was much more difficult and more frustrating than the first problem. Some students who had expressed anxiety at the beginning of the course testified that those feelings came rushing back when they tackled the Locker Problem, but that with the help of their classmates they were able to overcome their anxiety and get an answer to the problem. Here are the two problems.

*Generalize the Pattern: A Mathematical Problem*



Sketch the next two steps if this pattern**.**

Describe, in words, what the pattern looks like? Describe in words how the pattern is changing as you move from one step to the next. Is there more than one way to do this? If so describe the different ways you see the pattern. How many squares would there be in the $10^{th}$ step of the problem? The $100^{th}$ step?

*The Locker Problem: A Mathematical Problem*

As the school year begins, so does an absurd ritual at a local high school. There are 1000 lockers, all closed, and 1000 students. The lockers are numbered from 1 to 1000.

1. The first student entering the building goes along and opens every locker.
2. The second student goes along and closes every second locker beginning with locker #2.
3. The third student then goes along and **changes the state** (from open to close, closed to open) of every third locker beginning with locker #3.
4. The fourth student goes along and changes the state of every fourth locker beginning with locker #4.
    - What determines if a locker changes state? For instance, when does locker #18 change?
    - If this procedure is continued until all 1000 students have passed by all 1000 lockers, which lockers are open? Identify these lockers by their #s.
    - Pose a mathematical explanation for this phenomenon.

The mathematical explorations chosen could have been used as one of the mathematical problems as well. The difference between the two kinds of experiences was more in what the students had to do with the experiences (write it up formally or contribute to a large group discussion).

The first exploration was similar in kind to the first mathematics problem: look for a pattern, count the squares, and try to generalize what is happening. The second exploration—factors of 30030—called for different strategies. Some students used brute force; i.e., using a calculator to labouriously divide 30030 successively by 2, 3, 4, 5, … to determine if each number 'divided equally'. At this juncture in the course many students reasoned (after all, Sandy gave us the problem!) that there must be a pattern, and aggressively sought out ways to systematically determine what the factors were. In a few cases, the challenge provided a few students with an opportunity to explore the use of Excel, a spreadsheet program that was on their laptop computers but which few students had ever used. Here are two of the six mathematical explorations presented to the students.

*Squares on a Checkerboard: A Mathematic Exploration*

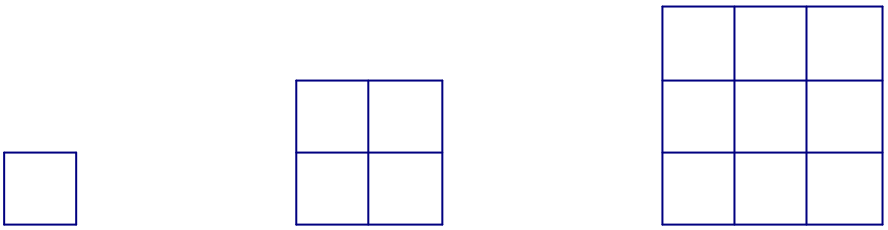


Figure 1 Figure 2 Figure 3

Look at Figure 1, how many squares are there?
Look at Figure 2. If you follow the grid lines, how many squares can you find?
(Hint: There are more than just 4)
If you follow the grid lines, how many squares are there in Figure 3?
(Hint: There are more than 10 and the squares can overlap.)
Use the ideas you developed from above to determine the number of squares on an 8 x 8 checkerboard.
What patterns do you see?
What generalizations can you make?

*Factors of large numbers: A Mathematical Exploration*

Factors of a number are other numbers which when multiplied together yields the original number. For example, 2 and 12 are factors of 24. "Why," you might well ask. The reason 2 and 12 are a factor of 24 is because when you multiply 2 and 12, the result is 24.
Some other factors of 24 are: 3 and 8 1 and 24 4 and 6 2, 3 and 4
The prime factors of 24 are: 2, 2, 2,and 3
The entire list of factors of 24 includes: 1, 2, 3, 4, 6, 8, 12, and 24.

**The task you are to explore is to list all the factors for the number 30 030.**
I'll get you started: 2 is a factor and so is 10.
Now go to work. Find the other factors of 30 030.
Remember to keep a record of the work you do. What strategies did you use to make sure you found all the factors? Are you sure you got all the factors? How would you convince someone else you got all the factors? How did you feel as you were doing the assignment? Did you get frustrated? Bored? Did you feel satisfaction when you found another factor that you hadn't seen before? If you got stuck, how did you get un-stuck?

Each successive *Elluminate* session held over the two terms when the mathematics education courses

were taught improved from the previous session. Students and the instructor got used to the *Elluminate* software, discovered how to use the application sharing tool to paste work onto the white board, and became more and more proficient in carrying on spirited discussions in small and large groups once the technique of making effective use of the microphone was mastered. The course designers and instructors were not sure at the outset of the program how much, if at all, the *Elluminate* sessions would contribute to the course delivery, and hence were pleasantly surprised at how well received and useful the sessions were. There was not much anxiety about the face-to-face sessions though the time slots seem to require that a lot of activities and experiences be compressed into a short period of time. The fact that these sessions only took place once a month was also a concern. Let's look now at how the f2f sessions evolved.

