A STUDY OF SEVENTH-GRADE STUDENTS' LEARNING FRACTIONS IN CHINA

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Abstract: Though many scholars think fractions are the most complex mathematical concepts, it seems that students had no difficulties on learning fractions in China. I want to find out the real status of learning fractions in China. Investigation questionnaire was designed; ***I*** analyzed and discussed students' learning fractions on fraction's meaning, size, operations, division, equivalence and problem solving. At last, ***I*** found that students are good at recognizing forms of representation of problems and equivalent fractions, their competency of computation on fraction is good, but their understanding of fraction has some deficiencies, and they are familiar with routine problem solving, however, they are puzzled with complex un-routine problem.
Key words: fractions; operation; division; problem solving

Introduction

Some different information is often heard of about students' learning fractions in Europe or America. Whether I study the literature or touch with a foreigner who is a European or American, I know that foreign students do not well in learning fractions, include their teachers, they have some difficulties on comprehending fractions' conception. Some students and teachers make some errors that I have never thought. For example, $\frac{1}{3}+\frac{1}{4}=$_____, most of their answers was: $\frac{1}{3}+\frac{1}{4}=\frac{2}{7}$. Another case, 21 U.S. teachers tried to calculate $1\frac{3}{4}\div\frac{1}{2}=$ _____. Only nine (43%) completed their computations and reached the correct answer (Liping, Ma， 1999) . When **I** saw the result, **I** can't believe it. **I** think that all the primary school mathematics teachers should do it well, because it is a basic mathematical competency. Every primary pupil can get right answer in China.

For most new learner, fractions are among the most complex mathematical concepts that children encounter in their years (Boulet, 1998; Davis et al., 1993). Some scholars thought that learning fractions is probably one of the most serious obstacles to the mathematical maturation of children (Behr et al., 1993).Although during the last three decades several factors have been identified as contributing to students' difficulties in learning fractions, researchers and scholars agree that one of the predominant factors contributing to the complexities of teaching and learning fractions lies in the fact that fractions comprise a multifaceted construct (Brousseau et al., 2004; Kieren, 1993; Lamon, 1999). Fractions are recognized to be a difficult topic in school-mathematics. The most important causes are given as:

- Fractions are used less often in daily life and are less easily described than natural numbers.
- The written form of the fractions is comparatively complicated.
- It is not easy to put the fractions in order of size on the number line.
- For the arithmetic of fractions there exist many rules, and these are more complicated than those for natural numbers. If these rules are introduced too early, there is a danger of their being used mechanically and without thought (cf., e.g. Bigalke and Hademann,1978).

Piaget, Inhelder, and Szeminska (1960); Peck and Jencks(1981); Pothier and Sawada (1983,

1990); and Streefland (1979, 1984, 1990), among others, have documented the importance of partitioning in the understanding of fractional numbers. McLellan and Dewey (1908) also recognized the importance of "the mental operation of rhythmic parting and wholing" (p. 83) in the generation of units of measurement and fractional units. "Parting and Wholing" seem to be essential mental operations in the development of children's fractional meanings. In the context of natural numbers, the "wholing" operation or unitizing operation has been documented by Steffe (1986, 1988, 1990, 1991) and Steffe and Cobb (1988). Fractions are “the expression of the part-whole relationship” (Pitkethly and Hunting, 1996, p. 6), a phenomenological source for rational numbers (Freudenthal, 1983), Although the importance of partition and its concomitant emerging units have been suggested in several studies, there have not been studies that focus on the importance of the unitizing operation in the generation of fractional-number units and the relationship between children's ways of operating with natural numbers and their ways of generating fractional numbers and operations with them(Adalirasaenz-ludlow, 1995).

It is easy to see that fraction learning is really a difficulty for most learners in Europe and America through the papers and literatures. What’s more, many scholars had done lots of studies on fraction learning and teaching. Most of them want to find out the reasons about the difficulties of fraction learning, and they also want to give some strategies on solve the problem. Some of studies are very serious and the results are very value for the school-mathematics education. But as **I** read the papers I can’t believe learning fraction will be so difficult. In China, our mathematics teachers, include primary teachers and high school teachers, never meet so many difficulties on learning fraction, and during fractions learning, students will have some problems on fraction’s meaning, fraction algorithm etc, but they never feel them very difficult. There are fewer obstacles in their fractions’ learning than in America. For Chinese students, only the applying problems about fractions will be a difficulty. As for Chinese mathematics teachers, understand fraction profoundly. They seldom make the mistakes as what the American teachers did. All of the 72 Chinese teachers computed correct and complete answers to the problem: $1\frac{3}{4} \div \frac{1}{2}$ (Liping, Ma，1999).

