

## **Modern Mathematics teaching proposals as seen in published textbooks in Brazil**

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### Abstract

We can say that Modern Mathematics Movement was the second international movement for renovating Mathematics teaching. The first one was spread out by IMUK (afterwards known as ICMI) when presided by Felix Klein, at the beginning of 20<sup>th</sup> century.

Modern Mathematics Movement, as we are referring to, corresponds to divulged shifting of course proposals, mainly in the 1960's, and it was, in a manner of speaking, an attempt to modernise Mathematics teaching, by means of modifying and updating teaching contents and methods at two levels of scholarship: primary and secondary (7-18 years old). One of the principal targets was to bring together teaching contents at secondary (15-18 years old) and tertiary levels (undergraduate courses).

In Brazil, professor Osvaldo Sangiorgi was one of its principal supporters and divulgators. He has acted as a teacher in different teaching levels – secondary and tertiary – and was the author of many adopted textbooks between 1950's and 1970's.

In this paper we have in mind to expose an analysis of professor Sangiorgi's appropriations over updating proposals for Mathematics teaching as propagated by his textbooks *Matemática curso moderno*. We restring ourselves to the study of numerical sets and operations contents at gymnasium initial series (11-12 years

old), as presented in his textbooks, and we have focused on methodological proposals as well.

### *Modern Mathematics Movement in Brazil: Osvaldo Sangiorgi's role*

We can say that Modern Mathematics Movement was the second international movement for renovating Mathematics teaching. The first one was spread out by IMUK (afterwards known as ICMI) when presided by Felix Klein, at the beginning of 20<sup>th</sup> century, aiming to focus Mathematics teaching on functional thinking.

Modern Mathematics Movement, as we are referring to, corresponds to divulged shifting of course proposals, mainly in the 1960's, and it was, in a manner of speaking, an attempt to modernise Mathematics teaching, by means of modifying and updating teaching contents and methods at two levels of scholarship: primary and secondary (7-18 years old). One of the principal targets was to bring together teaching contents at secondary (15-18 years old) and tertiary levels (undergraduate courses). Winds of advances in Mathematics research, mainly in Algebra field, compelled some famous mathematicians as Jean Dieudonné and Andre Lichnerowicz to engage in discussions and proposals about Mathematics teaching modernisation.

This movement has occurred in many countries, occidental (United States of America, European and Latin America countries, between them) as well as oriental (some Arabian countries). In Brazil, professor Osvaldo Sangiorgi was one of its principal supporters and divulgators. He has acted as a teacher in different teaching levels – secondary and tertiary – and was the author of many adopted textbooks between 1950's and 1970's in Brazil. Sangiorgi has attended a summer course at Kansas University in 1960. He had classes with professor George Springer, which afterwards came to Brazil to teach courses in GEEM (Grupo de Estudos do Ensino da Matemática<sup>1</sup>), founded by Osvaldo Sangiorgi in São Paulo, in 1961.

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<sup>1</sup> Mathematics Teaching Study Group (author's note).

GEEM had as targets, besides to provide an incentive to Modern Mathematics studies, also to promote updating courses for basic school Mathematics teachers. GEEM's members were textbooks authors; mathematicians; primary, secondary and tertiary levels teachers. Osvaldo Sangiorgi was president and spokesman of the group, kept a straight relationship with the press, gave a lot of depositions and wrote many articles for São Paulo's main newspapers. In these interviews and articles, Osvaldo Sangiorgi spread out Modern Mathematics and GEEM's enterprises.

### *Modern Mathematics in Brasil and Osvaldo Sangiorgi's textbooks*

In 1963, Sangiorgi<sup>[8]</sup> publishes, in Brazil, the first "Modern Mathematics" textbook, named *Matemática curso moderno*<sup>2</sup>. It was the first volume, for the 1<sup>st</sup> gymnasium series (11-12 years old, which means 6<sup>th</sup> year of our more recent fundamental scholarship), and started circulating from 1964 on; at the preface, author praises possibilities that *modern* Mathematics studies can arise, as presented in his textbook.