Instructional Methods and Mathematical Content: Face-to-Face Aspects

There were two instructional approaches used in the f2f sessions not used during the *Elluminate* sessions: the reading of mathematically oriented children's stories, and a hands-on-approach for using commercially bought or teacher made mathematical manipulatives. There were also two additional assignments that extended the pedagogical and mathematical content knowledge to the classrooms where the students were undertaking their practicum.

Dealing with these in reverse order, the manipulatives included items such as Cuisenaire rods, multi-base blocks, geoboards, Miras, metric and imperial measuring instruments, locking centimeter cubes, pattern blocks, and tangrams to name but a few. In most instances, the course instructor brought these materials with him from UHM when he flew over to Maui for the weekend sessions. These materials were used to further student understanding of and pedagogical approaches to teaching: computational operations on the real numbers, place value, numerical and geometric patterns (including tessellations), symmetry, perimeter and area, among many mathematical topics. While using these manipulatives to engage in mathematical tasks, the process standards as typically enunciated by the NCTM were addressed: problem solving, communication, reasoning and proof, connections, and representations. Post program reflections indicate that a greater emphasis was placed on problem solving, communication, reasoning, and representations, with a lesser focus on proof and connections.

The reading of a mathematically based children's story preceded most mathematical tasks the students engaged with during f2f sessions. The integration of mathematics and literature was a major thrust present in the courses. For example, prior to exploring geometrical ideas with tangrams, the children's book *Grandfather Tang's Story* was read to the students. As the instructor was reading, the students used their own set of tangram pieces to follow along with the story. When the story was completed, the students engaged in geometrical tasks involving the use of the tangrams. There are a wide variety of children's books now available which can be used with many elementary school mathematics topics. Students in the programs were provided with lists of such books, and experienced first hand how these books could be used to introduce and/or reinforce mathematical ideas and concepts. Examples of the great number of children's books useful in elementary school, and the source from where the author acquired his large collection of children's books, can be found on the ETACuisenaire website (http://www.etacuisenaire.com/).

The first of the two classroom assignments asked the prospective teachers to interview a child about the child's perception of mathematics. Student teachers worked with their assigned mentors to select a child to interview. The instructions provided for this assignment were, in part, as follows:

> Think of this in part as a conversation. When you sit down to talk to someone you often have a few initial questions in mind, but the interaction and the situation prompt you to ask other questions. Part of this assignment is about you having a conversation with a child about math and learning math. You should walk away from this assignment knowing more about this

child as a doer and learner of mathematics, and maybe having some new questions or ideas about teaching children mathematics.

Most of the student teachers came away from this assignment knowing much, much more about how children viewed mathematics. Some were amazed at the depth of understanding children had. On the other hand, some were very upset with the stereotypical image of mathematics possessed by older elementary school children, an image that implied that to do math was to follow the rules set by the teacher, whether one understood those rules or not.

The second assignment involved the student teachers working in pairs or triples to design and teach a mathematics lesson—in the format suggested by the Lesson Study approach—that included using manipulatives and a reading a children's story related to the lesson. The student teachers loved this assignment, testifying that it taught them the mathematics involved in the lesson, made them consider the pedagogy of how to bring the mathematics topic to life for the students, how to blend the children's book into the lesson, and how to make effective use of manipulatives to assist children whose learning style was suited to a hands on approach. They also appreciated observing each other's lessons as it is a luxury that most teachers never get to experience. The NCTM articles that the student teachers had read provided the basis for some of the lessons as these article included lesson ideas that had been developed by experienced elementary school mathematics teachers. The readings therefore supported and enhanced the lessons developed by the student teachers.

Challenges and Conclusions

The challenges presented by the creation of a hybrid model of teacher education program were mainly instructional; i.e., I had to modify old ways of teaching and invent new ways to engage the student teachers in meaningful mathematical explorations that fostered growth of their pedagogical and mathematical content knowledge. Lessons learnt during the first course (fall 2006) led to modifications in the offering of the second course (spring 2007). Both experiences led to further modifications for the new cohort of students who began the program in August 2007. The need to develop new ways of teachings and become comfortable using evolving electronic teaching aides energized and educated the instructor. Course evaluations from the student teachers that were initially fearful of their mathematical abilities had most of those fears alleviated. The other students uniformly testified that the courses met their needs and enhanced their ability to teach mathematics.

In terms of the research questions posed at the outset of the paper, namely: (a) What kind of mathematics could we present to pre-service teachers when physical contact time is restricted 24 hours per term? It was clear that challenging mathematical tasks appropriate for the prospective teachers, and in many cases, the students they taught could be offered and have a significant impact on both categories of learners' (b) What links or relationships could be forged using this mixed model between the kind of mathematics offered and the role of mathematical experiences of the pre-service teachers? The modeling done by the course instructors during f2f sessions and electronically was emulated successfully by the student teachers as reported in the lesson studies that they completed as a requirement of the program; and (c) Would the use of the innovative and creative computer approaches support the development of mathematical content knowledge of pre-service teachers? There was no doubt that the *Elluminate* sessions advanced the student teachers content knowledge. Monitoring of the small group discussions that took place during these sessions, plus the reporting back that occurred at the conclusion of each small group discussion, provided evidence that the student teachers were engaging with the mathematics, jointly working through multiple pathways to the solution of the mathematical challenges, and in the process expanding their mathematical awareness in significant ways.