But is it true that Chinese students do better on comprehending fractions than other countries’ students? Whether they have some difficulties during learning fraction? What are the difficulties? And what are the reasons? **I** read lots of literature in Chinese, and the extant literature in the area of fractional numbers shows me little about difficulties of fraction learning. Yet there are few investigations on fractional number learning. In schools, students are evaluated only by their marks in mathematical tests. However, marks can’t tell us real case about students’ learning. If **I** want to get more things about students’ learning on fraction, **I** must do some researches.

METHODS

The sample and questionnaire test

79 seventh-grade students were chosen as the sample from some public middle schools in Guangzhou, China. The public school at which the interviews took place can be considered representative of the general population of the Guangzhou area of the China. There were 56 boys and 23 girls; their ages are between 12 years and 14 years. The students were exposed to a traditional mathematics curriculum, guided by textbook instruction.

The fractional contents are very abundant, there are several different points about our test,

includes the meaning of fractional number; addition, subtraction, multiplication and division with fractions; comparison of size of fractional number; applying problems, etc. Main contents were selected from them to test students. Our test on the topic of fractions is set with several points in mind: Whole and part; Ratio; Arithmetic; Comparison size; Quotient; Equivalent fractions; Operation.

Questionnaire test is composed with three points: Operations with fractions; Form of representation of problems; Operational Forms.
In the test paper, there are nine big problems; every big problem is composed with several items. The tests included context-free items and items in context. Some items were adapted from previous studies.
Operations with fractions: recognizing equivalent fractions, ordering fractions, addition and subtraction, multiplication and division. For recognizing equivalent fractions, there includes 'reduction of fractions to a common denominator', 'reduction of a fraction', and operations. For examples, $\frac{(\quad)}{6}=\frac{35}{42}=\frac{10}{(\quad)}=\frac{(\quad)}{54}$, ordering fractions: $\frac{7}{12}$、$\frac{11}{18}$、$\frac{13}{24}$, order these proper fractions by decreasing order. Operation is covered by all items.

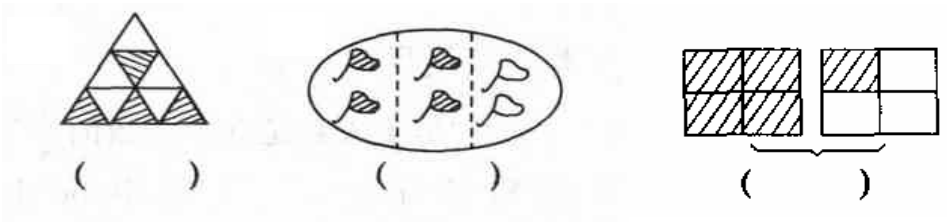

Figure1

Form of representation of problems: diagram, mathematical symbols, words expression. The expression of fractions is different on meaning of fractions, operations and applying problem. In test paper, in order to find how students understand the meaning of fraction and equivalent fractions, we set some diagram expression; for example, use fractions to express the shadow parts in figure 1.

On operations the test is depended on mathematical symbols and tables; we examine students' understanding on problem solving by words problem, which is based context. Operational Forms include: describing fractions in different form, and fractions in reverse form, determining whole from fractional part, addition, subtraction, multiplication and division with fractions. For different forms of representation of fractions, we set a table to test students. For example: Fill suitable real numbers in blanks.

Table1: Fill suitable real numbers in blanks.

Fraction	$\frac{3}{10}$	$\frac{7}{8}$			$\frac{1}{3}$
Decimal fraction	0.3		0.35		
Percent	30%			8.2%	

The interviews

Except of the test, we set up interviews about students' understanding of fraction. The questions of interviews mainly are divided into eight different aspects: (1)The interests of learning mathematics; (2) The understanding of 'unit 1'; (3) Applying fractions to solve problem; (4) The arithmetic; (5)Inverse reciprocal; (6) apply problems; (7) Distinguish true or false; (8) the difficulties in learning fractions. The interviews were carried out as clinical interviews. These individual interviews were described to the students as conversations about "what you know and think about numbers." During the interviews, each student was presented with a series of questions. The questions structured the interviews but also provided the flexibility necessary to a clinical interview. The interviews focused, not on a particular answer I was seeking from the students, but on the thinking and understanding that went into solving the various tasks. Following the clinical interview method, each student was presented with potentially conflicting situations during the course of the interviews, and the contradictions in their justifications or thinking were made explicit and explored.