Professor Sangiorgi's textbooks, mainly *Matemática curso moderno* collection, have spread out around the country, taking with them, to a lot of different localities, Sangiorgi's proposal for *modern* Mathematics teaching. A survey on the amount of books sold on this collection was done by Villela<sup>[11]</sup> 2008, showed *Matemática curso moderno* as a best seller in 1964 to 1973 at Companhia Editora Nacional, one of the most important textbooks publishing company in Brazil during this period.

A lot of works about Modern Mathematics Movement in Brazil show how textbooks are important as an instrument for divulging Mathematics teaching proposals. Mathematics textbooks are a day-by-day presence in teachers and students' lives,

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<sup>2</sup> Mathematics modern course (author's note).

much beyond five national congresses in Mathematics teaching and in-service courses for teachers, mainly offered by GEEM.

Many factors have contributed to professor Osvaldo Sangiorgi's textbook to become a *best-seller*, as classified by Valente<sup>[10]</sup>; among them we can sort out Sangiorgi good relationship with official entities and agencies in the Education area, his media visibility, frequent quotations in São Paulo's major newspapers, the fact that he already was a textbooks achieving author.

### *Sangiorgi's relationships with updating proposals for Mathematics teaching*

In this paper we have in mind to expose an analysis of professor Sangiorgi's appropriations over updating proposals for Mathematics teaching as propagated by his textbooks *Matemática curso moderno*. We restring ourselves to the study of numerical sets and operations contents at gymnasium initial series (11-12 years old), as presented in his textbooks, and we have focused on methodological proposals as well.

In discussing Sangiorgi's appropriations over the updating proposals we intend to establish a set of these, supported in some documents or books, which in turn characterize the Movement set of ideas and those professor had contact with.

The first book is "L'enseignement des mathématiques"<sup>3</sup>, which brings together texts from J. Piaget<sup>[4]</sup>, E. W. Beth, J. Dieudonné, A. Lichnerowicz, G. Choquet and C. Gattegno, and was published in 1955.

Professor Osvaldo Sangiorgi knew this book as we can see by his words in Annals of the 2<sup>nd</sup> National Congress in Mathematics Teaching<sup>[2]4</sup>, in Porto Alegre, in 1957. He does refer to book "*L'enseignement des mathématiques*" when argues that

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<sup>3</sup> "Mathematics teaching"

<sup>4</sup> Anais do II Congresso Nacional de Ensino da Matemática

Mathematics (classical or modern) must meld with secondary level scholarship programmes.

Royaumont Seminar<sup>[6]</sup>, which was held at the end of 1959, in France, and brought together about 50 representatives of 18 countries, is considered a mark for Modern Mathematics Movement. This Seminar and Dubrovnik meeting, due in 1960, carried on orientations for Mathematics teaching in secondary level of scholarship, either related to mathematical contents or teaching methods for these contents as well.

These orientations were systematized in the book *Un programme moderne de mathématiques pour l'enseignement secondaire*<sup>5</sup>, published by OECE<sup>[7]</sup> in 1961. This book was translated into portuguese by professor Jacy Monteiro (GEEM publications director) and published by GEEM, in 1965.

Mathematics teaching new proposals present a programme permeated with plenty of structuralist ideas, which were dominants at the time. Contents reveal an influence from Bourbaki Group conceptions and methods are based in Jean Piaget studies.

In a very short way we can say that, as curriculum contents are concerned, basically two guidelines can be brought to attention: emphasizing Mathematics cohesion; introducing new topics, the so named *modern* ones.

As teaching methods are concerned, orientations points out changes on teacher's and pupil's roles. Among the possible actions, some are valued: comprehension instead of mechanisation, learning by discovery, intuition as something that precedes deductive method. It is also valued experimental work as a step prior to abstraction.

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A Mathematics modern programme for secondary level of scholarship.