FINDINGS

Students' understanding of fractional number

According to the meaning of fraction, we started off with 'unit 1'. Students had learned that many objects can be regarded as the whole , and natural number 1 can be used to represent them, usually it is 'unit 1'. Students were asked to represent the shadow with fractional numbers in figure 1, they did well in the former two diagrams, the answers are $\frac{4}{9}$ and $\frac{2}{3}$, students knew well about the meaning of the fraction; but to the third diagram, they had some different results:$1\frac{1}{4}$, $4\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{8}$. The percentages of results are shown in the table 2.

Table 2: The results about figure 1

	$1\frac{1}{4}$	$4\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{8}$
Number of Students	34	1	1	43
percentages of the result	43%	1.3%	1.3%	54.4%

The reason is mislead by the diagram. From the results, students had two views: $1\frac{1}{4}$ and$\frac{5}{8}$. If we look the former shadow of four panes as "unit 1", the answer is $1\frac{1}{4}$, if look the two big panes as "unit 1" , the shadow represents $\frac{5}{8}$. Thus, students understand "unit 1" well. From the interview, We learned that most students thought that "unit 1" is "a whole", some even interpreted more particular, a whole was divided into two, three, …, the whole just is"unit 1". For the second question of the questionnaire, shade in $\frac{1}{6}$ of the circle, then shade in $\frac{3}{4}$ of the circle, then what fraction of the whole circle have you shaded in (figure 2)? students not only can shade in the $\frac{1}{6}$ and $\frac{1}{4}$ of circle, but can figure out all the shadows occupied $\frac{11}{12}$ of the circle , only two pupils

made mistake.

Through the analysis of the mathematics teaching and mathematics curriculum, we found that students comprehended profundity about the meaning of fraction which had close relation with their studying process. In grade three, when student started learning fraction, they knew one of the whole, and then some part of the whole. Teacher organized several activities, such as objects are divided into the same part, folding paper, division of the line segment and so on, it made students recognized that a whole was divided into several parts averagely, one part is one of the whole, and then students obtain the idea of one of the whole. By abstracting and generalizing of abundance intuitionistic examples, they knew some part of the whole, and then developed the conception of numerator and denominator. In the learning process, students understood fraction well step by step, it did not break away from the real world. So they caught on the conception of fraction more profundity.

The behaviors on students comparing the size of fractional numbers

Problem 3 is about the comparison of fractional numbers. There are 3 different items.

1. Arranging the fractions in degressive sequence: $\frac{7}{12}$、 $\frac{11}{18}$、 $\frac{13}{24}$.

2. Arranging the fractions in increased sequence: $2\frac{1}{3}$、 2.375、 $1\frac{7}{24}$.

3. Find out a fractional number between $\frac{1}{7}$ and $\frac{1}{8}$.

Students are familiar with the routine problems. Most of them finished item 1 and 2 correctly (Table3).

Table 3: Results of item 1 and 2

Item 1	Results	Right	$\frac{13}{24} > \frac{11}{18} > \frac{7}{12}$ (Wrong)	Others (Wrong)
	Number of students	69	5	5
	Percentage	87.3%	6.3%	6.3%
Item 2	Results	Right	$1\frac{7}{24} < 2.375 < 2\frac{1}{3}$ (Wrong)	$2.375 < 2\frac{1}{3} < 1\frac{7}{24}$ (Wrong)
	Number of students	72	3	4
	Percentage	91.1%	3.8%	5.1%

All the wrong are related to the natural number. The fraction is introduced to students by dividing quantities to same parts; this meaning of fractions is given in the natural number context. When students will have to complete an inequality like $\frac{3}{8}$ （ ） $\frac{3}{5}$ with the correct ordering sign (<), this dominance of meaning will manifest itself in the choice of the '>'sign, because – as they reason- ' 8 is greater than 5'.(Leen, Streefland: 1978,)

There are lots of different results about item 3(table 4). It is an un-routine problem.

Table 4: Different results about item 3

Results	$\frac{15}{112}$	$\frac{3}{28}$	$\frac{2}{15}$	$\frac{3}{56}$	$\frac{6}{7}$	$\frac{2}{7}$	$\frac{2}{8}$	$\frac{1}{56}$	$\frac{7}{56}$	$\frac{7}{8}$	$\frac{9}{70}$	$\frac{1}{15}$	$\frac{27}{200}$
Numbers of students	20	6	11	1	1	3	2	7	1	1	1	2	1
Results	$\frac{73}{560}$	$\frac{39}{250}$	$\frac{4}{7}$	$\frac{1}{13}$	$\frac{14}{56}$	$\frac{11}{56}$	$\frac{10}{75}$	$\frac{1}{14}$	$\frac{3}{7}$	$\frac{6}{56}$	$\frac{15}{102}$	$\frac{71}{560}$	No result
Numbers of students	1	1	1	1	1	1	1	1	1	1	2	4	5

The right result is not single. $\frac{15}{112}$ is one of the right results. Others include: $\frac{31}{224}$, $\frac{43}{336}$ ….

The competency of students computing fractions

Students are skilled on operations of fraction. All the students calculate $\frac{8}{9}\div[(\frac{5}{6}-\frac{2}{3})\div\frac{1}{4}]\times 1\frac{3}{4}$ and $6\frac{4}{9}\div\frac{1}{3}$ correctly. During interview, a problem was given to students: In a programme, if input a number, the output will be five eighths of the number. Xiaoming inputs 176, what is the output? (figure 2.) All of their results are right.

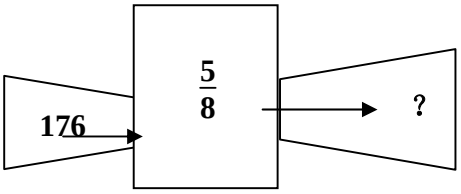

Figure2

Students' comprehension on the division of fractions

They have some deficiencies on comprehending the division of fractions. First a calculation problem is given: $6\frac{4}{9}\div\frac{1}{3}$. All of them got the right result. Then **I** asked them to answer the question: what is the meaning of $6\frac{4}{9}\div\frac{1}{3}$? The results are different kinds as shown in table 5.

Table 5: The results of what is the meaning of $6\frac{4}{9}\div\frac{1}{3}$

	Students' answers	Number of students	Percentage
1	How many $\frac{1}{3}$ in a $6\frac{4}{9}$ ($\frac{1}{3}$ and $6\frac{4}{9}$ are both nouns)?	26	32.9%
2	One factor multiplies another factor, the product is $6\frac{4}{9}$, first factor is $\frac{1}{3}$, what is another one?	10	12.7%
3	X is divided to 3 parts averagely, one is $6\frac{4}{9}$, what is X?	8	10.1%
4	$6\frac{4}{9}$ is divided to 3 parts averagely, what is one part?	7	8.9%
5	Blank	28	35.4%

During interview, some students are confused with division and multiplication. Though they calculated perfectly, their comprehension of division is not very good. This result showed that students have some problem with the calculated process and symbol.

Students' understanding on equivalence of fractional numbers

Students are tested with problems of the calculation of ratio, reduction of a fraction， reduction of fractions to a common denominator， and conversion of fractional number, fraction and percentage. All the students did well in test. Another question is to represent a fractional number with a diagram; only 3 students made mistakes, others all did correctly.

Students' competency on solving fractional applying problem

Students can do well with routine problems, but they have obstacles with some un-routine problems. They can solve most of simple real world problems about fraction, however, most of them failed in the question: find out a fractional number between $\frac{1}{7}$ and $\frac{1}{8}$. This shows that students' competency on solving fractional applying problem is not very good.

CONCLUSION

Students are good at recognizing forms of representation of problems and equivalent fractions.

To represent fraction whether with a diagram or with a mathematical symbol, students do well with the task. They understand reduction of a fraction，reduction of fractions to a common denominator，and conversion of fractional number, fraction and percentage well. They think that fraction is one of rational numbers and used to represent parts of the whole.

Students' competency of computation on fraction is good, but their understanding of fraction has some deficiencies.

Most students are good at calculation of fraction. But they have some difficulties on comprehension of 'operation'. During their learning, most activities are mechanical, they don't understand them really. This is related to Chinese educational evaluation system. In this system, 'test' is main mode.

Students are familiar with routine problem solving, but they are puzzled with complex problem.

Students do well with familiar fractional applying problems. They can find 'Unit 1' in a real world problem quickly, and then they choose suitable calculation. They said, the word 'is' 'summation' and 'than' are the key in solving problems.

But some complicated problems are obstacles for students. Most students think that applying problems are difficult during learning fraction. In a complex status, 'Unit 1' is hard to find out, especially, there are more than 2 different 'Unit 1'.

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