The collection of textbooks *Matemática curso moderno* besides textbooks, guides the teacher bringing tips about how to teach different subjects. There are recommendations for each chapter and in the end Sangiorgi presents the bibliographical references used. Analyzing these guides we observe that the the bibliographical references are basically French and American, overall with work from SMSG (School Mathematics Study Group).

### *Theoretical and methodological basis for this investigation*

The analysis that we present afterwards in this paper are supported by historical studies from authors, such as Michel de Certeau<sup>[5]</sup> and Roger Chartier<sup>[9]</sup>, who consider history as historian's production and *cultural practices* as objects. We consider Mathematics teaching and learning processes as *cultural practices*. In this sense, using textbooks as research source is one of the possible ways to study scholar practices during Modern Mathematics Movement times.

Professor Osvaldo Sangiorgi got hold of the ideas from what we call Modern Mathematics Movement. In his didactical collection *Matemática curso moderno* we can see them explicitly.

The notion of getting hold, as given by Chartier<sup>[9]</sup>, supports our investigation. By it, emphasis is done over the plurality of uses and interpretations besides the subject's creative liberty when in contact with texts, laws and regulations. To this author, getting hold means to produce a social history from the uses and interpretations, all of them in reference to theirs basic determinations and as inscribed in the specific practices who creates them.

Following Choppin's<sup>[1]</sup> ideas, we defend textbooks as very significant in teachers' grasps. To this author, textbook is not just a mirror; it also modifies reality to educate new generations. Textbook carry innovative proposals, provoking changes in practice.

*Sangiorgi's grasps of MMM proposals: Matemática curso moderno 1*

Textbook *Matemática curso moderno 1* has four chapters, the three first ones about numbers and operations and the last about geometry. In the first chapters natural numbers and positive rational numbers are studied.

Chapter 1 is divided in three parts. The first one is dedicated to "Set notion" and there was not such part in Sangiorgi's previous collection. The author introduces the first ideas about number by comparison between sets and gives an example of a one-to-one correspondence.

In the collection from the same author, however before MMM, the numerical idea is introduced linked with the counting process, the operation of counting objects of a set or single items from a group, gives origin to the numbers. The author already shows a concern in distinguish the symbols from the ideas, when he points out and represents the distinguishing ways of writing number and symbols.

First part of chapter 1 from modern collection is dedicated to distinguish number and numeral. To emphasise this, author presents some examples, showing with them that the same operation (transformation), when done over numbers or numerals, can produce different results. Although such examples can illustrate the difference between numbers and numerals, they seem to highlight less important facts, if we consider that they are done to students at the beginning of gymnasium (11-12 years old). The distinction between number and numeral has also been discussed in Mathematics teacher education courses given by GEEM.

In this chapter we can find transitive, symmetric and reflexive properties for equality and inequality relations, as well as their validity domains. Such properties are informed emphasising the difference between the symbols (numeral) and the ideas (number). There are no examples of application or use possibilities of these

properties. In the exercises the properties are explored in verifying whether it is a relation of equivalence. The explored relations are about daily situations, for instance, the relation of having the same mother is of equivalence for three brothers.

It is also presented to students the order structure in both sets, natural and positive rational numbers. The number line is explored aiming to make it possible intuitively seeing the order structure of natural numbers.

In this collection from the same author however before MMM, edited in the 50s only a few properties are presented to the students such as natural numbers, addition, the commutation and association for multiplication and the commutation and the distributive regarding the addition and their differences.

At the end of the first chapter, author brings numeral systems and bases to discussion. It is shown some numeral systems as egyptian, babylonian and roman, comparing them with base-10 numeral system and discussing some limitations of the old systems. Sangiorgi incites students to starting a Mathematics laboratory from an experience with counting in non-decimal bases. He describes in the textbook how to proceed to achieve this, giving suggestions for making a box to represent number systems' many classes. Two photos are included, each one showing students from different schools in São Paulo, when at work with the change of base experience.

In this first volume, Sangiorgi approaches natural and positive rational numbers systems, the last one called by him as fractional numbers, and their decimal representation.

Numerical sets' presentation emphasizes operations' structural properties study: addition–subtraction; multiplication–division; exponentiation with natural numbers–exponentiation with rational numbers. These operations are treated as inverses

and it is brought to discussion the validity of each one of the properties: closure, commutative, associative, existence of an identity element, existence of a inverse element.

The structural approach is extended to the study of subjects like maximum common divisor and minimum common multiple. Structural properties are discussed for each of these operations.

The application problems are isolated items into the chapters which present the two numerical sets studied in the first volume.

The 6<sup>th</sup> grade book (12 years old), Matemática curso moderno 2, starts with the study of positive rationals, which are simply called rationals. Operations with sets are retrieved and number line is explored for locating rationals. Rational structural properties are presented with their operations of addition and multiplication.

In the rate study it is explored in geometrical situations: segment units, area, volume and angles rates. In the same study we can find an item of special rates such as speed, demographical and other kinds of density. It follows the studies of proportions and percentages. Numbers and proportional magnitudes are simultaneously presented by means of problems. Simple interest, "the rule three", discount and exchange are also studied in this book.

In relation to numbers it is also presented relative integers and relative rationals with their respective structural properties.

In the book from the same author, from the same series however before matemática moderna, the focus is on operations such as Potentiation and Radication that are shown as a numerical set already studied in last series The calculus of the square root and cubic root is represented including techniques for the result of the round calculus. The irrational numbers are introduced in this series with mighty knowledge.

The literal calculus, the polynomial study and the algebraic fractions are also subjects of this book. The last chapter contains equations and inequalities of first degree and linear systems with two variables. In the appendix is found determined methods for the resolution of linear systems with three equations and three variable. The problems are restricted to the equation chapter.

### *Some considerations*

In 1962, GEEM<sup>[3]</sup> in cooperation with IBCEC (Instituto Brasileiro de Educação, Ciência e Cultura<sup>6</sup> – UNESCO<sup>7</sup>) has produced a book whose title is *Matemática Moderna para o ensino secundário*<sup>8</sup>. This book holds together articles from Brazilian and foreigners teachers, who discuss Modern Mathematics teaching at the secondary level of scholarship.

The first one of these articles is by Osvaldo Sangiorgi – *Introdução da Matemática Moderna no ensino secundário*<sup>9</sup>. In it we can identify some aspects of the Mathematics teaching reformulation that are worthy to professor Sangiorgi. It comes to attention his given importance to language issues. In a lot of his article's, parts he proclaims that it is urgently necessary to modernise language in Mathematics fundamental contents, and restates it. Such a modernisation in language means introducing the notion of set and utilising logical symbols that give essential precision to Mathematics.

As Sangiorgi emphasises the importance of language, we can identify his way of *get hold* of Mathematics teaching revising proposals, as discussed at the time.

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<sup>6</sup> Brazilian Institute of Education, Science and Culture (author's note).

<sup>7</sup> United Nations Educational, Scientific, and Cultural Organization (author's note).

<sup>8</sup> Secondary Teaching Modern Mathematics (author's note).

<sup>9</sup> Introducing Modern Mathematics at the secondary level (author's note).

Sangiorgi has made his representations of such proposals, and these ones, in addition to his experience as a teacher as well as a Mathematics textbooks' writer, have put together elements that have allowed his *getting hold* of those proposals as they appear in texts he writes, in interviews he gives, in textbooks he publishes and in the ways he divulges the Movement in Brazil.

Concerning the theory that supports Sangiorgi's sayings in his article, we can see that he explicitly quotes Piaget and Boole. Piaget's theory proves that exists a correspondence between algebraic structures and children's intelligence operational systems. And mathematician Boole's theory describes the existence of a thought algebra that, under the shape of structures, states itself by language and displays itself by grammar.

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The text presentation deserves to stand out, Unlike the previous collection, the books from Matematica curso modern tries to establish a dialogue with the student , along the chapters there many pictures and figures, diagrams, photos and reminders for he most important topics

This innovating collection has carved its mark in the Brazilian teaching system overall it was spread all over the country.

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